SCOPF Problem Formulation: Challenge 1

Grid Optimization Competition

August 28, 2018

1 Background

This document contains the official formulation that will be used for evaluation in Challenge 1 of the Grid Optimization (GO) Competition. Minor changes may occur within the formulation. Entrants will be notified when a new version is released. Changes are not expected to be of any significance, to cause a change in approach for the Entrants.

This formulation builds upon the formulation published in ARPA-E DE-FOA-0001952. **Entrants will be judged based on the current official formulation posted on the GO Competition website (this document, which is subject to change), not the formulation posted in DE-FOA-0001952.** Entrants are permitted and encouraged to use any alternative problem formulation and modeling convention within their own software (such as convex relaxation, decoupled power flow formulations, current-voltage formulations, etc.) in an attempt to produce an exact or approximate solution to this particular mathematical program. However, the judging of all submitted approaches must conform to the official formulation presented here.

The following mathematical programming problem is a type of a security-constrained (AC based) optimal power flow, or SCOPF. There are many ways to formulate the SCOPF problem; this document presents multiple equivalent options for specified constraints. Entrants are strongly encouraged to study this formulation precisely and to engage with the broader community if anything is not clear (please see the FAQs and forum on the GO Competition website, `https://gocompetition.energy.gov/`).

This SCOPF problem is defined to be an alternating current (AC) formulation, which is based on a bus-branch power system network model and considers security constraints. In general, Entrants are tasked with determining the optimal dispatch and control settings for power generation and grid control equipment in order to minimize the cost of operation, subject to pre- and post-contingency constraints. Feasible solutions must conform to operating standards including, but not limited to: minimum and maximum bus voltage magnitude limits, minimum and maximum real and reactive power generation from each generator, thermal transmission constraints, proxy stability constraints, and constraints to ensure the reliability of the system while responding to unexpected events (i.e., a contingency). Feasible solutions must also include contingency modeling to describe the response of generators and transmission elements to changes from the pre-contingency base case. This formulation allows for real and reactive power nodal violations as well as branch (transmission line and transformer) overloading; however, violations will be penalized in the objective function.

Features that are not modeled within this SCOPF include transformer tap settings, phase angle regulators, various flexible AC transmission system (FACTS) devices, or switchable shunts. Please note that shunts are included but we are not precisely modeling the binary nature of switchable shunts; rather, these are modeled using continuous variables.

Challenge 1 will include power system network models that vary in size of the network flow problem (number of nodes and branches) as well as the number of contingencies. The largest models will reach to at least the size of the largest regional transmission organization within the United States. The problem presented here is a single period problem. The modeling of the pre-contingency base case is a reflection of the first stage of a two-stage mathematical program whereas the post-contingency state represents the second stage. Unit commitment (the commitment/decommitment of generators) is not included in the formulation and fast-start generators are not acknowledged as an available post-contingency response; only units that are online initially may respond. Post-contingency generator response (i.e., activation of contingency reserve) is dictated by an offline policy: generators must follow their pre-defined particpation factor unless they reach an operational limit. As such, this problem does not fully optimize the second-stage recourse decision variables but rather forces them to follow this offline policy. Challenge 2 will likely expand upon Challenge 1 by considering more advanced modeling of power flow equipment (transformers, phase shifters, FACTS), the grid itself (e.g., bus-breaker models), and a more detailed representation of the flexibility available to respond during a contingency (e.g., generator response and modeling of recourse decision variables and limits).

2 Symbol reference

Units, notation, and the general nomenclature are given Tables (1, 2, 3, 4, 5, 6, 7, 8). These tables list sets, indices, subsets and special set elements, data parameters, and variables. As much as possible, the notation follows the following convention. (1) A symbol consists of a main letter with attached notation such as subscripts, superscripts, oversets, and undersets. (2) Two symbols with the same main letter but different attached notation are different symbols. (3) The main letters of symbols generally follow conventions common in the optimal power flow literature and the optimization literature, though other letters are used where there is no established convention.

Units of measurement are listed in Table (1). Attached notation convention is given in Table (2). Main letter convention is given in Table (3). Sets are given in Table (4). Indices are given in Table (5). Subsets and distinguished set elements are given in Table (6). Parameters are given in Table (7) and variables are given in Table (8).

