1 Background

This document contains the official formulation that will be used for evaluation in Challenge 1 of the Grid Optimization (GO) Competition. Minor changes may occur within the formulation. Entrants will be notified when a new version is released. Changes are not expected to be of any significance, to cause a change in approach for the Entrants.

This formulation builds upon the formulation published in ARPA-E DE-FOA-0001952. Entrants will be judged based on the current official formulation posted on the GO Competition website (this document, which is subject to change), not the formulation posted in DE-FOA-0001952. Entrants are permitted and encouraged to use any alternative problem formulation and modeling convention within their own software (such as convex relaxation, decoupled power flow formulations, current-voltage formulations, etc.) in an attempt to produce an exact or approximate solution to this particular mathematical program. However, the judging of all submitted approaches must conform to the official formulation presented here.

The following mathematical programming problem is a type of a security-constrained (AC based) optimal power flow, or SCOPF. There are many ways to formulate the SCOPF problem; this document presents multiple equivalent options for specified constraints. Entrants are strongly encouraged to study this formulation precisely and to engage with the broader community if anything is not clear (please see the FAQs and forum on the GO Competition website, https://gocompetition.energy.gov/).

This SCOPF problem is defined to be an alternating current (AC) formulation, which is based on a bus-branch power system network model and considers security constraints. In general, Entrants are tasked with determining the optimal dispatch and control settings for power generation and grid control equipment in order to minimize the cost of operation, subject to pre- and post-contingency constraints. Feasible solutions must conform to operating standards including, but not limited to: minimum and maximum bus voltage magnitude limits, minimum and maximum real and reactive power generation from each generator, thermal transmission constraints, proxy stability constraints, and constraints to ensure the reliability of the system while responding to unexpected events (i.e., a contingency). Feasible solutions must also include contingency modeling to describe the response of generators and transmission elements to changes from the pre-contingency base case. This formulation allows for real and reactive power nodal violations as well as branch (transmission line and transformer) overloading; however, violations will be penalized in the objective function.

Features that are not modeled within this SCOPF include transformer tap settings, phase angle regulators, various flexible AC transmission system (FACTS) devices, or switchable shunts. Please note that shunts are included but we are not precisely modeling the binary nature of switchable shunts; rather, these are modeled using continuous variables.

Challenge 1 will include power system network models that vary in size of the network flow problem (number of nodes and branches) as well as the number of contingencies. The largest
models will reach to at least the size of the largest regional transmission organization within the United States. The problem presented here is a single period problem. The modeling of the pre-contingency base case is a reflection of the first stage of a two-stage mathematical program whereas the post-contingency state represents the second stage. Unit commitment (the commitment/decommitment of generators) is not included in the formulation and fast-start generators are not acknowledged as an available post-contingency response; only units that are online initially may respond. Post-contingency generator response (i.e., activation of contingency reserve) is dictated by an offline policy: generators must follow their predefined participation factor unless they reach an operational limit. As such, this problem does not fully optimize the second-stage recourse decision variables but rather forces them to follow this offline policy. Challenge 2 will likely expand upon Challenge 1 by considering more advanced modeling of power flow equipment (transformers, phase shifters, FACTS), the grid itself (e.g., bus-breaker models), and a more detailed representation of the flexibility available to respond during a contingency (e.g., generator response and modeling of recourse decision variables and limits).
2 Symbol Reference

Units, notation, and the general nomenclature are given Tables 1, 2, 3, 4, 5, 6, 7, 8. These tables list sets, indices, subsets and special set elements, data parameters, and variables. As much as possible, the notation follows the following convention. (1) A symbol consists of a main letter with attached notation such as subscripts, superscripts, oversets, and undersets. (2) Two symbols with the same main letter but different attached notation are different symbols. (3) The main letters of symbols generally follow conventions common in the optimal power flow literature and the optimization literature, though other letters are used where there is no established convention.

Units of measurement are listed in Table 1. Attached notation convention is given in Table 2. Main letter convention is given in Table 3. Sets are given in Table 4. Indices are given in Table 5. Subsets and distinguished set elements are given in Table 6. Parameters are given in Table 7 and variables are given in Table 8.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>dimensionless. Dimensionless real number quantities are indicated by a unit of 1.</td>
</tr>
<tr>
<td>bin</td>
<td>binary. Binary quantities, i.e. taking values in {0, 1}, are indicated by a unit of bin.</td>
</tr>
<tr>
<td>h</td>
<td>hour. In the physical unit convention and the per unit convention, time is expressed in h.</td>
</tr>
<tr>
<td>deg</td>
<td>degree. In the physical unit convention, angles are expressed in deg.</td>
</tr>
<tr>
<td>USD</td>
<td>US dollar. Cost, penalty, and objective values are expressed in USD.</td>
</tr>
<tr>
<td>kV</td>
<td>kilovolt. In the physical unit convention, voltage magnitude is expressed in kV.</td>
</tr>
<tr>
<td>MVAR</td>
<td>megavolt-ampere-reactive. In the physical unit convention, reactive power is expressed in MVAR.</td>
</tr>
<tr>
<td>MVAR at 1 p.u. voltage</td>
<td>megavolt-ampere-reactive at unit voltage. In the physical unit convention, susceptance is expressed in MVAR at 1 p.u. voltage, meaning the susceptance is such as to yield a reactive power flow equal to the indicated amount when the voltage is equal to 1 p.u.</td>
</tr>
<tr>
<td>MW</td>
<td>megawatt. In the physical unit convention, real power is expressed in MW.</td>
</tr>
</tbody>
</table>
Table 1: Continued

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW at 1 p.u. voltage</td>
<td>megawatt at unit voltage. In the physical unit convention, conductance is expressed in MW at 1 p.u. voltage, meaning the conductance is such as to yield a real power flow equal to the indicated amount when the voltage is equal to 1 p.u.</td>
</tr>
<tr>
<td>p.u.</td>
<td>per unit. Real and reactive power, voltage magnitude, conductance, susceptance can be expressed in a per unit system under given base values, and the unit is denoted by p.u.</td>
</tr>
<tr>
<td>rad</td>
<td>radian. In the per unit convention, angles are expressed in rad.</td>
</tr>
</tbody>
</table>

Table 2: Attached notation

<table>
<thead>
<tr>
<th>example</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>subscript is used for an index in a set.</td>
</tr>
<tr>
<td>$w^*$</td>
<td>superscript is used for description of a symbol.</td>
</tr>
<tr>
<td>$\overline{w}$</td>
<td>overline is used for an upper bound.</td>
</tr>
<tr>
<td>$\underline{w}$</td>
<td>underline is used for a lower bound.</td>
</tr>
<tr>
<td>$\tilde{w}$</td>
<td>overset ~ indicates a unit base value.</td>
</tr>
<tr>
<td>$w^0$</td>
<td>superscript 0 indicates a value in a given operating point.</td>
</tr>
<tr>
<td>$w^o$</td>
<td>superscript o indicates the origin (from, sending) bus of a branch.</td>
</tr>
<tr>
<td>$w^d$</td>
<td>superscript d indicates the destination (to, receiving) bus of a branch.</td>
</tr>
<tr>
<td>$w^+$</td>
<td>superscript + indicates an upper bound violation, the positive part of an equality constraint violation, or an upper bound slack.</td>
</tr>
<tr>
<td>$w^-$</td>
<td>superscript – indicates a lower bound violation, or a lower bound slack.</td>
</tr>
<tr>
<td>$\hat{w}$</td>
<td>overset ^ indicates a value reported in a solution file, possibly in different units from the corresponding value in the model.</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>primitive index sets are denoted by a caligraphic capital letter.</td>
</tr>
<tr>
<td>$\mathcal{W}_2 \subset \mathcal{W}$</td>
<td>subsets are denoted by the same letter as the corresponding primitive index set, in italic.</td>
</tr>
<tr>
<td>$w \in \mathcal{W}$</td>
<td>set elements are denoted by the same letter as the corresponding primitive index set, in lower case italic.</td>
</tr>
<tr>
<td>$w'$</td>
<td>set elements are primed to denote different elements of the same set.</td>
</tr>
<tr>
<td>main letter</td>
<td>description</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>a</td>
<td>area</td>
</tr>
<tr>
<td>b</td>
<td>susceptance</td>
</tr>
<tr>
<td>c</td>
<td>cost, penalty, objective</td>
</tr>
<tr>
<td>e</td>
<td>transmission line (arc in a transmission network)</td>
</tr>
<tr>
<td>f</td>
<td>transformer (arc in a transmission network)</td>
</tr>
<tr>
<td>g</td>
<td>conductance (also generator: when $g$ appears with a subscript, it is a</td>
</tr>
<tr>
<td></td>
<td>conductance value)</td>
</tr>
<tr>
<td>g</td>
<td>generator (also conductance: when $g$ appears as an element of a set, it is</td>
</tr>
<tr>
<td></td>
<td>a generator)</td>
</tr>
<tr>
<td>h</td>
<td>point on a cost function</td>
</tr>
<tr>
<td>i</td>
<td>bus (node in a transmission network)</td>
</tr>
<tr>
<td>k</td>
<td>contingency</td>
</tr>
<tr>
<td>n</td>
<td>segment number for the piecewise linear penalty cost function for violations</td>
</tr>
<tr>
<td></td>
<td>(constraint relaxations)</td>
</tr>
<tr>
<td>p</td>
<td>real power</td>
</tr>
<tr>
<td>q</td>
<td>reactive power</td>
</tr>
<tr>
<td>R</td>
<td>apparent current rating (current magnitude); used as data label in the</td>
</tr>
<tr>
<td></td>
<td>Appendices, the interpretation is context dependent.</td>
</tr>
<tr>
<td>s</td>
<td>apparent power (power magnitude)</td>
</tr>
<tr>
<td>t</td>
<td>interpolation coefficient</td>
</tr>
<tr>
<td>v</td>
<td>voltage magnitude</td>
</tr>
<tr>
<td>x</td>
<td>binary variables</td>
</tr>
<tr>
<td>α</td>
<td>participation factor</td>
</tr>
<tr>
<td>Δ</td>
<td>post-contingency (adjusted) real power response for generators following</td>
</tr>
<tr>
<td>θ</td>
<td>bus voltage angle or transformer shift angle</td>
</tr>
<tr>
<td>λ</td>
<td>constraint violation penalty coefficient</td>
</tr>
<tr>
<td>σ</td>
<td>violation or slack variable relative to a constraint</td>
</tr>
<tr>
<td>τ</td>
<td>tap ratio</td>
</tr>
</tbody>
</table>
### Table 4: Primitive index sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{A}$</td>
<td>set of areas</td>
</tr>
<tr>
<td>$\mathcal{E}$</td>
<td>set of lines (nontransformer branches)</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>set of transformers (2-winding only)</td>
</tr>
<tr>
<td>$\mathcal{G}$</td>
<td>set of generators</td>
</tr>
<tr>
<td>$\mathcal{H}$</td>
<td>set of cost function sample points</td>
</tr>
<tr>
<td>$\mathcal{I}$</td>
<td>set of buses</td>
</tr>
<tr>
<td>$\mathcal{K}$</td>
<td>set of contingencies</td>
</tr>
<tr>
<td>$\mathcal{N}$</td>
<td>set of segments in the piecewise linear penalty cost function for violations (constraint relaxations)</td>
</tr>
</tbody>
</table>

### Table 5: Indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \in \mathcal{A}$</td>
<td>area indices</td>
</tr>
<tr>
<td>$e \in \mathcal{E}$</td>
<td>line indices</td>
</tr>
<tr>
<td>$f \in \mathcal{F}$</td>
<td>transformer indices</td>
</tr>
<tr>
<td>$g \in \mathcal{G}$</td>
<td>generator indices</td>
</tr>
<tr>
<td>$h \in \mathcal{H}$</td>
<td>cost function sample point indices</td>
</tr>
<tr>
<td>$i \in \mathcal{I}$</td>
<td>bus indices</td>
</tr>
<tr>
<td>$k \in \mathcal{K}$</td>
<td>contingency indices</td>
</tr>
<tr>
<td>$n \in \mathcal{N}$</td>
<td>segment indices for the piecewise linear penalty cost function for violations (constraint relaxations)</td>
</tr>
</tbody>
</table>

### Table 6: Subsets and distinguished set elements

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i \in \mathcal{A}$</td>
<td>area of bus $i$</td>
</tr>
<tr>
<td>$A_k \subseteq \mathcal{A}$</td>
<td>contingent areas in contingency $k$, i.e., areas containing a bus with a connected generator, line, or transformer that goes out of service in contingency $k$</td>
</tr>
</tbody>
</table>
Table 6: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \subset \mathcal{E}$</td>
<td>lines active in the base case</td>
</tr>
<tr>
<td>$E_i^d \subset \mathcal{E}$</td>
<td>lines with destination bus $i$, $E_i^d = {e \in E : i_e^d = i}$</td>
</tr>
<tr>
<td>$E_i^o \subset \mathcal{E}$</td>
<td>lines with origin bus $i$, $E_i^o = {e \in E : i_e^o = i}$</td>
</tr>
<tr>
<td>$E_k \subset \mathcal{E}$</td>
<td>lines active in contingency $k$</td>
</tr>
<tr>
<td>$E_{ik}^d \subset \mathcal{E}$</td>
<td>lines active in contingency $k$ with destination bus $i$, $E_{ik}^d = E_i^d \cap E_k$</td>
</tr>
<tr>
<td>$E_{ik}^o \subset \mathcal{E}$</td>
<td>lines active in contingency $k$ with origin bus $i$, $E_{ik}^o = E_i^o \cap E_k$</td>
</tr>
<tr>
<td>$F \subset \mathcal{F}$</td>
<td>transformers active in the base case</td>
</tr>
<tr>
<td>$F_i^d \subset \mathcal{F}$</td>
<td>transformers with destination bus $i$, $F_i^d = {f \in F : i_f^d = i}$</td>
</tr>
<tr>
<td>$F_i^o \subset \mathcal{F}$</td>
<td>transformers with origin bus $i$, $F_i^o = {f \in F : i_f^o = i}$</td>
</tr>
<tr>
<td>$F_k \subset \mathcal{F}$</td>
<td>transformers active in contingency $k$</td>
</tr>
<tr>
<td>$F_{ik}^d \subset \mathcal{F}$</td>
<td>transformers active in contingency $k$ with destination bus $i$, $F_{ik}^d = F_i^d \cap F_k$</td>
</tr>
<tr>
<td>$F_{ik}^o \subset \mathcal{F}$</td>
<td>transformers active in contingency $k$ with origin bus $i$, $F_{ik}^o = F_i^o \cap F_k$</td>
</tr>
<tr>
<td>$G \subset \mathcal{G}$</td>
<td>generators active in the base case</td>
</tr>
<tr>
<td>$G_i \subset \mathcal{G}$</td>
<td>generators connected to bus $i$, $G_i = {g \in G : i_g = i}$</td>
</tr>
<tr>
<td>$G_k \subset \mathcal{G}$</td>
<td>generators active in contingency $k$</td>
</tr>
<tr>
<td>$G_{ik}^P \subset \mathcal{G}$</td>
<td>generators participating in real power response in contingency $k$</td>
</tr>
<tr>
<td>$G_{ik} \subset \mathcal{G}$</td>
<td>generators active in contingency $k$ and connected to bus $i$, $G_{ik} = G_i \cap G_k$</td>
</tr>
<tr>
<td>$H_g \subset \mathcal{H}$</td>
<td>cost function sample points for generator $g$</td>
</tr>
<tr>
<td>$I_a \subset \mathcal{I}$</td>
<td>buses in area $a$, $I_a = {i \in \mathcal{I} : a_i = a}$</td>
</tr>
<tr>
<td>$i_e^d \in \mathcal{I}$</td>
<td>destination bus of line $e$</td>
</tr>
<tr>
<td>$i_e^o \in \mathcal{I}$</td>
<td>origin bus of line $e$</td>
</tr>
<tr>
<td>$i_f^d \in \mathcal{I}$</td>
<td>destination bus of transformer $f$</td>
</tr>
<tr>
<td>$i_f^o \in \mathcal{I}$</td>
<td>origin bus of transformer $f$</td>
</tr>
<tr>
<td>$i_g \in \mathcal{I}$</td>
<td>bus that generator $g$ is connected to</td>
</tr>
<tr>
<td>id$_g$</td>
<td>ID of generator $g$ (2-character string)</td>
</tr>
</tbody>
</table>