Table 1: Units of measurement

Unit	Description
1	dimensionless. Dimensionless real number quantities are indicated by a unit of 1.
bin	binary. Binary quantities, i.e. taking values in $\{0, 1\}$, are indicated by a unit of bin.
deg	degree. In the physical unit convention, angles are expressed in deg.
USD	US dollar. Cost, penalty, and objective values are expressed in USD.
kV	kilovolt. In the physical unit convention, voltage magnitude is expressed in kV.
MVAR	megavolt-ampere-reactive. In the physical unit convention, reactive power is expressed in MVAR.
MVAR at 1 p.u. voltage	megavolt-ampere-reactive at unit voltage. In the physical unit convention, susceptance is expressed in MVAR at 1 p.u. voltage, meaning the susceptance is such as to yield a reactive power flow equal to the indicated amount when the voltage is equal to 1 p.u.
MW	megawatt. In the physical unit convention, real power is expressed in MW.
MW at 1 p.u. voltage	megawatt at unit voltage. In the physical unit convention, conductance is expressed in MW at 1 p.u. voltage, meaning the conductance is such as to yield a real power flow equal to the indicated amount when the voltage is equal to 1 p.u.
p.u.	per unit. Real and reactive power, voltage magnitude, conductance, susceptance can be expressed in a per unit system under given base values, and the unit is denoted by p.u.
rad	radian. In the per unit convention, angles are expressed in rad.

Table 2: Attached notation

example	description
$w_{\square}$	subscript is used for an index in a set.
$w^{\square}$	superscript is used for description of a symbol.

Table 2: Continued

example	description
$\overline{w}$	overline is used for an upper bound.
$\underline{w}$	underline is used for a lower bound.
$\tilde{w}$	overset $\sim$ indicates a unit base value.
w^0	superscript 0 indicates a value in a given operating point.
w^o	superscript o indicates the origin (from, sending) bus of a branch.
w^d	superscript d indicates the destination (to, receiving) bus of a branch.
w^+	superscript $+$ indicates an upper bound violation, the positive part of an equality constraint violation, or an upper bound slack.
w^-	superscript $-$ indicates a lower bound violation, or a lower bound slack.
$\hat{w}$	overset $\hat{}$ indicates a value reported in a solution file, possibly in different units from the corresponding value in the model.
$\mathcal{W}$	primitive index sets are denoted by a caligraphic capital letter.
$W \subset \mathcal{W}$	subsets are denoted by the same letter as the corresponding primitive index set, in italic.
$w \in \mathcal{W}$	set elements are denoted by the same letter as the corresponding primitive index set, in lower case italic.
w'	set elements are primed to denote different elements of the same set.

Table 3: Main letter convention

main letter	description
a	area
b	susceptance
c	cost, penalty, objective
e	transmission line (arc in a transmission network)
f	transformer (arc in a transmission network)
g	conductance (also generator: when g appears with a subscript, it is a conductance value)
g	generator (also conductance: when g appears as an element of a set, it is a generator)
h	point on a cost function
i	bus (node in a transmission network)
k	contingency

Table 3: Continued

main letter	description
n	segment number for the piecewise linear penalty cost function for violations (constraint relaxations)
p	real power
q	reactive power
R	apparent current rating (current magnitude)
s	apparent power (power magnitude)
t	interpolation coeficient
v	voltage magnitude
x	binary variables
α	participation factor
Δ	post-contingency (adjusted) real power response for generators following
θ	bus voltage angle or transformer shift angle
λ	constraint violation penalty coefficient
σ	violation or slack variable relative to a constraint
τ	tap ratio

Table 4: Primitive index sets

Symbol	Description
$\mathcal{A}$	set of areas
$\mathcal{E}$	set of lines (nontransformer branches)
$\mathcal{F}$	set of transformers (2-winding only)
$\mathcal{G}$	set of generators
$\mathcal{H}$	set of cost function sample points
$\mathcal{I}$	set of buses
$\mathcal{K}$	set of contingencies
$\mathcal{N}$	set of segments in the piecewise linear penalty cost function for violations (constraint relaxations)

Table 5: Indices

Symbol	Description
$a, a' \in \mathcal{A}$	area indices
$e, e' \in \mathcal{E}$	line indices
$f, f' \in \mathcal{F}$	transformer indices
$g, g' \in \mathcal{G}$	generator indices
$h, h' \in \mathcal{H}$	cost function sample point indices
$i, i' \in \mathcal{I}$	bus indices
$k, k' \in \mathcal{K}$	contingency indices
$n, n' \in \mathcal{N}$	segment indices for the piecewise linear penalty cost function for violations (constraint relaxations)

Table 6: Subsets and distinguished set elements

Symbol	Description
$a_i \in \mathcal{A}$	area of bus i
$A_k \subset \mathcal{A}$	contingent areas in contingency k, i.e., areas containing a bus with a connected generator, line, or transformer that goes out of service in contingency k
$E \subset \mathcal{E}$	lines active in the base case
$E_i^d \subset \mathcal{E}$	lines with destination bus i, $E_i^d = \{e \in E : i_e^d = i\}$
$E_i^o \subset \mathcal{E}$	lines with origin bus i, $E_i^o = \{e \in E : i_e^o = i\}$
$E_k \subset \mathcal{E}$	lines active in contingency k
$E_{ik}^d \subset \mathcal{E}$	lines active in contingency k with destination bus i
$E_{ik}^o \subset \mathcal{E}$	lines active in contingency k with origin bus i, $E_{ik}^o = E_i^o \cap E_k$
$F \subset \mathcal{F}$	transformers active in the base case
$F_i^d \subset \mathcal{F}$	transformers with destination bus i, $F_i^d = \{f \in F : i_f^d = i\}$
$F_i^o \subset \mathcal{F}$	transformers with origin bus i, $F_i^o = \{f \in F : i_f^o = i\}$
$F_k \subset \mathcal{F}$	transformers active in contingency k
$F_{ik}^d \subset \mathcal{F}$	transformers active in contingency k with destination bus i, $F_{ik}^d = F_i^d \cap F_k$
$F_{ik}^o \subset \mathcal{F}$	transformers active in contingency k with origin bus i, $F_{ik}^o = F_i^o \cap F_k$
$G \subset \mathcal{G}$	generators active in the base case
$G_i \subset \mathcal{G}$	generators connected to bus i, $G_i = \{g \in G : i_g = i\}$
$G_k \subset \mathcal{G}$	generators active in contingency k
$G_k^P \subset \mathcal{G}$	generators participating in real power response in contingency k

Table 6: Continued

Symbol	Description
$G_{ik} \subset \mathcal{G}$	generators active in contingency k and connected to bus i, $G_{ik} = G_i \cap G_k$
$H_g \subset \mathcal{H}$	cost function sample points for generator g
$I_a \subset \mathcal{I}$	buses in area a, $I_a = \{i \in \mathcal{I} : a_i = a\}$
$i_e^d \in \mathcal{I}$	destination bus of line e
$i_e^o \in \mathcal{I}$	origin bus of line e
$i_f^d \in \mathcal{I}$	destination bus of transformer f
$i_f^o \in \mathcal{I}$	origin bus of transformer f
$i_g \in \mathcal{I}$	bus that generator g is connected to

Table 7: Data parameters

Symbol	Description
b_e	line e series susceptance (p.u.)
b_e^{CH}	line e total charging susceptance (p.u.)
b_f	transformer f series susceptance (p.u.)
b_f^M	transformer f magnetizing susceptance (p.u.)
$\overline{b}_i^{CS}$	bus i maximum controllable shunt susceptance (p.u.)
$\underline{b}_i^{CS}$	bus i minimum controllable shunt susceptance (p.u.)
b_i^{FS}	bus i fixed shunt susceptance (p.u.)
c^{slack}	the objective value of a certain easily constructed feasible solution (USD)
c_{gh}	generation cost of generator g at sample point h (USD)
g_e	line e series conductance (p.u.)
g_f	transformer f series conductance (p.u.)
g_f^M	transformer f magnetizing conductance (p.u.)
g_i^{FS}	bus i fixed shunt conductance (p.u.)
M	a large constant used in the big-M mixed integer programming formulation of generator real power contingency response (p.u.)
$\underline{M}$	a constant such that for any $M \geq \underline{M}$ the mixed integer programming formulation of generator real power response is valid. (p.u.)
M^P	a large constant used in the big-M mixed integer programming formulation of generator real power contingency response (p.u.)

Table 7: Continued

Symbol	Description
$\underline{M}^P$	a constant such that for any $M^P \geq \underline{M}^P$ the mixed integer programming formulation of generator real power response is valid. (p.u.)
M^Q	a large constant used in the big-M mixed integer programming formulation of generator reactive power contingency response (p.u.)
$\underline{M}^Q$	a constant such that for any $M^Q \geq \underline{M}^Q$ the mixed integer programming formulation of generator reactive power response is valid. (p.u.)
M^v	a large constant used in the big-M mixed integer programming formulation of generator reactive power contingency response (p.u.)
$\underline{M}^v$	a constant such that for any $M^v \geq \underline{M}^v$ the mixed integer programming formulation of generator reactive power response is valid. (p.u.)
$\overline{p}_g$	generator g real power maximum (p.u.)
$\underline{p}_g$	generator g real power minimum (p.u.)
p_{gh}	real power output of generator g at sample point h (p.u.)
p_i^L	bus i constant real power load (p.u.)
$\overline{q}_g$	generator g reactive power maximum (p.u.)
$\underline{q}_g$	generator g reactive power minimum (p.u.)
q_i^L	bus i constant reactive power load (p.u.)
$\overline{R}_e$	line e apparent current maximum in base case (p.u.)
$\overline{R}_e^K$	line e apparent current maximum in contingencies
$\tilde{s}$	system power base (MVA)
$\overline{s}_f$	transformer f apparent power maximum in base case (p.u.)
$\overline{s}_f^K$	transformer f apparent power maximum in contingencies (p.u.)
$\tilde{v}_i$	bus i voltage base (kV)
$\overline{v}_i$	bus i voltage magnitude maximum in the base case (p.u.)
$\underline{v}_i$	bus i voltage magnitude minimum in the base case (p.u.)
$\overline{v}_i^K$	bus i voltage magnitude maximum in contingencies (p.u.)
$\underline{v}_i^K$	bus i voltage magnitude minimum in contingencies (p.u.)
α_g	participation factor of generator g in real power contingency response (1)
δ	weight on base case in objective (1)
θ_f	transformer f phase angle (rad)
λ_n^P	objective coefficient on real power power constraint violations for segment n in the piecewise linear penalty cost function (USD/p.u.)