Table 7: Data parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_e$</td>
<td>line $e$ series susceptance (p.u.)</td>
</tr>
</tbody>
</table>
Table 7: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_e^{CH}$</td>
<td>line $e$ total charging susceptance (p.u.)</td>
</tr>
<tr>
<td>$b_f$</td>
<td>transformer $f$ series susceptance (p.u.)</td>
</tr>
<tr>
<td>$b_f^M$</td>
<td>transformer $f$ magnetizing susceptance (p.u.)</td>
</tr>
<tr>
<td>$\bar{b}_i^{CS}$</td>
<td>bus $i$ maximum controllable shunt susceptance (p.u.)</td>
</tr>
<tr>
<td>$\underline{b}_i^{CS}$</td>
<td>bus $i$ minimum controllable shunt susceptance (p.u.)</td>
</tr>
<tr>
<td>$b_i^{FS}$</td>
<td>bus $i$ fixed shunt susceptance (p.u.)</td>
</tr>
<tr>
<td>$c_{slack}$</td>
<td>the objective value of a certain easily constructed feasible solution (USD/h)</td>
</tr>
<tr>
<td>$c_{gh}$</td>
<td>generation cost of generator $g$ at sample point $h$ (USD/h)</td>
</tr>
<tr>
<td>$g_e$</td>
<td>line $e$ series conductance (p.u.)</td>
</tr>
<tr>
<td>$g_f$</td>
<td>transformer $f$ series conductance (p.u.)</td>
</tr>
<tr>
<td>$g_f^M$</td>
<td>transformer $f$ magnetizing conductance (p.u.)</td>
</tr>
<tr>
<td>$g_i^{FS}$</td>
<td>bus $i$ fixed shunt conductance (p.u.)</td>
</tr>
<tr>
<td>$M$</td>
<td>a large constant used in the big-M mixed integer programming formulation of generator real power contingency response (p.u.)</td>
</tr>
<tr>
<td>$M$</td>
<td>a constant such that for any $M \geq M$ the mixed integer programming formulation of generator real power response is valid. (p.u.)</td>
</tr>
<tr>
<td>$M^P$</td>
<td>a large constant used in the big-M mixed integer programming formulation of generator real power contingency response (p.u.)</td>
</tr>
<tr>
<td>$M^P$</td>
<td>a constant such that for any $M^P \geq M^P$ the mixed integer programming formulation of generator real power response is valid. (p.u.)</td>
</tr>
<tr>
<td>$M^Q$</td>
<td>a large constant used in the big-M mixed integer programming formulation of generator reactive power contingency response (p.u.)</td>
</tr>
<tr>
<td>$M^Q$</td>
<td>a constant such that for any $M^Q \geq M^Q$ the mixed integer programming formulation of generator reactive power response is valid. (p.u.)</td>
</tr>
<tr>
<td>$M^v$</td>
<td>a large constant used in the big-M mixed integer programming formulation of generator reactive power contingency response (p.u.)</td>
</tr>
<tr>
<td>$M^v$</td>
<td>a constant such that for any $M^v \geq M^v$ the mixed integer programming formulation of generator reactive power response is valid. (p.u.)</td>
</tr>
<tr>
<td>$\bar{p}_g$</td>
<td>generator $g$ real power maximum (p.u.)</td>
</tr>
<tr>
<td>$p_{-g}$</td>
<td>generator $g$ real power minimum (p.u.)</td>
</tr>
<tr>
<td>$p_{gh}$</td>
<td>real power output of generator $g$ at sample point $h$ (p.u.)</td>
</tr>
<tr>
<td>$p_i^L$</td>
<td>bus $i$ constant real power load (p.u.)</td>
</tr>
<tr>
<td>$\bar{q}_g$</td>
<td>generator $g$ reactive power maximum (p.u.)</td>
</tr>
</tbody>
</table>
Table 7: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_g$</td>
<td>generator $g$ reactive power minimum (p.u.)</td>
</tr>
<tr>
<td>$q_i^L$</td>
<td>bus $i$ constant reactive power load (p.u.)</td>
</tr>
<tr>
<td>$\bar{R}_e$</td>
<td>line $e$ apparent current maximum in base case (p.u.)</td>
</tr>
<tr>
<td>$\bar{R}_{ek}$</td>
<td>line $e$ apparent current maximum in contingencies</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>system power base (MVA)</td>
</tr>
<tr>
<td>$\bar{s}_f$</td>
<td>transformer $f$ apparent power maximum in base case (p.u.)</td>
</tr>
<tr>
<td>$\bar{s}_f^K$</td>
<td>transformer $f$ apparent power maximum in contingencies (p.u.)</td>
</tr>
<tr>
<td>$\bar{v}_i$</td>
<td>bus $i$ voltage base (kV)</td>
</tr>
<tr>
<td>$\bar{v}_i$</td>
<td>bus $i$ voltage magnitude maximum in the base case (p.u.)</td>
</tr>
<tr>
<td>$\bar{v}_i$</td>
<td>bus $i$ voltage magnitude minimum in the base case (p.u.)</td>
</tr>
<tr>
<td>$\bar{v}_i^K$</td>
<td>bus $i$ voltage magnitude maximum in contingencies (p.u.)</td>
</tr>
<tr>
<td>$\bar{v}_i^K$</td>
<td>bus $i$ voltage magnitude minimum in contingencies (p.u.)</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>participation factor of generator $g$ in real power contingency response (1)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>weight on base case in objective (1)</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>transformer $f$ phase angle (rad)</td>
</tr>
<tr>
<td>$\lambda_n^P$</td>
<td>objective coefficient on real power power constraint violations for segment $n$ in the piecewise linear penalty cost function (USD/p.u.-h)</td>
</tr>
<tr>
<td>$\lambda_n^Q$</td>
<td>objective coefficient on real power power constraint violations for segment $n$ in the piecewise linear penalty cost function (USD/p.u.-h)</td>
</tr>
<tr>
<td>$\lambda_n^S$</td>
<td>objective coefficient on apparent power constraint violations for segment $n$ in the piecewise linear penalty cost function (USD/p.u.-h)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{en}$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for line $e$ apparent current rating violation (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{fn}$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for transformer $f$ apparent power rating violation (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{ekn}$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for line $e$ contingency $k$ apparent current rating violation (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{fkn}$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for transformer $f$ contingency $k$ apparent power rating violation (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{im}^+$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ real power balance violation positive part, i.e., excess real power flowing into bus $i$ (p.u.)</td>
</tr>
</tbody>
</table>
Table 7: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\sigma}<em>{P</em>{in}}^-$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ real power balance violation negative part, i.e., excess real power flowing out of bus $i$ (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}<em>{Q</em>{in}}^+$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ reactive power balance violation positive part (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}<em>{Q</em>{in}}^-$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ reactive power balance violation negative part (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{ikn}^+$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ contingency $k$ real power balance violation positive part (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{ikn}^-$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ contingency $k$ real power balance violation negative part (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{ikn}^+$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ contingency $k$ reactive power balance violation positive part (p.u.)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{ikn}^-$</td>
<td>segment $n$ upper bound corresponding to the piecewise linear penalty cost function for bus $i$ contingency $k$ reactive power balance violation negative part (p.u.)</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>transformer $f$ tap ratio (1)</td>
</tr>
</tbody>
</table>

Table 8: Optimization variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{CS_i}$</td>
<td>bus $i$ controllable shunt susceptance (p.u.)</td>
</tr>
<tr>
<td>$b_{CS_{ik}}$</td>
<td>bus $i$ contingency $k$ controllable shunt susceptance (p.u.)</td>
</tr>
<tr>
<td>$c$</td>
<td>total objective (USD/h)</td>
</tr>
<tr>
<td>$c_g$</td>
<td>generation cost of generator $g$ (USD/h)</td>
</tr>
<tr>
<td>$c^\sigma$</td>
<td>total constraint violation penalty in base case (USD/h)</td>
</tr>
<tr>
<td>$c^\sigma_k$</td>
<td>total constraint violation penalty in contingency $k$ (USD/h)</td>
</tr>
<tr>
<td>$p_{ce}^d$</td>
<td>line $e$ real power from destination bus into line (p.u.)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$p^e_o$</td>
<td>line $e$ real power from origin bus into line (p.u.)</td>
</tr>
<tr>
<td>$p^d_k$</td>
<td>line $e$ contingency $k$ real power from destination bus into line (p.u.)</td>
</tr>
<tr>
<td>$p^o_k$</td>
<td>line $e$ contingency $k$ real power from origin bus into line (p.u.)</td>
</tr>
<tr>
<td>$p^d_f$</td>
<td>transformer $f$ real power from destination bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$p^o_f$</td>
<td>transformer $f$ real power from origin bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$p^d_{f,k}$</td>
<td>transformer $f$ contingency $k$ real power from destination bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$p^o_{f,k}$</td>
<td>transformer $f$ contingency $k$ real power from origin bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$p_g$</td>
<td>generator $g$ real power output (p.u.)</td>
</tr>
<tr>
<td>$p_{g,k}$</td>
<td>generator $g$ contingency $k$ real power output (p.u.)</td>
</tr>
<tr>
<td>$q^d_e$</td>
<td>line $e$ reactive power from destination bus into line (p.u.)</td>
</tr>
<tr>
<td>$q^o_e$</td>
<td>line $e$ reactive power from origin bus into line (p.u.)</td>
</tr>
<tr>
<td>$q^d_{e,k}$</td>
<td>line $e$ contingency $k$ reactive power from destination bus into line (p.u.)</td>
</tr>
<tr>
<td>$q^o_{e,k}$</td>
<td>line $e$ contingency $k$ reactive power from origin bus into line (p.u.)</td>
</tr>
<tr>
<td>$q^d_f$</td>
<td>transformer $f$ reactive power from destination bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$q^o_f$</td>
<td>transformer $f$ reactive power from origin bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$q^d_{f,k}$</td>
<td>transformer $f$ contingency $k$ reactive power from destination bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$q^o_{f,k}$</td>
<td>transformer $f$ contingency $k$ reactive power from origin bus into transformer (p.u.)</td>
</tr>
<tr>
<td>$q_g$</td>
<td>generator $g$ reactive power output (p.u.)</td>
</tr>
<tr>
<td>$q_{g,k}$</td>
<td>generator $g$ contingency $k$ reactive power output (p.u.)</td>
</tr>
<tr>
<td>$t_{g,h}$</td>
<td>coefficient of sample point $h$ for generator $g$ solution as a point on generation cost function (1)</td>
</tr>
<tr>
<td>$v_i$</td>
<td>bus $i$ voltage magnitude (p.u.)</td>
</tr>
<tr>
<td>$v_{i,k}$</td>
<td>bus $i$ contingency $k$ voltage magnitude (p.u.)</td>
</tr>
<tr>
<td>$x_{g,k}^{P+}$</td>
<td>generator $g$ contingency $k$ binary variable indicating positive slack in upper bound on real power output (bin)</td>
</tr>
<tr>
<td>$x_{g,k}^{P-}$</td>
<td>generator $g$ contingency $k$ binary variable indicating positive slack in lower bound on real power output (bin)</td>
</tr>
<tr>
<td>$x_{g,k}^{Q+}$</td>
<td>generator $g$ contingency $k$ binary variable indicating positive slack in upper bound on reactive power output (bin)</td>
</tr>
</tbody>
</table>
Table 8: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{gk}$</td>
<td>generator $g$ contingency $k$ binary variable indicating positive slack in lower bound on reactive power output (bin)</td>
</tr>
<tr>
<td>$\Delta_k$</td>
<td>contingency $k$ scale factor on generator participation factors defining generator real power contingency response (p.u.)</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>bus $i$ voltage angle (rad)</td>
</tr>
<tr>
<td>$\theta_{ik}$</td>
<td>bus $i$ contingency $k$ voltage angle (rad)</td>
</tr>
<tr>
<td>$\sigma_{en}^S$</td>
<td>line $e$ apparent current rating violation for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{ekn}^S$</td>
<td>line $e$ contingency $k$ apparent current rating violation for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{fn}^S$</td>
<td>transformer $f$ apparent power rating violation for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{fkn}^S$</td>
<td>transformer $f$ contingency $k$ apparent power rating violation for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{in}^{P+}$</td>
<td>bus $i$ real power balance violation positive part, i.e., excess real power flowing into bus $i$, for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{in}^{P-}$</td>
<td>bus $i$ real power balance violation negative part, i.e., excess real power flowing out of bus $i$, for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{in}^{Q+}$</td>
<td>bus $i$ reactive power balance violation positive part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{in}^{Q-}$</td>
<td>bus $i$ reactive power balance violation negative part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{P+}$</td>
<td>bus $i$ contingency $k$ real power balance violation positive part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{P-}$</td>
<td>bus $i$ contingency $k$ real power balance violation negative part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{Q+}$</td>
<td>bus $i$ contingency $k$ reactive power balance violation positive part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{Q-}$</td>
<td>bus $i$ contingency $k$ reactive power balance violation negative part for segment $n$ in the piecewise linear penalty cost function (p.u.)</td>
</tr>
</tbody>
</table>
3 Model Formulation

3.1 Objective Definition

The objective (for minimization) is the sum of generator real power output costs in the base case, and a weighted sum of soft constraint violation penalties in the base case and contingencies:

\[ c = \sum_{g \in G} c_g + \delta c^\sigma + \frac{(1 - \delta)}{|K|} \sum_{k \in K} c_k^\sigma \]  

(1)

Generator real power output cost is defined by a cost function given as a set of sample points, modeled by interpolating the cost to the sample points in the cost space and the real power output to the sample points in the real power output space, with common interpolation coefficients. Generator cost interpolation to sample points:

\[ c_g = \sum_{h \in H_g} c_{gh} t_{gh} \quad \forall g \in G \]  

(2)

Generator real power output interpolation to sample points:

\[ \sum_{h \in H_g} p_{gh} t_{gh} = p_g \quad \forall g \in G \]  

(3)

Generator cost interpolation coefficient bounds:

\[ 0 \leq t_{gh} \quad \forall g \in G, h \in H_g \]  

(4)

Generator cost interpolation coefficient normalization:

\[ \sum_{h \in H_g} t_{gh} = 1 \quad \forall g \in G \]  

(5)

The total constraint violation penalty in the base case and in contingencies includes penalties on violations of bus real and reactive power balance, penalties on violations of line apparent current ratings, and penalties on violations of transformer apparent power ratings. The penalty is given by a piecewise linear cost function for the violation cost terms - where a small penalty price is applied to minor violations followed by a more stringent penalty price for moderate violations and then an extremely severe penalty for all remaining violations (i.e., it is likely to have 3 segments, indexed by \( n \) in this formulation). The first two segments will follow existing industry practices related to constraint relaxations and the chosen penalty prices. The last price will be substantially higher to encourage the approach to not have significant violations. The penalties in the base case and contingencies are given by:

\[ c^\sigma = \sum_{n \in N} \left[ \lambda_n^P \sum_{i \in I} (\sigma_{i n}^{P+} + \sigma_{i n}^{P-}) + \lambda_n^Q \sum_{i \in I} (\sigma_{i n}^{Q+} + \sigma_{i n}^{Q-}) + \lambda_n^S \sum_{e \in E} \sigma_{en}^S + \lambda_n^S \sum_{f \in F} \sigma_{fn}^S \right] \]  

(6)
\[ c_k^g = \sum_{n \in N} \left[ \lambda_n^p \sum_{i \in I} (\sigma_{ikn}^p + \sigma_{ikn}^p) + \lambda_n^q \sum_{i \in I} (\sigma_{ikn}^q + \sigma_{ikn}^q) \right. \]
\[ \left. + \lambda_n^s \sum_{e \in E_k} \sum_{n \in N} \sigma_{ekn}^s + \lambda_n^s \sum_{f \in F_k} \sum_{n \in N} \sigma_{fkn}^s \right] \forall k \in K \]

(7)

Violations in each segment of the piecewise linear cost function are represented by overall violation variables (i.e., not indexed by \( n \)) for the remaining constraints in this formulation:

\[ \sigma_{i}^p = \sum_{n \in N} \sigma_{in}^p \quad \forall i \in I \]

(8)

\[ \sigma_{ik}^p = \sum_{n \in N} \sigma_{ikn}^p \quad \forall i \in I, k \in K \]

(9)

\[ \sigma_{i}^p = \sum_{n \in N} \sigma_{in}^p \quad \forall i \in I \]

(10)

\[ \sigma_{ik}^p = \sum_{n \in N} \sigma_{ikn}^p \quad \forall i \in I, k \in K \]

(11)

\[ \sigma_{i}^q = \sum_{n \in N} \sigma_{in}^q \quad \forall i \in I \]

(12)

\[ \sigma_{ik}^q = \sum_{n \in N} \sigma_{ikn}^q \quad \forall i \in I, k \in K \]

(13)

\[ \sigma_{i}^p = \sum_{n \in N} \sigma_{in}^p \quad \forall i \in I \]