Table 7: Continued

Symbol	Description
λ^Q_n	objective coefficient on real power power constraint violations for segment n in the piecewise linear penalty cost function (USD/p.u.)
λ^S_n	objective coefficient on apparent power constraint violations for segment n in the piecewise linear penalty cost function (USD/p.u.)
$\overline{\sigma}^S_{en}$	segment n upper bound corresponding to the piecewise linear penalty cost function for line e apparent current rating violation (p.u.)
$\overline{\sigma}^S_{fn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for transformer f apparent power rating violation (p.u.)
$\overline{\sigma}^S_{ekn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for line e contingency k apparent current rating violation (p.u.)
$\overline{\sigma}^S_{fkn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for transformer f contingency k apparent power rating violation (p.u.)
$\overline{\sigma}^{P+}_{in}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i real power balance violation positive part, i.e., excess real power flowing into bus i (p.u.)
$\overline{\sigma}^{P-}_{in}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i real power balance violation negative part, i.e., excess real power flowing out of bus i (p.u.)
$\overline{\sigma}^{Q+}_{in}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i reactive power balance violation positive part (p.u.)
$\overline{\sigma}^{Q-}_{in}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i reactive power balance violation negative part (p.u.)
$\overline{\sigma}^{P+}_{ikn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i contingency k real power balance violation positive part (p.u.)
$\overline{\sigma}^{P-}_{ikn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i contingency k real power balance violation negative part (p.u.)
$\overline{\sigma}^{Q+}_{ikn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i contingency k reactive power balance violation positive part (p.u.)
$\overline{\sigma}^{Q-}_{ikn}$	segment n upper bound corresponding to the piecewise linear penalty cost function for bus i contingency k reactive power balance violation negative part (p.u.)
τ_f	transformer f tap ratio (1)

Table 8: Optimization variables

Symbol	Description
b_i^{CS}	bus i controllable shunt susceptance (p.u.)
b_{ik}^{CS}	bus i contingency k controllable shunt susceptance (p.u.)
c	total objective (USD)
c_g	generation cost of generator g (USD)
c^σ	total constraint violation penalty in base case (USD)
c_k^σ	total constraint violation penalty in contingency k (USD)
p_e^d	line e real power from destination bus into line (p.u.)
p_e^o	line e real power from origin bus into line (p.u.)
p_{ek}^d	line e contingency k real power from destination bus into line (p.u.)
p_{ek}^o	line e contingency k real power from origin bus into line (p.u.)
p_f^d	transformer f real power from destination bus into transformer (p.u.)
p_f^o	transformer f real power from origin bus into transformer (p.u.)
p_{fk}^d	transformer f contingency k real power from destination bus into transformer (p.u.)
p_{fk}^o	transformer f contingency k real power from origin bus into transformer (p.u.)
p_g	generator g real power output (p.u.)
p_{gk}	generator g contingency k real power output (p.u.)
q_e^d	line e reactive power from destination bus into line (p.u.)
q_e^o	line e reactive power from origin bus into line (p.u.)
q_{ek}^d	line e contingency k reactive power from destination bus into line (p.u.)
q_{ek}^o	line e contingency k reactive power from origin bus into line (p.u.)
q_f^d	transformer f reactive power from destination bus into transformer (p.u.)
q_f^o	transformer f reactive power from origin bus into transformer (p.u.)
q_{fk}^d	transformer f contingency k reactive power from destination bus into transformer (p.u.)
q_{fk}^o	transformer f contingency k reactive power from origin bus into transformer (p.u.)
q_g	generator g reactive power output (p.u.)
q_{gk}	generator g contingency k reactive power output (p.u.)
t_{gh}	coefficient of sample point h for generator g solution as a point on generation cost function (1)

Table 8: Continued

Symbol	Description
v_i	bus i voltage magnitude (p.u.)
v_{ik}	bus i contingency k voltage magnitude (p.u.)
x_{gk}^{P+}	generator g contingency k binary variable indicating positive slack in upper bound on real power output (bin)
x_{gk}^{P-}	generator g contingency k binary variable indicating positive slack in lower bound on real power output (bin)
x_{gk}^{Q+}	generator g contingency k binary variable indicating positive slack in upper bound on reactive power output (bin)
x_{gk}^{Q-}	generator g contingency k binary variable indicating positive slack in lower bound on reactive power output (bin)
Δ_k	contingency k scale factor on generator participation factors definining generator real power contingency response (p.u.)
θ_i	bus i voltage angle (rad)
θ_{ik}	bus i contingency k voltage angle (rad)
σ_{en}^{S}	line e apparent current rating violation for segment n in the piecewise linear penalty cost function (p.u.)
σ_{ekn}^{S}	line e contingency k apparent current rating violation for segment n in the piecewise linear penalty cost function (p.u.)
σ_{fkn}^{S}	transformer f contingency k apparent power rating violation for segment n in the piecewise linear penalty cost function (p.u.)
σ_{in}^{P+}	bus i real power balance violation positive part, i.e., excess real power flowing into bus i, for segment n in the piecewise linear penalty cost function (p.u.)
σ_{in}^{P-}	bus i real power balance violation negative part, i.e., excess real power flowing out of bus i, for segment n in the piecewise linear penalty cost function (p.u.)
σ_{in}^{Q+}	bus i reactive power balance violation positive part for segment n in the piecewise linear penalty cost function (p.u.)
σ_{in}^{Q-}	bus i reactive power balance violation negative part for segment n in the piecewise linear penalty cost function (p.u.)
σ_{ikn}^{P+}	bus i contingency k real power balance violation positive part for segment n in the piecewise linear penalty cost function (p.u.)
σ_{ikn}^{P-}	bus i contingency k real power balance violation negative part for segment n in the piecewise linear penalty cost function (p.u.)
σ_{ikn}^{Q+}	bus i contingency k reactive power balance violation positive part for segment n in the piecewise linear penalty cost function (p.u.)

Table 8: Continued

Symbol	Description
σ_{ikn}^{Q-}	bus i contingency k reactive power balance violation negative part for segment n in the piecewise linear penalty cost function (p.u.)

3 Model formulation

3.1 Objective definition

The objective (for minimization) is the sum of generator real power output costs in the base case, and a weighted sum of soft constraint violation penalties in the base case and contingencies:

$$c = \sum_{g \in G} c_g + \delta c^{\sigma} + (1 - \delta)/|\mathcal{K}| \sum_{k \in \mathcal{K}} c_k^{\sigma} \tag{1}$$

Generator real power output cost is defined by a cost function given as a set of sample points, modeled by interpolating the cost to the sample points in the cost space and the real power output to the sample points in the real power output space, with common interpolation coefficients. Generator cost interpolation to sample points:

$$c_g = \sum_{h \in H_g} c_{gh} t_{gh} \quad \forall g \in G \tag{2}$$

Generator real power output interpolation to sample points:

$$\sum_{h \in H_g} p_{gh} t_{gh} = p_g \quad \forall g \in G \tag{3}$$

Generator cost interpolation coefficient bounds:

$$0 \leq t_{gh} \quad \forall g \in G, h \in H_g \tag{4}$$

Generator cost interpolation coefficient normalization:

$$\sum_{h \in H_g} t_{gh} = 1 \quad \forall g \in G \tag{5}$$

The total constraint violation penalty in the base case and in contingencies includes penalties on violations of bus real and reactive power balance, penalties on violations of line apparent current ratings, and penalties on violations of transformer apparent power ratings. The penalty is given by a piecewise linear cost function for the violation cost terms - where a small penalty price is applied to minor violations followed by a more stringent penalty price for moderate violations and then an extremely severe penalty for all remaining violations (i.e., it is likely to have 3 segments, indexed by n in this formulation). The first two segments will follow existing industry practices related to constraint relaxations and the chosen penalty prices. The last price will be substantially higher to encourage the approach to not have significant violations. The penalties in the base case and contingencyies are given by:

$$c^{\sigma} = \sum_{n \in N} \left[\lambda_n^P \sum_{i \in I} (\sigma_{in}^{P+} + \sigma_{in}^{P-}) + \lambda_n^Q \sum_{i \in I} (\sigma_{in}^{Q+} + \sigma_{in}^{Q-}) \quad + \lambda_n^S \sum_{e \in E} \sigma_{en}^S + \lambda_n^S \sum_{f \in F} \sigma_{fn}^S \right] \tag{6}$$

$$\begin{aligned} c_k^{\sigma} = \sum_{n \in N} \Bigg[& \lambda_n^P \sum_{k \in K, i \in I} (\sigma_{ikn}^{P+} + \sigma_{ikn}^{P-}) + \lambda_n^Q \sum_{k \in K, i \in I} (\sigma_{ikn}^{Q+} + \sigma_{ikn}^{Q-}) \\ & + \lambda_n^S \sum_{k \in K, e \in E_k} \sigma_{ekn}^S + \lambda_n^S \sum_{k \in K, f \in F_k} \sigma_{fkn}^S \Bigg] \quad \forall k \in \mathcal{K} \end{aligned} \tag{7}$$