(14)

\[ \sigma_{ik}^p = \sum_{n \in N} \sigma_{ikn}^p \quad \forall i \in I, k \in K \]

(15)

\[ \sigma_{e}^s = \sum_{n \in N} \sigma_{en}^s \quad \forall e \in E \]

(16)
\begin{align}
  \sigma^S_{ek} &= \sum_{n \in N} \sigma^S_{ekn} \quad \forall e \in E, k \in K \\
  \sigma^S_f &= \sum_{n \in N} \sigma^S_{fn} \quad \forall f \in F \tag{18} \\
  \sigma^S_{fk} &= \sum_{n \in N} \sigma^S_{fkn} \quad \forall f \in F, k \in K \tag{19}
\end{align}

Bounds on violation variables for each segment are established based on the following equations. Each segment slack variable is non-negative with a lower bound of zero and an upper bound that varies based on the \( n \) segment. While the presented formulation here is generic, the GO Competition Challenge 1 will likely set the upper bounds for each segment to be a percentage of the corresponding transmission transfer limit (line current limit or transformer power limit) or a fixed MW and MVar value for the node balance violations.

\begin{align}
  0 &\leq \sigma^P_{im} \leq \sigma^P_{in} \quad \forall i \in \mathcal{I}, n \in N \tag{20} \\
  0 &\leq \sigma^P_{ikn} \leq \sigma^P_{in} \quad \forall i \in \mathcal{I}, n \in N, k \in \mathcal{K} \tag{21} \\
  0 &\leq \sigma^P_{im} \leq \sigma^P_{in} \quad \forall i \in \mathcal{I}, n \in N \tag{22} \\
  0 &\leq \sigma^P_{ikn} \leq \sigma^P_{in} \quad \forall i \in \mathcal{I}, n \in N, k \in \mathcal{K} \tag{23} \\
  0 &\leq \sigma^Q_{im} \leq \sigma^Q_{in} \quad \forall i \in \mathcal{I}, n \in N \tag{24} \\
  0 &\leq \sigma^Q_{ikn} \leq \sigma^Q_{in} \quad \forall i \in \mathcal{I}, n \in N, k \in \mathcal{K} \tag{25} \\
  0 &\leq \sigma^Q_{im} \leq \sigma^Q_{in} \quad \forall i \in \mathcal{I}, n \in N \tag{26} \\
  0 &\leq \sigma^Q_{ikn} \leq \sigma^Q_{in} \quad \forall i \in \mathcal{I}, n \in N, k \in \mathcal{K} \tag{27}
\end{align}
\begin{align*}
0 \leq \sigma_{en}^{S+} &\leq \sigma_{en}^{S+} \quad \forall e \in E, n \in N \\ 
0 \leq \sigma_{ekn}^{S+} &\leq \sigma_{en}^{S+} \quad \forall e \in E, n \in N, k \in K \\ 
0 \leq \sigma_{fn}^{S+} &\leq \sigma_{fn}^{S+} \quad \forall f \in F, n \in N \\ 
0 \leq \sigma_{fkn}^{S+} &\leq \sigma_{fn}^{S+} \quad \forall f \in F, n \in N, k \in K
\end{align*}

3.2 Primary Optimization Variable Bounds in the Base Case

Bounds on voltage in the base case are given by:
\[ v_i \leq v_i \leq \bar{v}_i \quad \forall i \in \mathcal{I} \tag{32} \]

Bounds on real power in the base case are given by:
\[ p_g \leq p_g \leq \bar{p}_g \quad \forall g \in G \tag{33} \]

No real power is produced by generators that are not active in the base case:
\[ p_g = 0 \quad \forall g \in \mathcal{G} \setminus G \tag{34} \]

Bounds on reactive power in the base case are given by:
\[ q_g \leq q_g \leq \bar{q}_g \quad \forall g \in G \tag{35} \]

No reactive power is produced by generators that are not active in the base case:
\[ q_g = 0 \quad \forall g \in \mathcal{G} \setminus G \tag{36} \]

Bounds on shunt susceptance in the base case are given by:
\[ b_i^{CS} \leq b_i^{CS} \leq \bar{b}_i^{CS} \quad \forall i \in \mathcal{I} \tag{37} \]
3.3 Line Flow Definitions in the Base Case

Real and reactive power flows into a line at the origin buses in the base case are defined by:

\[
po_e = ge v_{oe}^2 \\
+ (-ge \cos(\theta_{oe} - \theta_{ie}) - be \sin(\theta_{oe} - \theta_{ie}))v_{oe}v_{ie} \quad \forall e \in E
\]

\[
qo_e = -(be + b_{eCH}/2)v_{oe}^2 \\
+ (be \cos(\theta_{oe} - \theta_{ie}) - ge \sin(\theta_{oe} - \theta_{ie}))v_{oe}v_{ie} \quad \forall e \in E
\] (38)

Real and reactive power flows into a line at the destination buses in the base case are defined by:

\[
pd_e = ge v_{de}^2 \\
+ (-ge \cos(\theta_{de} - \theta_{i'd}) - be \sin(\theta_{de} - \theta_{i'd}))v_{de}v_{i'd} \quad \forall e \in E
\]

\[
qd_e = -(be + b_{eCH}/2)v_{de}^2 \\
+ (be \cos(\theta_{de} - \theta_{i'd}) - ge \sin(\theta_{de} - \theta_{i'd}))v_{de}v_{i'd} \quad \forall e \in E
\] (39)

3.4 Transformer Flow Definitions in the Base Case

Real and reactive power flows into a transformer at the origin buses in the base case are defined by:

\[
pof = (gf/\tau_f^2 + g_M^f)v_{of}^2 \\
+ (-gf/\tau_f \cos(\theta_{of} - \theta_{i'o} - \theta_f) - b_f/\tau_f \sin(\theta_{of} - \theta_{i'o} - \theta_f))v_{of}v_{i'o} \quad \forall f \in F
\]

\[
qof = -(b_f/\tau_f^2 + b_M^f)v_{of}^2 \\
+ (b_f/\tau_f \cos(\theta_{of} - \theta_{i'o} - \theta_f) - g_f/\tau_f \sin(\theta_{of} - \theta_{i'o} - \theta_f))v_{of}v_{i'o} \quad \forall f \in F
\] (42)

Real and reactive power flows into a transformer at the destination buses in the base case are defined by:

\[
pdf = gf v_{df}^2 \\
+ (-gf/\tau_f \cos(\theta_{df} - \theta_{i'd} + \theta_f) - b_f/\tau_f \sin(\theta_{df} - \theta_{i'd} + \theta_f))v_{df}v_{i'd} \quad \forall f \in F
\]

\[
qdf = -b_f v_{df}^2 \\
+ (b_f/\tau_f \cos(\theta_{df} - \theta_{i'd} + \theta_f) - g_f/\tau_f \sin(\theta_{df} - \theta_{i'd} + \theta_f))v_{df}v_{i'd} \quad \forall f \in F
\] (45)
3.5 Bus Power Balance Constraints in the Base Case

Bus real power balance constraints ensure that all real power output from generators at a given bus sum to all real power flows into other grid components at the bus. Nonegative variables $\sigma_i^{P+}$ and $\sigma_i^{P-}$ represent the positive and negative parts of the net imbalance. These constraint violation variables (also called slack variables) appear in the objective with penalty coefficients.

$$\sum_{g \in G_i} p_g - p_i^L - g_i^F S v_i^2$$

$$- \sum_{e \in E_i} p_e^o - \sum_{f \in F_i^o} p_f^o - \sum_{e \in E_i} p_e^d - \sum_{f \in F_i^d} p_f^d = \sigma_i^{P+} - \sigma_i^{P-} \quad \forall i \in I$$

$$\sigma_i^{P+} \geq 0 \quad \forall i \in I$$

$$\sigma_i^{P-} \geq 0 \quad \forall i \in I$$

Bus reactive power balance constraints are similar with soft constraint violation variables $\sigma_i^{Q+}$ and $\sigma_i^{Q-}$:

$$\sum_{g \in G_i} q_g - q_i^L - (-b_i^F S - b_i^C S) v_i^2$$

$$- \sum_{e \in E_i} q_e^o - \sum_{f \in F_i^o} q_f^o - \sum_{e \in E_i} q_e^d - \sum_{f \in F_i^d} q_f^d = \sigma_i^{Q+} - \sigma_i^{Q-} \quad \forall i \in I$$

$$\sigma_i^{Q+} \geq 0 \quad \forall i \in I$$

$$\sigma_i^{Q-} \geq 0 \quad \forall i \in I$$

3.6 Line Current Ratings in the Base Case

Line current ratings in the base case at the origin bus, with soft constraint violation variables $\sigma_e^S$, are given by:

$$\sqrt{(p_e^o)^2 + (q_e^o)^2} \leq R_e v_i^e + \sigma_e^S \quad \forall e \in E$$

$$\sigma_e^S \geq 0 \quad \forall e \in E$$

Line current ratings in the base case at the destination bus, with soft constraint violation variables $\sigma_e^S$, are given by:

$$\sqrt{(p_e^d)^2 + (q_e^d)^2} \leq R_e v_i^d + \sigma_e^S \quad \forall e \in E$$
3.7 Transformer Power Ratings in the Base Case

Transformer power ratings in the base case at the origin bus, with soft constraint violation variables $\sigma_f^S$, are given by:

$$\sqrt{(p_f^o)^2 + (q_f^o)^2} \leq \bar{s}_f + \sigma_f^S \quad \forall f \in F$$  \hspace{1cm} (55)

$$\sigma_f^S \geq 0 \quad \forall f \in F$$  \hspace{1cm} (56)

Transformer power ratings in the base case at the destination bus, with soft constraint violation variables $\sigma_f^S$, are given by:

$$\sqrt{(p_f^d)^2 + (q_f^d)^2} \leq \bar{s}_f + \sigma_f^S \quad \forall f \in F$$  \hspace{1cm} (57)

3.8 Primary Optimization Variable Bounds in Contingencies

Bounds on voltage in each contingency are given by:

$$v^K_i \leq v_{ik} \leq \bar{v}^K_i \quad \forall k \in K, i \in I$$  \hspace{1cm} (58)

Bounds on real power generation in each contingency are given by:

$$p_g \leq p_{gk} \leq \bar{p}_g \quad \forall k \in K, g \in G_k$$  \hspace{1cm} (59)

No real power is produced by generators that are not active in each contingency:

$$p_{gk} = 0 \quad \forall k \in K, g \in G \setminus G_k$$  \hspace{1cm} (60)

Bounds on reactive power generation in each contingency are given by:

$$q_g \leq q_{gk} \leq \bar{q}_g \quad \forall k \in K, g \in G_k$$  \hspace{1cm} (61)

No reactive power is produced by generators that are not active in each contingency:

$$q_{gk} = 0 \quad \forall k \in K, g \in G \setminus G_k$$  \hspace{1cm} (62)

Bounds on shunt susceptance in each contingency are given by:

$$b_{i}^{CS} \leq b_{ik}^{CS} \leq \bar{b}_i^{CS} \quad \forall k \in K, i \in I$$  \hspace{1cm} (63)
3.9 Line Flow Definitions in Contingencies

Real and reactive power flows into a line at the origin bus in each contingency are defined by:

\[ p_{ek}^{o} = g_{e} v_{i_{e}}^{2} \]
\[ + (-g_{e} \cos(\theta_{i_{e} k} - \theta_{i_{d} k}) - b_{e} \sin(\theta_{i_{e} k} - \theta_{i_{d} k})) v_{i_{e} k} v_{i_{d} k} \quad \forall k \in \mathcal{K}, e \in E_{k} \]  
\[ q_{ek}^{o} = -(b_{e} + b_{e}^{CH}/2) v_{i_{e} k}^{2} \]
\[ + (b_{e} \cos(\theta_{i_{e} k} - \theta_{i_{d} k}) - g_{e} \sin(\theta_{i_{e} k} - \theta_{i_{d} k})) v_{i_{e} k} v_{i_{d} k} \quad \forall k \in \mathcal{K}, e \in E_{k} \]  

Real and reactive power flows into a line at the destination bus in each contingency are defined by:

\[ p_{ek}^{d} = g_{e} v_{i_{d}}^{2} \]
\[ + (-g_{e} \cos(\theta_{i_{d} k} - \theta_{i_{e} k}) - b_{e} \sin(\theta_{i_{d} k} - \theta_{i_{e} k})) v_{i_{d} k} v_{i_{e} k} \quad \forall k \in \mathcal{K}, e \in E_{k} \]  
\[ q_{ek}^{d} = -(b_{e} + b_{e}^{CH}/2) v_{i_{d} k}^{2} \]
\[ + (b_{e} \cos(\theta_{i_{d} k} - \theta_{i_{e} k}) - g_{e} \sin(\theta_{i_{d} k} - \theta_{i_{e} k})) v_{i_{d} k} v_{i_{e} k} \quad \forall k \in \mathcal{K}, e \in E_{k} \]  

3.10 Transformer Flow Definitions in Contingencies

Real and reactive power flows into a transformer at the origin bus in each contingency are defined by:

\[ p_{fk}^{o} = (g_{f}/\tau_{f}^{2} + g_{f}^{M}) v_{i_{f}}^{2} \]
\[ + (-g_{f}/\tau_{f} \cos(\theta_{i_{f} k} - \theta_{i_{d} k} - \theta_{f}) - b_{f}/\tau_{f} \sin(\theta_{i_{f} k} - \theta_{i_{d} k} - \theta_{f})) v_{i_{f} k} v_{i_{d} k} \quad \forall k \in \mathcal{K}, f \in F_{k} \]  
\[ q_{fk}^{o} = -(b_{f}/\tau_{f}^{2} + b_{f}^{M}) v_{i_{f}}^{2} \]
\[ + (b_{f}/\tau_{f} \cos(\theta_{i_{f} k} - \theta_{i_{d} k} - \theta_{f}) - g_{f}/\tau_{f} \sin(\theta_{i_{f} k} - \theta_{i_{d} k} - \theta_{f})) v_{i_{f} k} v_{i_{d} k} \quad \forall k \in \mathcal{K}, f \in F_{k} \]  

Real and reactive power flows into a transformer at the destination bus in each contingency are defined by:

\[ p_{fk}^{d} = g_{f} v_{i_{d}}^{2} \]
\[ + (-g_{f}/\tau_{f} \cos(\theta_{i_{d} k} - \theta_{i_{f} k} + \theta_{f}) - b_{f}/\tau_{f} \sin(\theta_{i_{d} k} - \theta_{i_{f} k} + \theta_{f})) v_{i_{d} k} v_{i_{f} k} \quad \forall k \in \mathcal{K}, f \in F_{k} \]  
\[ q_{fk}^{d} = -b_{f} v_{i_{d} k}^{2} \]
\[ + (b_{f}/\tau_{f} \cos(\theta_{i_{d} k} - \theta_{i_{f} k} + \theta_{f}) - g_{f}/\tau_{f} \sin(\theta_{i_{d} k} - \theta_{i_{f} k} + \theta_{f})) v_{i_{d} k} v_{i_{f} k} \quad \forall k \in \mathcal{K}, f \in F_{k} \]
3.11 Bus Power Balance Constraints in Contingencies

Bus real power balance constraints in each contingency with soft constraint violation variables $\sigma_{ik}^{P+}$ and $\sigma_{ik}^{P-}$ are given by:

$$\sum_{g \in G_{ik}} p_{gk} - p_{i}^{L} - g_{i}^{FS} v_{ik}^2 - \sum_{e \in E_{ik}^o} p_{ek}^o - \sum_{f \in F_{ik}^o} p_{fk}^o - \sum_{e \in E_{ik}^d} p_{ek}^d - \sum_{f \in F_{ik}^d} p_{fk}^d = \sigma_{ik}^{P+} - \sigma_{ik}^{P-} \quad \forall k \in K, i \in I$$

(72)

$$\sigma_{ik}^{P+} \geq 0 \quad \forall k \in K, i \in I$$

(73)

$$\sigma_{ik}^{P-} \geq 0 \quad \forall k \in K, i \in I$$

(74)

Bus reactive power balance constraints in each contingency with soft constraint violation variables $\sigma_{ik}^{Q+}$ and $\sigma_{ik}^{Q-}$ are given by:

$$\sum_{g \in G_{ik}} q_{gk} - q_{i}^{L} - (-b_{i}^{FS} - b_{ik}^{CS}) v_{ik}^2 - \sum_{e \in E_{ik}^o} q_{ek}^o - \sum_{f \in F_{ik}^o} q_{fk}^o - \sum_{e \in E_{ik}^d} q_{ek}^d - \sum_{f \in F_{ik}^d} q_{fk}^d = \sigma_{ik}^{Q+} - \sigma_{ik}^{Q-} \quad \forall k \in K, i \in I$$

(75)

$$\sigma_{ik}^{Q+} \geq 0 \quad \forall k \in K, i \in I$$

(76)

$$\sigma_{ik}^{Q-} \geq 0 \quad \forall k \in K, i \in I$$

(77)

3.12 Line Current Ratings in Contingencies

Line current ratings at the origin bus in each contingency with soft constraint violation variables $\sigma_{ek}^{S}$ are modeled by:

$$\sqrt{(p_{ek}^o)^2 + (q_{ek}^o)^2} \leq R_{e}^K v_{ek} + \sigma_{ek}^{S} \quad \forall k \in K, e \in E_k$$

(78)

$$\sigma_{ek}^{S} \geq 0 \quad \forall k \in K, e \in E_k$$

(79)

Line current ratings at the destination bus in each contingency with soft constraint violation variables $\sigma_{ek}^{S}$ are modeled by:

$$\sqrt{(p_{ek}^d)^2 + (q_{ek}^d)^2} \leq R_{e}^K v_{i_ek} + \sigma_{ek}^{S} \quad \forall k \in K, e \in E_k$$

(80)
3.13 Transformer Power Ratings in Contingencies

Transformer power ratings in each contingency at the origin bus with soft constraint violation variables $\sigma^S_f k$, are given by:

$$\sqrt{(p^o_{fk})^2 + (q^o_{fk})^2} \leq \pi^K_f + \sigma^S_{fk} \quad \forall k \in \mathcal{K}, f \in F_k$$ (81)

$$\sigma^S_{fk} \geq 0 \quad \forall k \in \mathcal{K}, f \in F_k$$ (82)

Transformer power ratings in each contingency at the destination bus with soft constraint violation variables $\sigma^S_f k$, are given by:

$$\sqrt{(p^d_{fk})^2 + (q^d_{fk})^2} \leq \pi^K_f + \sigma^S_{fk} \quad \forall k \in \mathcal{K}, f \in F_k$$ (83)

3.14 Generator Real Power Contingency Response

The real power output $p_{gk}$ of a generator $g$ in a contingency $k$ is subject to constraints linking it to the base case value $p_g$.

A generator that is online in a contingency but is not selected to respond to that contingency maintains its real power output from the base case:

$$p_{gk} = p_g \quad \forall k \in \mathcal{K}, g \in G_k \setminus G^P_k$$ (84)

A generator that does respond to a given contingency adjusts its real power output according to its predefined (offline) participation factor until it hits an operational bound (min or max capacity). The real power output of a responding generator $g$ in contingency $k$ is $p_g + \alpha_g \Delta_k$, if the generator is following its required participation factor. The actual real power output $p_{gk}$ must be equal to this value unless the generator were to violate its min production or maximum capacity. The generator must operate at its lower bound when the predefined participation factor response would force it to violate its min production level and the generator must operate at its maximum capacity when the predefined participation factor response would force it to violate its upper bound.

Given this conceptual definition of generator real power contingency response, we give several mathematical formulations, including a formulation using logical constraints, a formulation using the projection operator, and a big-M mixed integer programming formulation.

3.14.1 Logical Formulation

In this section we formulate generator real power contingency response using a disjunction of linear constraints. This formulation most clearly expresses the constraints that we want
to impose on the solution.

\[
\begin{cases}
\{p_g \leq p_{gk} \leq p_g \quad \text{and} \quad p_{gk} = p_g + \alpha_g \Delta_k\} & \text{or} \\
\{p_{gk} = p_g \quad \text{and} \quad p_{gk} \leq p_g + \alpha_g \Delta_k\} & \text{or} \\
\{p_{gk} = p_g \quad \text{and} \quad p_{gk} \geq p_g + \alpha_g \Delta_k\}
\end{cases} \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(85)

3.14.2 Projection Formulation

In this section we formulate generator real power contingency response using the projection operator \(\Pi\). This formulation is equivalent to the logic based presentation in the preceding section but it may not be easy to implement in standard optimization tools.

\[
p_{gk} = \Pi_{[p_g, p_g]}(p_g + \alpha_g \Delta_k) \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(86)

Equation [86] is an equivalent reformulation of [85].

3.14.3 Mixed Integer Programming Formulation

In this section, we reformulate the generator real power contingency response using the big-M mixed integer programming (MIP) technique. This approach requires the determination of a large multiplier, the big-M value, and that value must be sufficiently large enough to ensure that the MIP formulation is equivalent to the preceding two formulations. Such an MIP formulation is easier to implement within standard optimization modeling tools. First let \(M^P\) and \(M\) denote large positive constants, left unspecified here. Then introduce binary variables \(x_{gk}^{P+}\) and \(x_{gk}^{P-}\):

\[
x_{gk}^{P+} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(87)

\[
x_{gk}^{P-} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(88)

Then, equation [89] is written such that it is inactive if \(x_{gk}^{P+} = 1\). When \(x_{gk}^{P+} = 0\), equations [59 89] force \(p_{gk} = \bar{p}_g\).

\[
\bar{p}_g - p_{gk} \leq M^P x_{gk}^{P+} \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(89)

Equation [90] is written such that it is inactive if \(x_{gk}^{P-} = 1\). When \(x_{gk}^{P-} = 0\), equations [59 90] force \(p_{gk} = \bar{p}_g\).

\[
p_{gk} - \bar{p}_g \leq M^P x_{gk}^{P-} \quad \forall k \in \mathcal{K}, g \in G_k^P
\]

(90)
Equation (91) is written such that it is inactive if \( x_{pk}^{P+} = 0 \). When \( x_{pk}^{P+} = 1 \), \( p_{pk} \) is forced to be equal to or greater than the desired real power response, \( p_g + \alpha_g \Delta_k \), dictated by the predefined participation factor, \( \alpha_g \).

\[
p_g + \alpha_g \Delta_k - p_{pk} \leq M(1 - x_{pk}^{P+}) \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{91}
\]

Equation (92) is written such that it is inactive if \( x_{pk}^{P-} = 0 \). When \( x_{pk}^{P-} = 1 \), \( p_{pk} \) is forced to be equal to or less than the desired real power response, \( p_g + \alpha_g \Delta_k \), dictated by the predefined participation factor, \( \alpha_g \).

\[
p_{pk} - p_g - \alpha_g \Delta_k \leq M(1 - x_{pk}^{P-}) \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{92}
\]

Since there exists \( M^P \) and \( M \) such that for all \( M^P \geq M \) and \( M \geq M_g \), equations (87)–(88) create an equivalent reformulation of (85). Table (3.14.3) shows simplified equations (89)–(92) for all combinations of binary variables \( x_{pk}^{P+} \) and \( x_{pk}^{P-} \). The gray highlighted cells are the inactive constraints under the specified solution for the binary variables. The first row, when \( x_{pk}^{P+} = 1 \) and \( x_{pk}^{P-} = 1 \), represents the case when the generator precisely follows the predefined participation factor response, which is the first state described by (85). The second row, when \( x_{pk}^{P+} = 0 \) and \( x_{pk}^{P-} = 1 \), represents the case when the participation factor response would require the generator to violate its maximum capacity; therefore, the generator operates instead at its max output. This is the second state defined by (85). The third row, when \( x_{pk}^{P+} = 1 \) and \( x_{pk}^{P-} = 0 \), represents the case when the participation factor response would require the generator to violate its minimum capacity; therefore, the generator operates instead at its min output. This is the third and final state defined by (85). The last row represents an invalid (infeasible) solution for the binary variables; based on the defined equations, it is not possible for both binary variables to take on a value of zero. This infeasibility is directly imposed as can be seen by the resulting inequalities in the last row that simultaneously force the generator’s real power production to be below its min capacity and above its max capacity. Entrants may choose to add a combinatorial cut that directly excludes this state, though such a constraint is not necessary to obtain a valid solution.

Table 9: Equations (89)–(92) under all combinations of binary variables \( x_{pk}^{P+} \) and \( x_{pk}^{P-} \) values. All equations in Table (3.14.3) are written \( \forall k \in \mathcal{K}, g \in G_k^P \). \( M^P \) and \( M \) are sufficiently large such that when they appear in the simplified equations (89)–(92), the constraints will not be binding in any feasible solution; these constraints have been shaded gray in Table (3.14.3).

<table>
<thead>
<tr>
<th>( x_{pk}^{P+} )</th>
<th>( x_{pk}^{P-} )</th>
<th>(89)</th>
<th>(90)</th>
<th>(91)</th>
<th>(92)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( p_{pk} \geq -M^P + \bar{p}_g )</td>
<td>( p_{pk} \leq M^P + \bar{p}_g )</td>
<td>( p_{pk} \geq p_g + \alpha_g \Delta_k )</td>
<td>( p_{pk} \leq p_g + \alpha_g \Delta_k )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( p_{pk} \geq \bar{p}_g )</td>
<td>( p_{pk} \leq M^P + \bar{p}_g )</td>
<td>( p_{pk} \geq -M + p_g + \alpha_g \Delta_k )</td>
<td>( p_{pk} \leq p_g + \alpha_g \Delta_k )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( p_{pk} \geq -M^P + \bar{p}_g )</td>
<td>( p_{pk} \leq \bar{p}_g )</td>
<td>( p_{pk} \geq p_g + \alpha_g \Delta_k )</td>
<td>( p_{pk} \leq M + p_g + \alpha_g \Delta_k )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( p_{pk} \geq \bar{p}_g )</td>
<td>( p_{pk} \leq \bar{p}_g )</td>
<td>( p_{pk} \geq -M + p_g + \alpha_g \Delta_k )</td>
<td>( p_{pk} \leq M + p_g + \alpha_g \Delta_k )</td>
</tr>
</tbody>
</table>
3.15 Generator Reactive Power Contingency Response

A generator $g$ that is active during a contingency $k$ tries to maintain the base case (pre-contingency) voltage magnitude at its bus, $i_g$, by adjusting its reactive power output. The generator will do all that it can to maintain this voltage magnitude. If the bus voltage magnitude drops below its base case magnitude, then the generator reactive power must be at its upper bound, reflecting that it has exhausted its ability to increase voltage. Similarly if the bus voltage magnitude is higher than the base case magnitude, then the generator reactive power must be at its lower bound. In power systems, this is referred to as PV/PQ switching; the generator’s bus is providing adequate voltage control then it is acting as a PV bus as there is sufficient reactive power to maintain the voltage. If there is insufficient reactive power to maintain the voltage, the bus is deemed to be a PQ bus in this case and the reactive power injection is fixed (in this special case, the reactive power is fixed to either the generator’s reactive power lower or upper bound).

Given this conceptual definition of generator reactive power contingency response, we give several mathematical formulations, including a formulation using logical constraints, a formulation using min and max functions, and a big-M mixed integer programming formulation.

3.15.1 Formulation with Logical Constraints

In this section, we formulate generator reactive power contingency response using a disjunction of linear constraints.

\[
\begin{align*}
\{q_g & \leq q_{gk} \leq \bar{q}_g \text{ and } v_{i_gk} = v_{i_g}\} \text{ or } \\
\{q_{gk} = \bar{q}_g \text{ and } v_{i_gk} \leq v_{i_g}\} \text{ or } \\
\{q_{gk} = q_g \text{ and } v_{i_gk} \geq v_{i_g}\}
\end{align*}
\forall k \in K, g \in G_k \tag{93}
\]

3.15.2 Formulation with Min and Max Operators

In this section, we formulate generator reactive power contingency response using min and max functions.

\[
\min\{\max\{0, v_{i_g} - v_{i_gk}\}, \max\{0, \bar{q}_g - q_{gk}\}\} = 0 \quad \forall k \in K, g \in G_k \tag{94}
\]

\[
\min\{\max\{0, v_{i_gk} - v_{i_g}\}, \max\{0, q_{gk} - q_g\}\} = 0 \quad \forall k \in K, g \in G_k \tag{95}
\]

Equations (94, 95) are an equivalent reformulation of (93).
3.15.3 Formulation with Binary Variables

In this section, we formulate generator reactive power contingency response using the big-M mixed integer programming technique. First let $M^Q$ and $M^v$ be large positive constants. Then introduce binary variables $x_{Qg}^+$ and $x_{Qg}^-$:

\[ x_{Qg}^+ \in \{0, 1\} \quad \forall k \in K, g \in G_k \quad (96) \]

\[ x_{Qg}^- \in \{0, 1\} \quad \forall k \in K, g \in G_k \quad (97) \]

Then equation (98) is written such that it is inactive if $x_{ik}^Q = 1$. When $x_{ik}^Q = 0$, equations (96,98) force $q_{gk} = q_g$.

\[ \bar{q}_g - q_{gk} \leq M^Q x_{Qg}^+ \quad \forall k \in K, g \in G_k \quad (98) \]

Equation (99) is written such that it is inactive if $x_{ik}^Q = 1$. When $x_{ik}^Q = 0$, equations (96,99) force $q_{gk} = q_g$.

\[ q_{gk} - q_g \leq M^Q x_{Qg}^- \quad \forall k \in K, g \in G_k \quad (99) \]

The following two equations then handle the voltage.

Equation (100) is inactive if $x_{ik}^Q = 0$. When $x_{ik}^Q = 1$, $v_{ikg}$ is bounded below by $v_{ig}$.

\[ v_{ig} - v_{ikg} \leq M^v (1 - x_{Qg}^+) \quad \forall k \in K, g \in G_k \quad (100) \]

For equation (101), it is inactive if $x_{ik}^Q = 0$. When $x_{ik}^Q = 1$, $v_{ikg}$ is bounded above by $v_{ig}$.

\[ v_{ikg} - v_{ig} \leq M^v (1 - x_{Qg}^-) \quad \forall k \in K, g \in G_k \quad (101) \]

Since there exists $M^Q$ and $M^v$ such that for all $M^Q \geq M^Q$ and $M^v \geq M^v$, the equations (96,97,98,99,100,101) are an equivalent reformulation of (93). Table (3.15.3) shows simplified equations (98,99,100,101) for all combinations of binary variables $x_{P}^+$ and $x_{P}^-$. The gray highlighted cells are the inactive constraints under the specified solution for the binary variables.

The first row, when $x_{Qg}^+ = 1$ and $x_{Qg}^- = 1$, the generator has sufficient enough reactive power support to maintain the bus voltage magnitude at the pre-contingency voltage set point, i.e., $v_{ikg} = v_{ig}$; this corresponds to the first state described by (93). The second row represents the second state in (93); with $x_{Qg}^+ = 0$ and $x_{Qg}^- = 1$, $q_{gk} = \bar{q}_g$ due to (61,98) and $v_{ikg}$ must be bounded above by $v_{ig}$, (101), signifying that the generator ran out of reactive...
Table 10: Equations (3.15.3) shows simplified equations (98, 99, 100, 101) under all combinations of binary variables $x_{Q^+}^g$ and $x_{Q^-}^g$ values. All equations in Table (3.15.3) are written \( \forall k \in K, g \in G_k \). $M^Q$ and $M^v$ are sufficiently large such that when appear in the simplified equations (3.15.3) shows simplified equations (98, 99, 100, 101), the constraints will not be binding in any feasible solution; these constraints have been shaded gray in Table (3.15.3).

<table>
<thead>
<tr>
<th>$x_{Q^+}^g$</th>
<th>$x_{Q^-}^g$</th>
<th>(98)</th>
<th>(99)</th>
<th>(100)</th>
<th>(101)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$q_g \geq -M^Q + q_g$</td>
<td>$q_g \leq M^Q + q_g$</td>
<td>$v_{i,k} \geq v_{i,g}$</td>
<td>$v_{i,k} \leq v_{i,g}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$q_g \geq \bar{q}_g$</td>
<td>$q_g \leq M^Q + q_g$</td>
<td>$v_{i,k} \geq -M^v + v_{i,g}$</td>
<td>$v_{i,k} \leq v_{i,g}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$q_g \geq -M^Q + \bar{q}_g$</td>
<td>$q_g \leq q_g$</td>
<td>$v_{i,k} \geq v_{i,g}$</td>
<td>$v_{i,k} \leq M^v + v_{i,g}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$q_g \geq \bar{q}_g$</td>
<td>$q_g \leq q_g$</td>
<td>$v_{i,k} \geq -M^v + v_{i,g}$</td>
<td>$v_{i,k} \leq M^v + v_{i,g}$</td>
</tr>
</tbody>
</table>

power support and could not maintain the post-contingency bus voltage magnitude at the pre-contingency voltage set point. The third row represents the third state in (93); with $x_{Q^+}^g = 1$ and $x_{Q^-}^g = 0$, $q_g = q_g$ due to (61,99) and $v_{i,k}$ must be bounded below by $v_{i,g}$, (100), signifying that the generator could not reduce the post-contingency bus voltage magnitude to the pre-contingency voltage set point. The last row represents an invalid (infeasible) solution for the binary variables; based on the defined equations, it is not possible for both binary variables to take on a value of zero. This infeasibility is directly imposed as can be seen by the resulting inequalities in the last row that simultaneously force the generator’s reactive power production to be below its min capacity and above its max capacity. Entrants could choose to add a combinatorial cut that directly excludes this state, though such a constraint is not necessary to obtain a valid solution.