Violations in each segment of the piecewise linear cost function are represented by overall violation variables (i.e., not indexed by n) for the remaining constraints in this formulation:

$$\sigma_i^{P+} = \sum_{n \in N} \sigma_{in}^{P+} \quad \forall i \in I \tag{8}$$

$$\sigma_{ik}^{P+} = \sum_{n \in N} \sigma_{ikn}^{P+} \quad \forall i \in I, k \in K \tag{9}$$

$$\sigma_i^{P-} = \sum_{n \in N} \sigma_{in}^{P-} \quad \forall i \in I \tag{10}$$

$$\sigma_{ik}^{P-} = \sum_{n \in N} \sigma_{ikn}^{P-} \quad \forall i \in I, k \in K \tag{11}$$

$$\sigma_i^{Q+} = \sum_{n \in N} \sigma_{in}^{Q+} \quad \forall i \in I \tag{12}$$

$$\sigma_{ik}^{Q+} = \sum_{n \in N} \sigma_{ikn}^{Q+} \quad \forall i \in I, k \in K \tag{13}$$

$$\sigma_i^{Q-} = \sum_{n \in N} \sigma_{in}^{Q-} \quad \forall i \in I \tag{14}$$

$$\sigma_{ik}^{Q-} = \sum_{n \in N} \sigma_{ikn}^{Q-} \quad \forall i \in I, k \in K \tag{15}$$

$$\sigma_{e}^{S} = \sum_{n \in N} \sigma_{en}^{S} \quad \forall e \in E \tag{16}$$

$$\sigma_{ek}^{S} = \sum_{n \in N} \sigma_{enk}^{S} \quad \forall e \in E, k \in K \tag{17}$$

$$\sigma_{f}^{S} = \sum_{n \in N} \sigma_{fn}^{S} \quad \forall f \in F \tag{18}$$

$$\sigma_{fk}^{S} = \sum_{n \in N} \sigma_{fnk}^{S} \quad \forall f \in F, k \in K \tag{19}$$

Bounds on violation variables for each segment are established based on the following equations. Each segment slack variable is non-negative with a lower bound of zero and an upper bound that varies based on the n segment. While the presented formulation here is generic, the GO Competition Challenge 1 will likely set the upper bounds for each segment to be a percentage of the corresponding transmission transfer limit (line current limit or transformer power limit) or a fixed MW and MVAr value for the node balance violations.

$$0 \leq \sigma_{in}^{P+} \leq \overline{\sigma}_{in}^{P+} \quad \forall i \in I, n \in N \tag{20}$$

$$0 \leq \sigma_{ink}^{P+} \leq \overline{\sigma}_{in}^{P+} \quad \forall i \in I, n \in N, k \in K \tag{21}$$

$$0 \leq \sigma_{in}^{P-} \leq \overline{\sigma}_{in}^{P-} \quad \forall i \in I, n \in N \tag{22}$$

$$0 \leq \sigma_{ink}^{P-} \leq \overline{\sigma}_{in}^{P-} \quad \forall i \in I, n \in N, k \in K \tag{23}$$

$$0 \leq \sigma_{in}^{Q+} \leq \overline{\sigma}_{in}^{Q+} \quad \forall i \in I, n \in N \tag{24}$$

$$0 \leq \sigma_{ink}^{Q+} \leq \overline{\sigma}_{in}^{Q+} \quad \forall i \in I, n \in N, k \in K \tag{25}$$

$$0 \leq \sigma_{in}^{Q-} \leq \overline{\sigma}_{in}^{Q-} \quad \forall i \in I, n \in N \tag{26}$$

$$0 \leq \sigma_{ink}^{Q-} \leq \overline{\sigma}_{in}^{Q-} \quad \forall i \in I, n \in N, k \in K \tag{27}$$

$$0 \leq \sigma_{en}^{S+} \leq \overline{\sigma}_{en}^{S+} \quad \forall e \in E, n \in N \tag{28}$$

$$0 \leq \sigma_{enk}^{S+} \leq \overline{\sigma}_{en}^{S+} \quad \forall e \in E, n \in N, k \in K \tag{29}$$

$$0 \leq \sigma_{fn}^{S+} \leq \overline{\sigma}_{fn}^{S+} \quad \forall f \in F, n \in N \tag{30}$$

$$0 \leq \sigma_{fnk}^{S+} \leq \overline{\sigma}_{fn}^{S+} \quad \forall f \in F, n \in N, k \in K \tag{31}$$

3.2 Primary optimization variable bounds in the base case

Bounds on voltage in the base case are given by:

$$\underline{v}_i \leq v_i \leq \overline{v}_i \quad \forall i \in \mathcal{I} \tag{32}$$