### 3.16 Optimization Model

The objective is to minimize $c$.

The variables are: $(c, c_g, c^\sigma, c^\sigma_k, t_{gh}, \Delta_k, v_i, \theta_i, b^{CS}_i, \sigma^{P+}_i, \sigma^{P-}_i, \sigma^{Q+}_i, \sigma^{Q-}_i, \sigma^{S}_i, \sigma^S_i, p_{g}, q_{g}, p_{e}, q_{e}, p_{f}, q_{f}, v_{ik}, \theta_{ik}, b^{CS}_{ik}, \sigma^{P+}_{ik}, \sigma^{P-}_{ik}, \sigma^{Q+}_{ik}, \sigma^{Q-}_{ik}, \sigma^{S}_{ik}, \sigma^S_{ik}, p_{gk}, q_{gk}, p_{ek}, q_{ek}, p_{fk}, q_{fk}, q_{d}^e, q_{d}^f)$.  

The constraints are: $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93)$.  

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3.17 Deviations from the Focused Formulation Presented in ARPA-E DE-FOA-0001952

The formulation presented in ARPA-E DE-FOA-0001952 served to give applicants an idea of the structure of the full formulation presented here. In general, this official formulation expands upon the focused formulation presented in ARPA-E DE-FOA-0001952 by including transformers and shunt elements and providing detailed mathematical structure for generator real and reactive power response to contingencies. However, this formulation incorporates some notational deviations from the focused formulation presented in ARPA-E DE-FOA-0001952, which are noted below.

1. In the focused formulation presented in ARPA-E DE-FOA-0001952, slack variables representing soft constraint violations are represented by $s$. In this formulation, these variables are represented by $\sigma$.

2. In the focused formulation presented in ARPA-E DE-FOA-0001952, the objective function explicitly contains cost terms for real power generation in the base case as well as explicitly contains penalties for nodal real and reactive power violations and branch overloading in the base case and contingencies. In this formulation, the objective function includes the cost of real power generation in the base case ($c_g$) as well as dummy variables for the cost of real and reactive nodal violations and branch overloading in the base case ($c_\sigma$) and the cost of real and reactive nodal violations and branch overloading in the contingencies ($c_{\sigma k}$). The dummy variables $c_\sigma$ and $c_{\sigma k}$ are explicitly defined in equations (6) and (7).

3. In the focused formulation presented in ARPA-E DE-FOA-0001952, superscripts $+$ and $-$ are used to represent origin (+) and destination (−) buses. In this formulation, the superscripts $o$ and $d$ are used to denote origin ($o$) and destination ($d$) buses.
A  Input Data Format

A.1  Introduction

The Grid Optimization Competition (GO Competition) Challenge 1 uses a set of four files for the input data of each problem instance. The input files are formatted according to industry standard data file formats used with a popular power system software package. This section gives a description of these formats. This information is intended for use by GO Competition Challenge 1 Entrants. The industry standard data formats include some data elements that are not used by the GO Competition Challenge 1 and do not otherwise need to be understood by Entrants. These elements are not described in detail, so this section cannot be viewed as a full format specification of the industry standard formats. Rather it is a format specification of the aspects of these formats that are relevant to the GO Competition Challenge 1.

A.1.1  Input Data Files

The data for each problem instance are contained in the following input data files:

  case.raw  Power Flow Raw Data File (RAW)
  case.rop  Generator Cost Data File (ROP)
  case.inl  Unit Inertia and Governor Response Data File (INL)
  case.con  Contingency Description Data File (CON)

A.1.2  General Format of Data Files

Each data file is structured as a sequence of one or more data sections in a fixed order. The RAW and ROP data files have multiple sections, the INL and CON files have only one. Each section is a table. Each section table has a number of records. Each record has a number of lines. Typically, each record has exactly one line, and exceptions to this are identified in this document. Each line is a comma-separated list of fields, ending with a newline or endline character after the last field, not a comma. Blank space characters within a line are ignored. Each field can be either a number or a string enclosed in matching quotes. The end of each section is indicated by a line with initial character ‘0’ (numeric zero), although this is not strictly enforced for the last section which may terminate with an end of file. In some cases
a line with an initial character ‘0’ does not mark the end of a section but instead constitutes actual data, and these exceptions are identified in this document. The sections appear in a prescribed order, and empty sections are not skipped. The multi-section data files are terminated by a line with initial character ‘Q’, although this too is not strictly enforced and the file may simply terminate with end of file.

The CON file is somewhat different. The multi-line records are terminated with the characters END in columns 1-3 instead of an initial zero. Like the INL file, it too is just one section and therefore no section end line is required but the last line may be END. It uses string tokens separated by spaces. Keywords are used to structure each record and each line in each record. Commas and quote marks are not needed and therefore are not permitted.

A.1.3 Restrictions on Data File Formats Imposed by the GO Competition Challenge 1

The standard data file formats allow for many alternatives. As much as possible these alternatives are not allowed by the GO Competition Challenge 1.

Many features of the standard format are not used by the GO Competition Challenge 1. These include certain data sections, fields, and flag values. Features that are not used by the GO Competition Challenge 1 are identified as such in this document.

The standard format provides default values for some fields. The GO Competition Challenge 1 does not use default values, so fields whose values are used by the GO Competition Challenge 1 are not left blank. String fields that are not used by the GO Competition Challenge 1 are filled by an empty string (‘’). Numeric fields that are not used by the GO Competition Challenge 1 are filled by a digit 0.

The standard format allows fields to be separated by commas or spaces. The GO Competition Challenge 1 uses only comma separators, not spaces. The Contingency Case Description Data File is an exception, as it uses spaces as token separators.

The standard format allows strings to be enclosed in matching single quotes or in matching double quotes. The GO Competition Challenge 1 uses matching single quotes only. The Contingency Case Description Data File is an exception, as the string tokens do not contain spaces or special characters, and therefore need not be enclosed by quotes, so the GO Competition Challenge 1 requires that no quotes be used here.

The standard format allows for strings to be used instead of numbers in some fields containing names or identifiers of power system elements. For the purposes of the GO Competition Challenge 1, all fields allowed by the standard format to be numbers must be numbers. String fields are identified as such in this document.
A.1.4 Notation Used in Input Data Format Description

Individual data items are annotated in this document to indicate special ways they are to be treated.

* An asterisk (*) is used to indicate that a particular data item is used by the GO Competition Challenge 1.

X A field name is underlined to indicate that the field is a key field in the table being described. Concatenating all key fields in a given table record yields a single key so each record in the table has a unique key. For example, the key fields in the Generator Data section are I and ID, so each generator has a unique value of (I, ID), though two different generators may have the same value of I or the same value of ID.

' ' A field name is enclosed in matching single quotes (' ') to indicate that the field contains string data.

1 ' ' A field name is enclosed in matching single quotes (' ') with a preceding superscript 1 if the field contains short string data, i.e. a string of one or two characters, all upper case letters or digits.

, Field names are separated by commas (,) in the description of the table format to indicate that fields are separated by commas in the table. The Contingency Description Data File uses tokens separated by spaces, not commas, so commas are not used in the description of its format.

Please note that not all data items mentioned in the following sections are used by the GO Competition Challenge 1. Fields which are not used by the Go Competition Challenge 1 are not marked with an asterisk and can be ignored by Entrants. They are listed in this document because they are fields in the input files.

A.1.5 Reading the Data Files

The general structure of the data files, together with the restrictions placed by the GO Competition Challenge 1, imply that the data files can be read easily by CSV packages in a variety of programming languages. We note, for example, the CSV packages used by Python and Julia, Java has equivalent options, and MATLAB has csvread (though this is limited to numeric data only, and entrants would need to work around this restriction). The GO Competition team has written a Python module, data.py, to read and parse these files.
This code is available to entrants at https://gocompetition.energy.gov/challenges/challenge-1/evaluation/. This code is also used by the solution evaluation procedure.

A.2 Power Flow Raw Data File (case.raw)

The sections (data tables) are:

*Case Identification Data
*Bus Data
*Load Data
*Fixed Bus Shunt Data
*Generator Data
*Non-Transformer Branch Data
*Transformer Data
Area Interchange Data
Two-Terminal DC Transmission Line Data
Voltage Source Converter (VSC) DC Transmission Line Data
Transformer Impedance Correction Tables
Multi-Terminal DC Transmission Line Data
Multi-Section Line Grouping Data
Zone Data
Interarea Transfer Data
Owner Data
FACTS Device Data
*Switched Shunt Data
GNE Device Data
Induction Machine Data
A.2.1 Case Identification Data

The Case Identification Data section consists of a single record with exactly 3 lines, in the following format:

IC, *SBASE, REV, XFRRAT, NXFRAT, BASFRQ

‘CASELINE2’
‘CASELINE3’

The Case Identification Data fields are:

IC
*SBASE System MVA base, in MVA.
REV
XFRRAT
NXFRAT
BASFRQ
‘CASELINE2’
‘CASELINE3’

A.2.2 Bus Data

Each bus in the system is described by a bus record with the following format:

*EVHI, *EVLO
The Bus Data fields are:

*I* Bus number Allowable range integer 1 through 999997. Referenced by fields I, J, and BUS in the Load Data (Section A.2.3), Fixed Bus Shunt Data (Section A.2.4), Generator Data (Section A.2.5), Non-Transformer Branch Data (Section A.2.6), Transformer Data (Section A.2.7), Switched Shunt Data (Section A.2.8), and Generator Dispatch Units (Section A.3.1) sections, the Unit Inertia and Governor Response Data File (Section A.4), and the Branch Out-of-Service Event (Section A.5.1), and Generator Out-of-Service Event (section A.5.2), records.

‘NAME’ –
BASKV –
IDE –
*AREA Area number of the area containing bus I. Positive integer.
ZONE –
OWNER –
*VM bus voltage magnitude in pu.
*VA bus voltage angle in degrees.
*NVHI Normal voltage magnitude high limit in pu.
*NVLO Normal voltage magnitude low limit in pu.
*EVHI Emergency voltage magnitude high limit in pu.
*EVLO Emergency voltage magnitude low limit in pu.

A.2.3 Load Data

Each load record has the following format:

*I*, *ID*, *STATUS, AREA, ZONE, *PL, *QL, IP, IQ, YP, YQ, OWNER, SCALE, INTRPT
The Load Data fields are:

*I*  
Bus number. Refers to field I in the Bus Data section (Section [A.2.2]).

*I+ID*  
Identifier. One- or two-character string used to distinguish among multiple loads at a single bus.

*STATUS*  
Status, binary. 1 indicates in service, 0 out of service.

AREA  
–

ZONE  
–

*PL*  
Active power component of constant power load, in MW.

*QL*  
Reactive power component of constant power load, in Mvar.

IP  
–

IQ  
–

YP  
–

YQ  
–

OWNER  
–

SCALE  
–

INTRPT  
–

A.2.4 Fixed Bus Shunt Data

Each fixed shunt is represented by a record in the following format:


The Fixed Bus Shunt Data fields are:

*I*  
Bus number. Refers to field I in the Bus Data section (Section [A.2.2]).

*I+ID*  
Identifier. One- or two-character string used to distinguish among multiple fixed shunts at a single bus.
A.2.5 Generator Data

Each generator is represented by a record in the following format:


The Generator Data fields are:

*I bus number. Refers to field I in the Bus Data section (section A.2.2).
*1:ID' identifier. one- or two-character string used to distinguish among multiple generators at a bus. Referenced by fields ID and GENID in the Generator Dispatch Units section (Section A.3.1), the Unit Inertia and Governor Response Data File (Section A.4), and the Generator Out-of-Service Event record (Section A.5.2).
*PG generator real power output, in MW.
*QG generator reactive power output, in MVar.
*QT maximum generator reactive power output, in Mvar.
*QB minimum generator reactive power output, in Mvar.
VS –
IREG –
MBASE –
ZR –
ZX –
RT –
XT –
GTAP –
*STAT generator status, binary. 1 indicates in service, 0 out of service.
RMPCT –
*RPT maximum generator active power output, MW.
*PB minimum generator active power output, MW.
O1 –
F1 –
O2 –
F2 –
O3 –
F3 –
O4 –
F4 –
WMOD –
WPF –

A.2.6 Non-Transformer Branch Data

Each non-transformer branch is represented by a record in the following format:

LEN, O1, F1, O2, F2, O3, F3, O4, F4

The Non-Transformer Branch Data fields are:

*I  origin bus number. Refers to field I in the Bus Data section (Section A.2.2).
*J  destination bus number. not equal to I. Refers to field I in the Bus Data section (Section A.2.2).
**CKT** circuit identifier. One- or two-character string used to distinguish among multiple branches between I and J. Referenced by field CKT in the Branch Out-of-Service record (Section A.5.1). If the first character of CKT is one of @, *, >, then the branch is treated in a special way. The GO Competition Challenge 1 does not use these special treatments, and CKT should be consistent with this. Among all branches (non-transformer or transformer) between buses I and J (from I to J or from J to I), the value of CKT identifies each branch uniquely.

*R* branch resistance, pu.

*X* branch reactance, pu. Allowable range nonzero real numbers.

*B* total branch charging susceptance, pu.

*RATEA* line rating in the base case, current expressed as MVA at bus base voltage of origin and destination buses. Origin and destination buses of a non-transformer branch must have equal base voltages.

RATEB –

*RATEC* line rating in contingency cases, current expressed as MVA at bus base voltage of origin and destination buses. Origin and destination buses of a non-transformer branch must have equal base voltages.

GI –

BI –

GJ –

BJ –

*ST* status, binary. 1 indicates in service, 0 out of service.

MET –

LEN –

O1 –

F1 –

O2 –

F2 –

O3 –

F3 –

O4 –

F4 –
A.2.7 Transformer Data

Any three-winding transformer can be represented as a configuration of two-winding transformers, and all three-winding transformers present in the problem instances considered by GOComp are represented in this way. Therefore the RAW data file will contain only two-winding transformers, and the data formant of a three-winding transformer is not described here.

Each (two-winding) transformer is represented by a record consisting of four lines of data in the following format:


*R12, *X12, SBASE12
*WINDV1, NOMV1, *ANG1, *RATA1, RATB1, *RATC1, COD1, CONT1, RMA1, RMI1, VMA1, VMI1, NTP1, TAB1, CR1, CX1, CNXA1
*WINDV2, NOMV2

The Transformer Data fields are:

*I
origin (winding 1) bus number. References field I in the Bus Data section (Section A.2.2).

*J
destination (winding 2) bus number. References field I in the Bus Data section (Section A.2.2).

K
–

*1‘CKT’
circuit identifier. one- or two-character string used to distinguish among multiple branches between I and J. Referenced by field CKT in the Branch Out-of-Service record (Section A.5.1).