Bounds on real power in the base case are given by:

$$\underline{p}_g \leq p_g \leq \overline{p}_g \quad \forall g \in G \tag{33}$$

No real power is produced by generators that are not active in the base case:

$$p_g = 0 \quad \forall g \in \mathcal{G} \setminus G \tag{34}$$

Bounds on reactive power in the base case are given by:

$$\underline{q}_g \leq q_g \leq \overline{q}_g \quad \forall g \in G \tag{35}$$

No reactive power is produced by generators that are not active in the base case:

$$q_g = 0 \quad \forall g \in \mathcal{G} \setminus G \tag{36}$$

Bounds on shunt susceptance in the base case are given by:

$$\underline{b}_i^{CS} \leq b_i^{CS} \leq \overline{b}_i^{CS} \quad \forall i \in \mathcal{I} \tag{37}$$

3.3 Line flow definitions in the base case

Real and reactive power flows into a line at the origin buses in the base case are defined by:

$$\begin{aligned} p^o_e &= g_e v^2_{i^o_e} \\ &\quad + (-g_e \cos(\theta_{i^o_e} - \theta_{i^d_e}) - b_e \sin(\theta_{i^o_e} - \theta_{i^d_e})) v_{i^o_e} v_{i^d_e} \quad \forall e \in E \end{aligned} \tag{38}$$

$$\begin{aligned} q^o_e &= -(b_e + b^{CH}_e/2) v^2_{i^o_e} \\ &\quad + (b_e \cos(\theta_{i^o_e} - \theta_{i^d_e}) - g_e \sin(\theta_{i^o_e} - \theta_{i^d_e})) v_{i^o_e} v_{i^d_e} \quad \forall e \in E \end{aligned} \tag{39}$$

Real and reactive power flows into a line at the destination buses in the base case are defined by:

$$\begin{aligned} p^d_e &= g_e v^2_{i^d_e} \\ &\quad + (-g_e \cos(\theta_{i^d_e} - \theta_{i^o_e}) - b_e \sin(\theta_{i^d_e} - \theta_{i^o_e})) v_{i^o_e} v_{i^d_e} \quad \forall e \in E \end{aligned} \tag{40}$$

$$\begin{aligned} q^d_e &= -(b_e + b^{CH}_e/2) v^2_{i^d_e} \\ &\quad + (b_e \cos(\theta_{i^d_e} - \theta_{i^o_e}) - g_e \sin(\theta_{i^d_e} - \theta_{i^o_e})) v_{i^o_e} v_{i^d_e} \quad \forall e \in E \end{aligned} \tag{41}$$

3.4 Transformer flow definitions in the base case

Real and reactive power flows into a transformer at the origin buses in the base case are defined by:

$$\begin{aligned} p^o_f &= (g_f/\tau^2_f + g^M_f) v^2_{i^o_f} \\ &\quad + (-g_f/\tau_f \cos(\theta_{i^o_f} - \theta_{i^d_f} - \theta_f) - b_f/\tau_f \sin(\theta_{i^o_f} - \theta_{i^d_f} - \theta_f)) v_{i^o_f} v_{i^d_f} \quad \forall f \in F \end{aligned} \tag{42}$$

$$\begin{aligned} q^o_f &= -(b_f/\tau^2_f + b^M_f) v^2_{i^o_f} \\ &\quad + (b_f/\tau_f \cos(\theta_{i^o_f} - \theta_{i^d_f} - \theta_f) - g_f/\tau_f \sin(\theta_{i^o_f} - \theta_{i^d_f} - \theta_f)) v_{i^o_f} v_{i^d_f} \quad \forall f \in F \end{aligned} \tag{43}$$

Real and reactive power flows into a transformer at the destination buses in the base case are defined by:

$$\begin{aligned} p^d_f &= g_f v^2_{i^d_f} \\ &\quad + (-g_f/\tau_f \cos(\theta_{i^d_f} - \theta_{i^o_f} + \theta_f) - b_f/\tau_f \sin(\theta_{i^d_f} - \theta_{i^o_f} + \theta_f)) v_{i^o_f} v_{i^d_f} \quad \forall f \in F \end{aligned} \tag{44}$$

$$\begin{aligned} q^d_f &= -b_f v^2_{i^d_f} \\ &\quad + (b_f/\tau_f \cos(\theta_{i^d_f} - \theta_{i^o_f} + \theta_f) - g_f/\tau_f \sin(\theta_{i^d_f} - \theta_{i^o_f} + \theta_f)) v_{i^o_f} v_{i^d_f} \quad \forall f \in F \end{aligned} \tag{45}$$

3.5 Bus power balance constraints in the base case

Bus real power balance constraints ensure that all real power output from generators at a given bus sum to all real power flows into other grid components at the bus. Nonegative variables σ_i^{P+} and σ_i^{P-} represent the positive and negative parts of the net imbalance. These constraint violation variables (also called slack variables) appear in the objective with penalty coefficients.