CW
–

CZ
–

CM
–

*MAG1
transformer magnetizing conductance connected to ground at bus I, in pu on system MVA base
*MAG2 transformer magnetizing susceptance connected to ground at bus I, in pu on system MVA base

NMETR –
‘NAME’ –
*STAT status, binary. 1 indicates in service, 0 out of service.
O1 –
F1 –
O2 –
F2 –
O3 –
F3 –
O4 –
F4 –
‘VECGRP’ –
*R12 transformer resistance, in pu on system MVA base and winding voltage base
*X12 transformer reactance, in pu on system MVA base and winding voltage base
SBASE12

*WINDV1 winding 1 off-nominal turns ratio in pu of winding 1 bus voltage base.
NOMV1 –
*ANG1 winding 1 phase shift angle in degress. allowable range \((-180.0,180.0]\)
*RATA1 winding 1 three-phase power rating, in MVA
RATB1 –
*RATC1 three-phase power rating in contingencies, in MVA
COD1 –
CONT1 –
RMA1 –
RMI1 –
VMA1 –
VMI1 –
NTP1 –
TAB1 –
CR1 –
CX1 –
CNXA1 –
*WINDV2 winding 2 off-nominal turns ratio in pu of winding 2 bus voltage base.
NOMV2 –

A.2.8 Switched Shunt Data

Each switched shunt (i.e. a bus with a set of switched shunt blocks connected to it) is represented by a record in the following format:

*I, MODSW, ADJM, *STAT, VSWHI, VSWLO, SWREM, RMPCT, ‘RMIDNT’,
*B8

The Switched Shunt Data fields are:

*I bus number of bus that this shunt is connected to. References field I in the Bus Data section (Section [A.2.2]).
MODSW –
ADJM –
*STAT status, binary. 1 indicates in service, 0 out of service.
VSWHI –
VSWLO –
SWREM –
RMPCT –
‘RMIDNT’ –
*BINIT Initial switched shunt susceptance, in Mvar at unit voltage
*N1 number of steps in block 1, nonnegative integer
A.3 Generator Cost Data File (case.rop)

Sections:

Modification Code
Bus Voltage Attributes
Adjustable Bus Shunts
Bus Loads
Adjustable Bus Load Tables
*Generator Dispatch Units
*Active Power Dispatch Tables
Generator Reserve Units
A.3.1 Generator Dispatch Units

Each generator dispatch data record is in the following format:

\begin{verbatim}
*BUS, *1GENID', DISP, *DSPTBL
\end{verbatim}

The fields are

- **BUS**: Bus number. Refers to field I in the Bus Data section (Section A.2.2)
- **1GENID**: 1- or 2-character string used to identify a generator at a bus having multiple generators
- **DISP**: –
- **DSPTBL**: Active power dispatch table number, positive integer, refers to field TBL in the Active Power Dispatch Tables section (Section A.3.2)
A.3.2 Active Power Dispatch Tables

Each active power dispatch table record is in the following format:

```
*TBL, PMAX, PMIN, FUELCAST, CTYP, STATUS, *CTBL
```

The fields are:

- **TBL**: Active power dispatch table number, positive integer. Referenced by DSPTBL field in Generator Dispatch Units section (Section A.3.1).
- **PMAX**: –
- **PMIN**: –
- **FUELCAST**: –
- **CTYP**: –
- **STATUS**: –
- **CTBL**: Cost curve table number, positive integer, refers to field LTBL in the Piecewise Linear Cost Curve Tables section (Section A.3.3), referenced by field DSPTBL in the Generator Dispatch Units section (Section A.3.1).

A.3.3 Piecewise Linear Cost Curve Tables

Each piecewise linear cost curve table record is a multiline record in the following format:

```
*LTBL, 'LABEL', *NPAIRS
*X1, *Y1
...
*XN, *YN
```
where the number of lines is $NPAIRS + 1$. The fields are:

- **LTBL**: Piecewise linear cost table number. Positive integer referenced by field CTBL in the Active Power Dispatch Tables section (Section A.3.2).
- **‘LABEL’**: –
- **NPAIRS**: Number of points (X,Y) on the cost function provided in this record, positive integer
- **Xi**: X-coordinate of pair $i$ in 1, . . . , $NPAIRS$, in MW. Note that $Xi$ may take the value 0. This does not indicate a section end line. The value of $NPAIRS$ indicates the number of lines containing $(Xi, Yi)$, and therefore the number of lines in which an initial character ‘0’ does not indicate a section end line.
- **Yi**: Y-coordinate of pair $i$ in 1, . . . , $NPAIRS$, in USD/h

### A.4 Unit Inertia and Governor Response Data File (case.inl)

There is just one section. Each nonzero generator participation factor is listed in a record in the following format:


The fields are:

- $*I$: bus number. Refers to field I in Bus Data section (Section A.2.2).
- $*1:\text{ID}'$: identifier. Refers to field ID in Generator Data section (Section A.2.5).
- H: –
- PMAX: –
- PMIN: –
Governor permanent droop, in pu on MBASE base. Nonnegative real number. Note: In the industry standard data format adopted by the GO Competition, this field is referred to as governor permanent droop, and its value is expressed in the per unit convention. However the GO Competition treats this field as a dimensionless real number and takes it as the participation factor $\alpha_g$ for generator $g = (I, ID)$ in post-contingency redispatch.

A.5 Contingency Description Data File (case.con)

The contingency description data file consists of multiple lines of text. The text consists of string tokens separated by blank space. The string tokens do not contain blank space, so no quotation marks are needed or allowed. This file can be read as CSV with blank space as the separator.

Each string token is either a keyword or a data field. The keywords valid for this data format (with those used by the GO Competition Challenge 1 indicated by *), are:

*CONTINGENCY
*END
DISCONNECT
*OPEN
TRIP
*BRANCH
LINE
*FROM
*BUS
*TO
*CIRCUIT
CKT
CLOSE
THREEWINDING
AT
BLOCK
TWOTERMDC
MULTITERMDC
VSCDC
FACTS
SET
MW
AMPS
KV
GENERATION
LOAD
SHUNT
PERCENT
DISPATCH
MVAR
CHANGE
ALTER
MODIFY
BY
INCREASE
RAISE
DECREASE
REDUCE
MOVE
*REMOVE
MACHINE
INDUCMACHINE
*UNIT
ADD
SWSHUNT
TIE
SINGLE
BREAKER
DOUBLE
BUSDOUBLE
PARALLEL
IN
AREA
ZONE
OWNER
SYSTEM
SUBSYSTEM
3WLOWVOLTAGE
SKIP

Each contingency case is defined in a block structure, which consists of $N + 2$ lines of text, where $N$ is an integer in $\{0, \ldots, 32\}$, in the following format:

Start Line
Event Line 1
...
Event Line N
End Line

The line after a contingency End Line can be either: (1) another contingency Start Line indicating the start of a new contingency, or (2) another End Line indicating the end of the Contingency Case Description Data File.

The format of an End Line is

END
The format of a Start Line is

CONTINGENCY *LABEL

The non-keyword Start Line and fields are:

*LABEL  contingency case identifier, unquoted string

Event lines can take several different formats. the GO Competition Challenge 1 uses only a narrow set of formats, corresponding to the following types of events:

Branch Out-of-Service Event
Generator Out-of-Service Event

The GO Competition Challenge 1 uses only contingencies defined by either exactly one generator outage, or exactly one line outage, or exactly one transformer outage. Therefore each contingency case has at most 1 event line.

A.5.1 Branch Out-of-Service Event

A non-transformer or two-winding transformer branch may be placed out of service with an event line in the following format:

OPEN BRANCH FROM BUS *I TO BUS *J CIRCUIT *1CKT
The branch out-of-service event fields are:

*\( I \) origin bus number. Refers to field I in Bus Data section (Section A.2.2).
*\( J \) destination bus number. Refers to field I in Bus Data section (Section A.2.2).
*\( ^{1} \text{CKT} \) circuit identifier, 1- or 2-character string with only digits and upper case letters. Refers to field CKT in Non-transformer Branch Data (Section A.2.6) and Transformer Data (Section A.2.7) sections.

A.5.2 Generator Out-of-Service Event

A generator can be placed out of service by an event line in the following format:

```plaintext
REMOVE UNIT *\(^{1}\text{ID} \) FROM BUS *\( I \)
```

The generator out-of-service event fields are:

*\( I \) bus number. Refers to field I in Bus Data section (Section A.2.2).
*\( ^{1}\text{ID} \) generator identifier within bus I. Refers to field ID in Generator Data section (Section A.2.5).
B Data Construction Reference Table

Sections (C) and (D) contain detailed instructions on the construction of the data of the competition problem formulation from the input data files. In this section we give a convenient summary of these instructions. For each data item in the competition problem formulation, Table (49) supplies references to the equations and sections of this document containing these instructions. Not all symbols used have an explicit definition provided in this document, and no reference is given for these.

Table 49: Index to detailed parameter definitions

<table>
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</thead>
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<tr>
<td>(E^d_i \subset \mathcal{E})</td>
<td>(E^d_i = {e \in E : i^d_e = i})</td>
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<td>(E^o_i \subset \mathcal{E})</td>
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<td>(E^o_{ik} \subset \mathcal{E})</td>
<td>(E^o_{ik} = E^o_i \cap E_k)</td>
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<td>(F^d_i \subset \mathcal{F})</td>
<td>(F^d_i = {f \in F : i^d_f = i})</td>
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<td>(F^o_i \subset \mathcal{F})</td>
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</tr>
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<td>(F^o_{ik} \subset \mathcal{F})</td>
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<td>(G \subset \mathcal{G})</td>
<td>Equations (104, 139)</td>
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<tr>
<td>(G_i \subset \mathcal{G})</td>
<td>(G_i = {g \in G : i_g = i})</td>
</tr>
<tr>
<td>Symbol</td>
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<td>$i^e \in I$</td>
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<tr>
<td>$i^f \in I$</td>
<td>Equation (151)</td>
</tr>
<tr>
<td>$i^g \in I$</td>
<td>Equation (150)</td>
</tr>
<tr>
<td>$i_g \in I$</td>
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</tr>
<tr>
<td>$id_g$</td>
<td>Equation (132)</td>
</tr>
<tr>
<td>$b_e$</td>
<td>Equation (144)</td>
</tr>
<tr>
<td>$b^{CH}_e$</td>
<td>Equation (145)</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Equation (155)</td>
</tr>
<tr>
<td>$b^M_f$</td>
<td>Equation (153)</td>
</tr>
<tr>
<td>$b^{CS}_i$</td>
<td>Equations (165) (166) (167)</td>
</tr>
<tr>
<td>$b^{CS}_i$</td>
<td>Equations (165) (166) (168)</td>
</tr>
<tr>
<td>$b^{FS}_i$</td>
<td>Equations (127) (129)</td>
</tr>
<tr>
<td>$c^{slack}$</td>
<td>Section (D.3)</td>
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<tr>
<td>$c_{gh}$</td>
<td>Equation (172)</td>
</tr>
<tr>
<td>$g_e$</td>
<td>Equation (143)</td>
</tr>
<tr>
<td>$g_f$</td>
<td>Equation (154)</td>
</tr>
<tr>
<td>$g^M_f$</td>
<td>Equation (152)</td>
</tr>
<tr>
<td>$g^{FS}_i$</td>
<td>Equations (126) (128)</td>
</tr>
<tr>
<td>$M$</td>
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<tr>
<td>$M^P$</td>
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<tr>
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<td>–</td>
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<tr>
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<td>–</td>
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<td>$M^u$</td>
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<tr>
<td>$M^u$</td>
<td>–</td>
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Table 49: Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Defining equations, tables, and text</th>
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</thead>
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<tr>
<td>( \bar{p}_g )</td>
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<tr>
<td>( p_g )</td>
<td>Equation (138)</td>
</tr>
<tr>
<td>( p_{gh} )</td>
<td>Equation (171)</td>
</tr>
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<td>( p^L )</td>
<td>Equations (122, 124)</td>
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<tr>
<td>( \bar{q}_g )</td>
<td>Equation (135)</td>
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<tr>
<td>( q_g )</td>
<td>Equation (136)</td>
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<tr>
<td>( q^L_i )</td>
<td>Equations (123, 125)</td>
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<td>( \overline{R}^K_e )</td>
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<td>Equation (112)</td>
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<tr>
<td>( \bar{s}^K_f )</td>
<td>Equation (159)</td>
</tr>
<tr>
<td>( \bar{v}_i )</td>
<td>Equation (118)</td>
</tr>
<tr>
<td>( v_i )</td>
<td>Equation (119)</td>
</tr>
<tr>
<td>( \bar{v}^K_i )</td>
<td>Equation (120)</td>
</tr>
<tr>
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<td>Table (50) and Equation (183)</td>
</tr>
<tr>
<td>( \lambda^Q_i )</td>
<td>Table (50) and Equation (184)</td>
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<tr>
<td>( \lambda^S_i )</td>
<td>Table (50) and Equation (185)</td>
</tr>
<tr>
<td>( \sigma^S_{en} )</td>
<td>Table (50) and Equation (186)</td>
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<td>( \sigma^S_{fn} )</td>
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<td>( \sigma^S_{ekn} )</td>
<td>Table (50) and Equation (188)</td>
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<tr>
<td>( \sigma^S_{fkn} )</td>
<td>Table (50) and Equation (189)</td>
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<tr>
<td>( \sigma^P^+_{in} )</td>
<td>Table (50) and Equation (190)</td>
</tr>
<tr>
<td>( \sigma^P^-_{in} )</td>
<td>Table (50) and Equation (191)</td>
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<tr>
<td>( \sigma^Q^+_{in} )</td>
<td>Table (50) and Equation (192)</td>
</tr>
<tr>
<td>( \sigma^Q^-_{in} )</td>
<td>Table (50) and Equation (193)</td>
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Table 49: Continued

<table>
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<tr>
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<tr>
<td>$\bar{\sigma}_{ikn}^-$</td>
<td>Table (50) and Equation (195)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{Q+}$</td>
<td>Table (50) and Equation (196)</td>
</tr>
<tr>
<td>$\sigma_{ikn}^{Q-}$</td>
<td>Table (50) and Equation (197)</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>Equation (156)</td>
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</table>
C Construction of Primitive Data from Input Data Files

This appendix explains how to form the data of the GO Competition Challenge 1 problem formulation from the raw data that is read from the input files. The format of the input files is described in section [A]. Names of fields read from the input files are written in upper case Roman font. E.g. SBASE is the name of the field of the Case Identification Data table of the RAW file that contains power units base, in MVA, and the value read from this field is denoted by \( \tilde{s} \) in the model formulation. These field names are specified in the input file description document.

To form the model data, we first specify initial values of the data in subsection (C.1). Then for each table contained in the input files, in subsections (C.2, C.3, C.4, C.5, C.6, C.7, C.8, C.9, C.10, C.11, C.12) we give instructions on modifying these values as each record is read from the table.

C.1 Initial Parameter Values prior to Reading Data

Prior to reading data, we initialize some parameters and sets to certain values. These initial values are updated as data is read. E.g. some sets are initialized as empty sets, and elements are added to them as they are read from the data files. This initialization is specified here:

\[
\mathcal{I} = \{\} \quad (102)
\]

\[
\mathcal{G} = \{\} \quad (103)
\]

\[
\mathcal{G} = \{\} \quad (104)
\]

\[
E = \{\} \quad (106)
\]

\[
\mathcal{F} = \{\} \quad (107)
\]
\[ F = \{\} \quad (108) \]

\[ \mathcal{H} = \{\} \quad (109) \]

\[ \mathcal{K} = \{\} \quad (110) \]

\[ \mathcal{A} = \{\} \quad (111) \]

### C.2 Case Identification Data from RAW

On reading field SBASE from the first record in the Case Identification Data section of the RAW file, set

\[ \tilde{s} = \text{SBASE} \quad (112) \]

### C.3 Bus Data from RAW

For each record in the Bus Data section of the RAW file, read fields I, AREA, VM, VA, NVHI, NVLO, EVHI, EVLO, then set \( i = I, a = \text{AREA} \), and set

\[ \mathcal{I} := \mathcal{I} \cup \{i\} \quad (113) \]

\[ \mathcal{A} := \mathcal{A} \cup \{a\} \quad (114) \]

\[ a_i = a \quad (115) \]

\[ v_i^0 = \text{VM} \quad (116) \]

\[ \theta_i^0 = \text{VA} \quad (117) \]
\[ v_i = \text{NVHI} \quad (118) \]

\[ v_i = \text{NVLO} \quad (119) \]

\[ v_i^K = \text{EVHI} \quad (120) \]

\[ v_i^K = \text{EVLO} \quad (121) \]

### C.4 Load Data from RAW

Initialize the load data by

\[ p_i^L = 0 \quad \text{for all } i \in \mathcal{I} \quad (122) \]

\[ q_i^L = 0 \quad \text{for all } i \in \mathcal{I} \quad (123) \]

For each record in the Load Data section of the RAW file, read fields I, STATUS, PL, QL. If STATUS = 1, then set \( i = \mathcal{I} \), and set

\[ p_i^L := p_i^L + \frac{\text{PL}}{\bar{s}} \quad (124) \]

\[ q_i^L := q_i^L + \frac{\text{QL}}{\bar{s}} \quad (125) \]

### C.5 Fixed Shunt Data from RAW

Initialize the fixed shunt parameters by

\[ g_i^{FS} = 0 \quad \text{for all } i \in \mathcal{I} \quad (126) \]

\[ b_i^{FS} = 0 \quad \text{for all } i \in \mathcal{I} \quad (127) \]
For each record in the Fixed Shunt Data section of the RAW file, read fields I, STATUS, GL, BL. If STATUS = 1, then set $i = I$, and set

$$g_i^{FS} := g_i^{FS} + GL/\tilde{s} \quad (128)$$

$$b_i^{FS} := b_i^{FS} + BL/\tilde{s} \quad (129)$$

C.6 Generator Data from RAW

For each record in the Generator Data section of the RAW file, read fields I, ID, PG, QG, QT, QB, STAT, PT, PB, then set $i = I$, $id = ID$ $g = (i, id)$, and set

$$\mathcal{G} := \mathcal{G} \cup \{g\} \quad (130)$$

$$i_g = i \quad (131)$$

$$id_g = id \quad (132)$$

$$p_g^0 = PG/\tilde{s} \quad (133)$$

$$q_g^0 = QG/\tilde{s} \quad (134)$$

$$q_g = QT/\tilde{s} \quad (135)$$

$$q_g = QB/\tilde{s} \quad (136)$$

$$p_g = PT/\tilde{s} \quad (137)$$

$$p_g = PB/\tilde{s} \quad (138)$$

and if STAT = 1 set

$$G := G \cup \{g\} \quad (139)$$
C.7 Line Data from RAW

For each record in the Nontransformer Branch Data section of the RAW file, read fields I, J, CKT, R, X, B, RATEA, RATEC, ST, then set \( i = I, \quad i' = J, \quad id = CKT \), and \( e = (i, i', id) \), and set

\[
E := E \cup \{e\} \quad (140)
\]

\[
i^o_e = i \quad (141)
\]

\[
i^d_e = i' \quad (142)
\]

\[
g_e = R/(R^2 + X^2) \quad (143)
\]

\[
b_e = -X/(R^2 + X^2) \quad (144)
\]

\[
b^{CH}_e = B \quad (145)
\]

\[
\overline{R}_e = RATEA/\bar{s} \quad (146)
\]

\[
\overline{R}^K_e = RATEC/\bar{s} \quad (147)
\]

and if \( ST = 1 \), set

\[
E := E \cup \{e\} \quad (148)
\]

C.8 Transformer Data from RAW

For each record in the Transformer Data section of the RAW file, read fields I, J, CKT, MAG1, MAG2, STAT, R12, X1-2, WINDV1, ANG1, RATA1, RATC1, WINDV2, then set \( i = I, \quad i' = J, \quad id = CKT \), \( f = (i, i', id) \), and set

\[
F := F \cup \{f\} \quad (149)
\]
\[ i_f^0 = i \quad (150) \]
\[ i_d^a = i' \quad (151) \]
\[ g_f^M = \text{MAG1} \quad (152) \]
\[ b_f^M = \text{MAG2} \quad (153) \]
\[ g_f = \frac{(R12)}{(R12^2 + (X12)^2)} \quad (154) \]
\[ b_f = -\frac{(X12)}{(R12^2 + (X12)^2)} \quad (155) \]
\[ \tau_f = \frac{\text{WINDV1}}{\text{WINDV2}} \quad (156) \]
\[ \theta_f = \text{ANG1} \ast \frac{\pi}{180} \quad (157) \]
\[ \pi_f = \frac{\text{RATA1}}{\bar{s}} \quad (158) \]
\[ \bar{s}_f^K = \frac{\text{RATC1}}{\bar{s}} \quad (159) \]

and if \text{STAT} = 1, set
\[ F := F \cup \{f\} \quad (160) \]
C.9 Switched Shunt Data from RAW

Initialize the switched shunt parameters by

\[ b_{i}^{CS} = 0 \]

for all \( i \in I \) \hspace{1cm} (161)

\[ b_{i}^{CS} = 0 \]

for all \( i \in I \) \hspace{1cm} (162)

\[ b_{i}^{CS0} = 0 \]

for all \( i \in I \) \hspace{1cm} (163)

For each record in the Switched Shunt Data section of the RAW file, read fields I, STAT, BINIT, N1, B1, ..., N8, B8. If STAT = 1, then set \( i = I \), and set

\[ b_{i}^{CS0} = \text{BINIT}/\bar{s} \] \hspace{1cm} (164)

\[ BL_1 = N1 \times B1/\bar{s} \]

\[ \ldots \]

\[ BL_8 = N8 \times B8/\bar{s} \] \hspace{1cm} (165)

\[ NBL = \begin{cases} \max\{l \in \{1, \ldots, 8\} : BL_{l'} \neq 0 \text{ for all } l' \in \{1, \ldots, l\} \} & \text{if } BL_1 \neq 0 \\ 0 & \text{if } BL_1 = 0 \end{cases} \] \hspace{1cm} (166)

\[ b_{i}^{CS} = \sum_{l:1 \leq l \leq NBL} \max\{0, BL_{l}\} \] \hspace{1cm} (167)

\[ b_{i}^{CS} = \sum_{l:1 \leq l \leq NBL} \min\{0, BL_{l}\} \] \hspace{1cm} (168)

C.10 Generator Cost Data from ROP

In the ROP file, for each record in the Generator Dispatch Units section, read fields BUS, GENID, and DSPTBL, then read fields TBL and CTBL from the record in the Active Power Dispatch Tables section with TBL = DSPTBL, then read fields LTBL, NPAIRS, X1,
Y1, ..., XN, YN from the record in the Piecewise Linear Cost Curve Tables section with LTBL = CTBL, then set \( i = I \), \( id = GENID \), \( g = (i, id) \), \( N = NPAIRS \), and set

\[
H_g = \{1, \ldots, N\}
\]

(169)

\[
\mathcal{H} := \mathcal{H} \cup H_g
\]

(170)

define \( p_{gh} \) for \( h \in H_g \) by

\[
p_{g,1} = \frac{X1}{\tilde{s}}
\]

\[
\ldots
\]

\[
p_{g,N} = \frac{XN}{\tilde{s}}
\]

and define \( c_{gh} \) for \( h \in H_g \) by

\[
c_{g,1} = Y1
\]

\[
\ldots
\]

\[
c_{g,N} = YN
\]

(171)

(172)

Note that \( X1, \ldots, XN \) are not reactance values, and \( Y1, \ldots, YN \) are not admittance values. These fields are the X and Y coordinates of points on a curve, as defined in the input file description, Section (A.3.3).

C.11 Participation Factor Data from INL

For each record in the INL file, read fields I, ID, R, and set \( i = I \), \( id = ID \), \( g = (i, id) \), then set

\[
\alpha_g = R
\]

(173)

Note that field R in the INL file is different from the branch resistance fields R in the RAW file and is not a resistance. In the industry standard data format adapted by the GO Competition, the INL field R represents the governor permanent droop of a generator. However, in the GO Competition the INL field R is used as the participation factor for a generator in post-contingency redispatch and is a dimensionless, nonnegative, real number.

C.12 Contingency Data from CON

In the CON file, read the Contingency Case Data Description records one at a time. For each record read, do the following:
Suppose the field LABEL is read from the start line of the record. Let \( k = \text{LABEL} \). Set

\[
K := K \cup \{k\} \tag{174}
\]

Prior to reading the rest of the record, set initial values, which will be updated as the record is read:

\[
E_k = E \tag{175}
\]

\[
F_k = F \tag{176}
\]

\[
G_k = G \tag{177}
\]

The rest of the record contains exactly one contingency event Line and then a contingency record end line. The contingency event is either (1) a Branch Out-of-Service Event or (2) a Generator Out-of-Service Event.

Suppose the Contingency Event is a Branch Out-of-Service Event. Then read fields I, J, CKT and set \( i = I, i' = J, id = \text{CKT} \). It can be assumed that either \((i, i', id) \in E\) or \((i, i', id) \in F\) but not both. If \((i, i', id) = e \in E\) then set

\[
E_k := E_k \setminus \{e\} \tag{178}
\]

If \((i, i', id) = f \in F\) then set

\[
F_k := F_k \setminus \{f\} \tag{179}
\]

If the Contingency Event is a Generator Out-of-Service Event read fields I, ID, set \( i = I, id = \text{ID}, g = (i, id) \) and set

\[
G_k := G_k \setminus \{g\} \tag{180}
\]
D Construction of Further Data

Some data for the GO Competition Challenge 1 model formulation is not directly translated from data read from the input files. In some cases this further data is defined in the symbol reference tables in the formulation, section (2), and no additional definition of this data appears here. In other cases the construction of this further data is more extensive or otherwise needs to be explained outside of the symbol reference tables. The definition this further data is given here.

D.1 Constraint Violation Penalty Parameters

The weight on the base case in the objective is given by

\[ \delta = \frac{1}{2} \]  

(181)

The set of penalty function segments is given by

\[ \mathcal{N} = \{1, 2, 3\} \]  

(182)

To define the constraint violation penalty function parameters, we define the these parameters in the data units convention in table (50).

Table 50: Constraint violation penalty function parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PViolMax_n )</td>
<td>MW</td>
<td>2</td>
<td>50</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( QViolMax_n )</td>
<td>MVar</td>
<td>2</td>
<td>50</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( SViolMax_n )</td>
<td>MVA</td>
<td>2</td>
<td>50</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( PViolCost_n )</td>
<td>USD/MW-h</td>
<td>( 1e3 )</td>
<td>( 5e3 )</td>
<td>( 1e6 )</td>
</tr>
<tr>
<td>( QViolCost_n )</td>
<td>USD/MVar-h</td>
<td>( 1e3 )</td>
<td>( 5e3 )</td>
<td>( 1e6 )</td>
</tr>
<tr>
<td>( SViolCost_n )</td>
<td>USD/MVA-h</td>
<td>( 1e3 )</td>
<td>( 5e3 )</td>
<td>( 1e6 )</td>
</tr>
</tbody>
</table>

Then we convert to the model units convention:

\[ \lambda_n^p = PViolCost_n \hat{s} \]  

(183)
\begin{align*}
\lambda^Q_n &= Q\text{ViolCost}_n \delta \tag{184} \\
\lambda^S_n &= S\text{ViolCost}_n \delta \tag{185} \\
\sigma^S_{en} &= S\text{ViolMax}_n / \delta \tag{186} \\
\sigma^S_{fn} &= S\text{ViolMax}_n / \delta \tag{187} \\
\sigma^S_{ekn} &= S\text{ViolMax}_n / \delta \tag{188} \\
\sigma^S_{fkn} &= S\text{ViolMax}_n / \delta \tag{189} \\
\sigma^P_{in} &= P\text{ViolMax}_n / \delta \tag{190} \\
\sigma^P_{i+} &= P\text{ViolMax}_n / \delta \tag{191} \\
\sigma^P_{i-} &= P\text{ViolMax}_n / \delta \tag{192} \\
\sigma^P_{ikn} &= P\text{ViolMax}_n / \delta \tag{193} \\
\sigma^P_{ik+} &= P\text{ViolMax}_n / \delta \tag{194} \\
\sigma^P_{ik-} &= P\text{ViolMax}_n / \delta \tag{195} \\
\sigma^Q_{ikn} &= Q\text{ViolMax}_n / \delta \tag{196} \\
\sigma^Q_{ik+} &= Q\text{ViolMax}_n / \delta \tag{197} \\
\sigma^Q_{ik-} &= Q\text{ViolMax}_n / \delta \\
\end{align*}

The parameters defined in table (50) appear in the Evaluation code [https://gocompetition.energy.gov/challenges/challenge-1/evaluation](https://gocompetition.energy.gov/challenges/challenge-1/evaluation) used to score solutions. The parameters appear as Python variables base_case.penalty_weight (eqn. (181)), and penalty_block.pow_ x_coef and penalty_block.pow_max (table (50)) in the routine evaluation.py.
D.2 Generator Real Power Contingency Response Data

Every contingency is defined by the outage of exactly one generator, line, or transformer. An area $a$ is contingent in a contingency $k$ if the generator, line or transformer that is out of service in $k$ is connected to a bus in area $a$. That is, the set $A_k$ of contingent areas in contingency $k$ is defined by:

$$A_k = \begin{cases} 
\{a_{ig}\} & \text{if } g \in G \setminus G_k \\
\{a_{ie}, a_{if}\} & \text{if } e \in E \setminus E_k \quad \forall k \in K \\
\{a_{if}, a_{if}\} & \text{if } f \in F \setminus F_k 
\end{cases}$$  \quad (198)

For a given contingency, the generators participating in real power response are those that are active and connected to buses in the contingent areas:

$$G_k^P = \{g \in G_k : a_{ig} \in A_k\} \quad \forall k \in K$$  \quad (199)

D.3 Construction of $c^{\text{slack}}$

A solution violating a hard constraint is assigned a score of $c^{\text{slack}}$ (see section (F.1) and scoring description at https://gocompetition.energy.gov/challenges/challenge-1/scoring/).

For this we construct a solution that is feasible to all hard constraints and set $c^{\text{slack}}$ as the resulting objective value. Here we specify this construction. A python code implementing the construction of this solution is available to entrants at https://github.com/GOCompetition/WorstCase.

For each variable in $v_i$, $p_g$, $q_g$, set the variable to the midpoint of its lower and upper bounds. Set $b^{CS} = 0$. Set $v_{ik} = v_i$, $b^{CS}_{ik} = b^{CS}$. For $g \in G_k$, set $p_{gk} = p_g$, $q_{gk} = q_g$. For $g \in G \setminus G_k$, set $p_{gk} = 0$, $q_{gk} = 0$. Set $\Delta_k = 0$. Evaluate generator costs. Evaluate flows on shunts, lines, and transformers. Evaluate line and transformer flow limit violations. Evaluate bus power imbalances. Note that generator real and reactive power contingency response constraints are satisfied. Apply penalty coefficients to all constraint violations. Compute the objective value as the sum of all generator costs and constraint violation penalties. Set $c^{\text{slack}}$ to this computed objective value.

D.4 Given Base Case Operating Point

A given base case operating point is available to Entrants in the data files. This point is represented by $v_i^0$, $q_i^0$, $b^{CS0}$, $p_g^0$, $q_g^0$. This point can be used as a start point for various algorithms. In general we make no assertion of feasibility or optimality for this point.
E  Solution Output

The solution is written to two solution files, a base case solution file solution1.txt and a contingency solution file solution2.txt (contingencies). The base case solution file contains the values of the primary optimization variables corresponding to the base case. The contingency solution file contains the values of the primary optimization variables corresponding to the contingencies.

E.1 Solution Normalization for Output Files

As in the data files, the values in the solution files are reported in the data unit convention, i.e. per unit voltage magnitude, deg, MW, MVAR, MVAR at 1 p.u. Specifically, the base case solution file contains \( \hat{v}_i \), \( \hat{\theta}_i \), \( \hat{b}^{CS}_i \), \( \hat{p}_g \), \( \hat{q}_g \), and the contingency solution file contains \( \hat{v}_{ik} \), \( \hat{\theta}_{ik} \), \( \hat{b}^{CS}_{ik} \), \( \hat{p}_{gk} \), \( \hat{q}_{gk} \), \( \hat{\Delta}_k \). The conversion from model units to data units is:

\[
\hat{v}_i = v_i \quad \forall i \in I
\]

\[
\hat{\theta}_i = (180/\pi)\theta_i \quad \forall i \in I
\]

\[
\hat{b}^{CS}_i = \tilde{sb}^{CS}_i \quad \forall i \in I
\]

\[
\hat{p}_g = \tilde{sp}_g \quad \forall g \in G
\]

\[
\hat{q}_g = \tilde{sq}_g \quad \forall g \in G
\]

\[
\hat{v}_{ik} = v_{ik} \quad \forall k \in K, i \in I
\]

\[
\hat{\theta}_{ik} = (180/\pi)\theta_{ik} \quad \forall k \in K, i \in I
\]

\[
\hat{b}^{CS}_{ik} = \tilde{sb}^{CS}_{ik} \quad \forall k \in K, i \in I
\]

\[
\hat{p}_{gk} = \tilde{sp}_{gk} \quad \forall k \in K, g \in G
\]

\[
\hat{q}_{gk} = \tilde{sq}_{gk} \quad \forall k \in K, g \in G
\]

\[
\hat{\Delta}_k = \tilde{s}\Delta_k \quad \forall k \in K
\]
E.2 Solution File Format Description

Each solution file contains a sequence of sections. Each section is marked by a start line starting with two hyphen characters (\texttt{--}). After the start line is a table of comma separated values, including a header row followed by a number of data rows. The sections appear in a fixed order specified in this document. The fields in each section data table appear in a fixed order specified in this document. The text in a section start line after \texttt{--} is not read by the solution evaluation code and therefore can be any text. The text in a section table header row is not read by the solution evaluation code and therefore can be any text.

The base case solution file contains the following sections, in order: the bus section, and the generator section.

Each data row of the bus section contains the following fields, in order: bus number $i$ (positive integer), voltage magnitude $\hat{v}_i$, voltage angle $\hat{\theta}_i$, controllable shunt susceptance $\hat{b}_i^{CS}$. Every bus $i \in \mathcal{I}$ should be reported in exactly one row of the bus section.

Each data row of the generator section contains the following fields, in order: bus number $i_g$ (positive integer), unit id $id_g$ (1- or 2-character string), real power output $\hat{p}_g$, reactive power output $\hat{q}_g$. Every generator $g \in \mathcal{G}$ should be reported in exactly one row of the generator section.

For each contingency $k \in \mathcal{K}$, the following for sections appear in the contingency solution file, in order: contingency section, bus section, generator section, delta section. After the delta section of one contingency, the contingency section of the next contingency starts.

The contingency section data table for contingency $k \in \mathcal{K}$ contains exactly one row, with exactly one field, containing $k$ (the string contingency label).

The bus section of contingency $k$ is formatted like the bus section of the base case solution file. Each record contains the following fields, in order: bus number $i$ (positive integer), voltage magnitude $\hat{v}_{ik}$, voltage angle $\hat{\theta}_{ik}$, controllable shunt susceptance $\hat{b}_{ik}^{CS}$. Every bus $i \in \mathcal{I}$ should be reported in exactly one row of the bus section.

The generator section of contingency $k$ is formatted like the generator section of the base case solution file. Each record contains the following fields, in order: bus number $i_g$ (positive integer), unit id $id_g$ (1- or 2-character string), real power output $\hat{p}_{gk}$, reactive power output $\hat{q}_{gk}$. Every generator $g \in \mathcal{G}$ should be reported in exactly one row of the generator section.

The delta section of contingency $k$ contains exactly one row, with exactly one field, $\Delta_k$. 


--bus section
i, v(p.u.), theta(deg), bcs(MVAR at v = 1 p.u.)
1, 1.01, 5.0, 10.0
2, 0.99, 0.0, 0.0
--generator section
i, id, p(MW), q(MVAR)
1, '1', 100.0, 10.0
1, '2', 50.0, 0.0

Figure 1: solution1.txt

E.3 Sample Solution Output Files

A sample solution1.txt file is shown in Figure 1 and a sample solution2.txt file is shown in Figure 2.
--contingency
label
'G-1-1'
--bus section
i, v(p.u.), theta(deg), bcs(MVAR at v = 1 p.u.)
1, 1.02, 5.0, 10.0
2, 0.98, 0.0, 0.0
--generator section
i, id, p(MW), q(MVAR)
1, '1', 150.0, -10.0
1, '2', 0.0, 0.0
--delta section
delta(MW)
50.0
--contingency
label
'L-1-2-1'
--bus section
i, v(p.u.), theta(deg), bcs(MVAR at v = 1 p.u.)
1, 1.005, 0.0, 0.0
2, 0.995, 0.0, 0.0
--generator section
i, id, p(MW), q(MVAR)
1, '1', 130.0, 0.0
1, '2', 40.0, 0.0
--delta section
delta(MW)
-30.0

Figure 2: solution2.txt
F Solution Evaluation

A solution is evaluated by a Python code written for the competition. This code is available to entrants at [https://gocompetition.energy.gov/challenges/challenge-1/evaluation](https://gocompetition.energy.gov/challenges/challenge-1/evaluation). The main procedure is contained in `evaluation.py`, and this module uses the data reading code `data.py`.

F.1 Procedure

The evaluation procedure is:

Read solution. The primary optimization variables and binary variables are read from the solution files in the data unit convention, i.e. as $\hat{v}_i$, $\hat{\theta}_i$, $\hat{b}_i^C$, $\hat{p}_g$, $\hat{q}_g$, $\hat{v}_{ik}$, $\hat{\theta}_{ik}$, $\hat{b}_{ik}^C$, $\hat{p}_{gk}$, $\hat{q}_{gk}$, $\hat{\Delta}_k$. If incorrect formatting prevents reading these values, then the solution is deemed infeasible.

Convert units. The primary optimization variables are converted to the model unit convention, i.e. to $v_i$, $\theta_i$, $b_i^C$, $p_g$, $q_g$, $v_{ik}$, $\theta_{ik}$, $b_{ik}^C$, $p_{gk}$, $q_{gk}$, $\Delta_k$, using (200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210).

Check bounds. Check bounds (32, 33, 34, 35, 36, 37, 58, 59, 60, 61, 62, 63) on $v_i$, $p_g$, $q_g$, $b_i^C$, $v_{ik}$, $p_{gk}$, $q_{gk}$, $b_{ik}^C$. If any violation $> 10^{-4}$ is found, then the solution is deemed infeasible.

Compute flows. Use (38, 39, 40, 41, 42, 43, 44, 45, 64, 65, 66, 67, 68, 69, 70, 71) to compute the values of $p_o^e$, $p_d^e$, $q_o^e$, $q_d^e$, $p_o^f$, $p_d^f$, $q_o^f$, $q_d^f$, $p_o^{ek}$, $q_o^{ek}$, $q_d^{ek}$, $p_o^{fk}$, $q_o^{fk}$, $q_d^{fk}$.

Compute flow bound violations. Use (52, 53, 54, 55, 56, 57, 78, 79, 80, 81, 82, 83) to compute minimal $\sigma^S_o^e$, $\sigma^S_o^f$, $\sigma^S_{ek}$, $\sigma^S_{fk}$.

Compute generator contingency real power output. Compute $p_{gk}$ from (84, 86), overwriting the value provided in the solution file. These computed values of $p_{gk}$ are used in assessing power balance constraints. Here we are using the projection formulation of generator real power response to contingencies, and not the other equivalent formulations.

Compute balance violations. Use (46, 47, 48, 49, 50, 51, 72, 73, 74, 75, 76, 77) to compute minimal $\sigma^+_i$, $\sigma^-_i$, $\sigma^Q_+$, $\sigma^Q_-$, $\sigma^P_+$, $\sigma^P_-$, $\sigma^Q_{ik}$, $\sigma^P_{ik}$.

Check generator reactive power contingency response. Check generator reactive power contingency response using the min/max constraints (94, 95). Here we are using the min/max formulation of generator reactive power response to contingencies, not the other equivalent formulations. If any violation $> 10^{-4}$ is found then the solution is deemed infeasible.

Compute the objective. Use (6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30) to compute minimal $c^o$. Use (7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31) to compute minimal $c^k$. Use (3, 4, 5) and convexity of generator cost functions to compute $t_{gh}$. Use (2) to compute...
Outputs. There are two main outputs: feasibility, which is a boolean value, and the objective \( c \), which is a real value in USD/h. In addition, a detailed output file (\texttt{details.csv}) is written in CSV format, giving the index and value of the maximum violation of each type of constraint in the base case and each contingency.

The detailed output file has a header row, then a row for the base case, then one row for each contingency. Each row is a comma separated list of fields. A description of each field is given in Table (51). The table gives references to the most relevant equations in the formulation, specified by the equation numbers in parentheses. Most of the fields come in (idx, val) pairs, where idx refers to the index or key of the maximum violation in a class of constraints, and val refers to the value of the maximum violation. Most of the fields apply to the individual contingency or base case specified in the current row. The one exception is the 'obj' field, which gives the cumulative objective value for the base case and all contingencies up through the current row.

<table>
<thead>
<tr>
<th>Field Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ctg</td>
<td>contingency label for the current contingency, empty if base case</td>
</tr>
<tr>
<td>infeas</td>
<td>binary indicator of infeasibility for the base case or contingency of the current row - 1 indicates infeasible</td>
</tr>
<tr>
<td>pen</td>
<td>penalty value (USD/h) on soft constraint violations in the base case or current contingency (1, 6, 7)</td>
</tr>
<tr>
<td>cost</td>
<td>generator cost value (USD/h) (base case only), 0 in contingencies (1, 2, 3, 4, 5)</td>
</tr>
<tr>
<td>obj</td>
<td>objective value (USD/h), equal to generator costs plus penalties, cumulative (i.e. base case plus all contingencies through the current row) (1)</td>
</tr>
<tr>
<td>vmax-idx</td>
<td>bus number of maximum violation of bus voltage magnitude upper bounds (32, 58)</td>
</tr>
<tr>
<td>vmax-val</td>
<td>value (p.u.) of maximum violation of bus voltage magnitude upper bounds (32, 58)</td>
</tr>
<tr>
<td>vmin-idx</td>
<td>bus number of maximum violation of bus voltage magnitude lower bounds (32, 58)</td>
</tr>
<tr>
<td>vmin-val</td>
<td>value (p.u.) of maximum violation of bus voltage magnitude lower bounds (32, 58)</td>
</tr>
<tr>
<td>bmax-idx</td>
<td>bus number of maximum violation of switched shunt susceptance upper bounds (37, 63)</td>
</tr>
<tr>
<td>bmax-val</td>
<td>value (p.u.) of maximum violation of switched shunt susceptance upper bounds (37, 63)</td>
</tr>
</tbody>
</table>
Table 51: Continued

<table>
<thead>
<tr>
<th>Field Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bmin-idx</td>
<td>bus number of maximum violation of switched shunt susceptance lower bounds</td>
</tr>
<tr>
<td>bmin-val</td>
<td>value (p.u.) of maximum violation of switched shunt susceptance lower bounds</td>
</tr>
<tr>
<td>pbal-idx</td>
<td>bus number of maximum violation of real power balance contraints (46, 72)</td>
</tr>
<tr>
<td>pbal-val</td>
<td>value (p.u.) of maximum violation of real power balance contraints (46, 72)</td>
</tr>
<tr>
<td>qbal-idx</td>
<td>bus number of maximum violation of reactive power balance contraints (49, 75)</td>
</tr>
<tr>
<td>qbal-val</td>
<td>value (p.u.) of maximum violation of reactive power balance contraints (49, 75)</td>
</tr>
<tr>
<td>pgmax-idx</td>
<td>bus and unit id of maximum violation of generator real power upper bounds</td>
</tr>
<tr>
<td>pgmax-val</td>
<td>value (p.u.) of maximum violation of generator real power upper bounds</td>
</tr>
<tr>
<td>pgmin-idx</td>
<td>bus and unit id of maximum violation of generator real power lower bounds</td>
</tr>
<tr>
<td>pgmin-val</td>
<td>value (p.u.) of maximum violation of generator real power lower bounds</td>
</tr>
<tr>
<td>qgmax-idx</td>
<td>bus and unit id of maximum violation of generator reactive power upper bounds</td>
</tr>
<tr>
<td>qgmax-val</td>
<td>value (p.u.) of maximum violation of generator reactive power upper bounds</td>
</tr>
<tr>
<td>qgmin-idx</td>
<td>bus and unit id of maximum violation of generator reactive power lower bounds</td>
</tr>
<tr>
<td>qgmin-val</td>
<td>value (p.u.) of maximum violation of generator reactive power lower bounds</td>
</tr>
<tr>
<td>qvg1-idx</td>
<td>bus and unit id of maximum violation of generator pv/pq switching constraints of type 1 (undervoltage implies reactive power at max) (94)</td>
</tr>
<tr>
<td>qvg1-val</td>
<td>value (p.u.) of maximum violation of generator pv/pq switching constraints of type 1 (undervoltage implies reactive power at max) (94)</td>
</tr>
<tr>
<td>qvg2-idx</td>
<td>bus and unit id of maximum violation of generator pv/pq switching constraints of type 2 (overvoltage implies reactive power at min) (95)</td>
</tr>
<tr>
<td>qvg2-val</td>
<td>value (p.u.) of maximum violation of generator pv/pq switching constraints of type 2 (overvoltage implies reactive power at min) (95)</td>
</tr>
<tr>
<td>Field Name</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>lineomax-idx</td>
<td>origin destination and circuit id of maximum violation of line origin flow bounds (52, 78)</td>
</tr>
<tr>
<td>lineomax-val</td>
<td>value (p.u.) of maximum violation of line origin flow bounds (52, 78)</td>
</tr>
<tr>
<td>linedmax-idx</td>
<td>origin destination and circuit id of maximum violation of line destination flow bounds (54, 80)</td>
</tr>
<tr>
<td>linedmax-val</td>
<td>value (p.u.) of maximum violation of line destination flow bounds (54, 80)</td>
</tr>
<tr>
<td>xfmromax-idx</td>
<td>origin destination and circuit id of maximum violation of transformer origin flow bounds (55, 83)</td>
</tr>
<tr>
<td>xfmromax-val</td>
<td>value (p.u.) of maximum violation of transformer origin flow bounds (55, 83)</td>
</tr>
<tr>
<td>xfmrdmax-idx</td>
<td>origin destination and circuit id of maximum violation of transformer destination flow bounds (57, 81)</td>
</tr>
<tr>
<td>xfmrdmax-val</td>
<td>value (p.u.) of maximum violation of transformer destination flow bounds (57, 81)</td>
</tr>
</tbody>
</table>

### F.2 Solution Score

Each submitted solution is assigned a score. If no solution is submitted within the required time limit, or the solution files are unreadable, or the solution is infeasible, then the score is $c_{slack}$. If the solution is feasible with objective value $c$, then the score is $\min(c, c_{slack})$. The scores of all the submitted algorithms on all the problem scenarios are the fundamental inputs to the algorithm scoring procedure, which is documented in [https://gocompetition.energy.gov/challenges/challenge-1/scoring/](https://gocompetition.energy.gov/challenges/challenge-1/scoring/)
G  Change Log

Changes to this document after its first posting will be listed here with references to where they appear in the document.

G.1  November 1, 2018

Added table of contents and appendices.

Added \(i_d\) to the symbol reference, in Table (6). This symbol is needed to write the solution files.

Fixed some unit descriptions. Some units were missing time, measured in hours (h). This has been added where needed. This change does not affect the equations of the formulation or of the processing of input data into the data of the formulation. It only affects the clarity of the interpretation of the variables and parameters of the model.

G.2  November 14, 2018

Corrected an error in the interpretation of the STATUS field (and other analogous fields with different names) in the Load, Fixed Shunt, and Switched Shunt sections of the RAW file. Previously the STATUS field was ignored. Now we specify that among all loads, fixed shunts, and switched shunts, only those with STATUS = 1 should be added to the model. The changed text appears in sections (C.4 C.5 C.9)

G.3  November 30, 2018

The interpretation of the R field of the INL file is clarified in sections (A.4 C.11). Specifically, this field is taken as the participation factor of a generator in post-contingency real power output response, which is a dimensionless, nonnegative, real number. The interpretation of R in Table 3 is excluded from its interpretation in the Appendices.

It can be assumed that any (I, ID, CKT) appearing in a record of the CON file appears also in the RAW file as either a line (non-transformer branch) or a transformer but not both. This assumption is posted in section (C.12).
G.4 March 29, 2019

The construction of $b^C_S$ and $b^C_i$ from data files (C.9) was missing an initialization step. This initialization step was added. The effect of this step is that, for any bus $i$, if there are no active switched shunt elements connected to $i$, then we have $b^C_S = 0$ and $b^C_i = 0$. Cf. equations (161, 162).

Documentation of the evaluation code output file (details.csv) was added to section (F.1). Cf. table (51).

An entry for $\sigma_{f_n}$ was added to table (8).

References to $K$ and $I$ were corrected to $\mathcal{K}$ and $\mathcal{I}$.

Multi-indices $ekn$ or $fkn$ were previously sometimes written as $enk$ or $fnk$. All instances of $enk$ (resp. $fnk$) have been corrected to $ekn$ ($fkn$).

The vague term “responsive” has been corrected to “active” at the start of section (3.15).

The symbol $G^P_k$ has been corrected to $G_k$ in the caption of table (3.15.3).

Equation (7) previously used $k$ incorrectly as both a free index and an index of summation. The summation was removed.

The treatment of hard constraint violations in the evaluation procedure is modified, where hard constraints are those constraints that do not use penalty variables. A tolerance of $10^{-4}$ p.u. will be used in determining whether these violations constitute infeasibility. Specifically, any violation of a hard constraint $> 10^{-4}$ p.u. will result in infeasibility, and any violation of a hard constraint $\leq 10^{-4}$ p.u. will be ignored. This is equivalent to truncating the floating point representation of the violation to 4 digits after the decimal point. The solution procedure is modified to reflect this practice. This change is made in the description of the evaluation procedure (appendix F.1) and in the evaluation code itself (https://github.com/GOCompetition/Evaluation).

G.5 April 9, 2019

In section (C.9), no initial value of $b^C_{i0}$ was provided, so this symbol was left undefined for buses $i$ with no fixed shunts present and in service. This has been corrected by specifying that $b^C_{i0}$ takes the initial value 0 prior to reading any switched shunt records.

In section (C.9), for each record for an in service switched shunt, a local variable $NBL$ is defined giving the number of valid blocks. The expression for $NBL$ was not well defined in the case where at least one of the fields $N1$ and $B1$ is 0. This has been corrected by specifying that $NBL = 0$ in this case and that the original definition of $NBL$ applies in the
case where $N_1$ and $B_1$ are both nonzero.