$$\begin{aligned} &\sum_{g \in G_i} p_g - p_i^L - g_i^{FS} v_i^2 \\ &\quad - \sum_{e \in E_i^o} p_e^o - \sum_{e \in E_i^d} p_e^d - \sum_{f \in F_i^o} p_f^o - \sum_{f \in F_i^d} p_f^d = \sigma_i^{P+} - \sigma_i^{P-} \quad \forall i \in \mathcal{I} \end{aligned} \tag{46}$$

$$\sigma_i^{P+} \geq 0 \quad \forall i \in \mathcal{I} \tag{47}$$

$$\sigma_i^{P-} \geq 0 \quad \forall i \in \mathcal{I} \tag{48}$$

Bus reactive power balance constraints are similar with soft constraint violation variables σ_i^{Q+} and σ_i^{Q-}:

$$\begin{aligned} &\sum_{g \in G_i} q_g - q_i^L - (-b_i^{FS} - b_i^{CS}) v_i^2 \\ &\quad - \sum_{e \in E_i^o} q_e^o - \sum_{e \in E_i^d} q_e^d - \sum_{f \in F_i^o} q_f^o - \sum_{f \in F_i^d} q_f^d = \sigma_i^{Q+} - \sigma_i^{Q-} \quad \forall i \in \mathcal{I} \end{aligned} \tag{49}$$

$$\sigma_i^{Q+} \geq 0 \quad \forall i \in \mathcal{I} \tag{50}$$

$$\sigma_i^{Q-} \geq 0 \quad \forall i \in \mathcal{I} \tag{51}$$

3.6 Line current ratings in the base case

Line current ratings in the base case at the origin bus, with soft constraint violation variables σ_e^S, are given by:

$$\sqrt{(p_e^o)^2 + (q_e^o)^2} \leq \overline{R}_e v_{i_e^o} + \sigma_e^S \quad \forall e \in E \tag{52}$$

$$\sigma_e^S \geq 0 \quad \forall e \in E \tag{53}$$

Line current ratings in the base case at the destination bus, with soft constraint violation variables σ_e^S, are given by:

$$\sqrt{(p_e^d)^2 + (q_e^d)^2} \leq \overline{R}_e v_{i_e^d} + \sigma_e^S \quad \forall e \in E \tag{54}$$

3.7 Transformer power ratings in the base case

Transformer power ratings in the base case at the origin bus, with soft constraint violation variables σ_f^S, are given by:

$$\sqrt{(p_f^o)^2 + (q_f^o)^2} \leq \overline{s}_f + \sigma_f^S \quad \forall f \in F \tag{55}$$

$$\sigma_f^S \geq 0 \quad \forall f \in F \tag{56}$$

Transformer power ratings in the base case at the destination bus, with soft constraint violation variables σ_f^S, are given by:

$$\sqrt{(p_f^d)^2 + (q_f^d)^2} \leq \overline{s}_f + \sigma_f^S \quad \forall f \in F \tag{57}$$

3.8 Primary optimization variable bounds in contingencies

Bounds on voltage in each contingency are given by:

$$\underline{v}_i^K \leq v_{ik} \leq \overline{v}_i^K \quad \forall k \in \mathcal{K}, i \in \mathcal{I} \tag{58}$$

Bounds on real power generation in each contingency are given by:

$$\underline{p}_g \leq p_{gk} \leq \overline{p}_g \quad \forall k \in \mathcal{K}, g \in G_k \tag{59}$$

No real power is produced by generators that are not active in each contingency:

$$p_{gk} = 0 \quad \forall k \in \mathcal{K}, g \in \mathcal{G} \setminus G_k \tag{60}$$

Bounds on reactive power generation in each contingency are given by:

$$\underline{q}_g \leq q_{gk} \leq \overline{q}_g \quad \forall k \in \mathcal{K}, g \in G_k \tag{61}$$

No reactive power is produced by generators that are not active in each contingency:

$$q_{gk} = 0 \quad \forall k \in \mathcal{K}, g \in \mathcal{G} \setminus G_k \tag{62}$$

Bounds on shunt susceptance in each contingency are given by:

$$\underline{b}_i^{CS} \leq b_{ik}^{CS} \leq \overline{b}_i^{CS} \quad \forall k \in \mathcal{K}, i \in \mathcal{I} \tag{63}$$

3.9 Line flow definitions in contingencies

Real and reactive power flows into a line at the origin bus each contingency are defined by:

$$\begin{aligned} p_{ek}^o &= g_e v_{i_e^o k}^2 \\ &\quad + (-g_e \cos(\theta_{i_e^o k} - \theta_{i_e^d k}) - b_e \sin(\theta_{i_e^o k} - \theta_{i_e^d k})) v_{i_e^o k} v_{i_e^d k} \quad \forall k \in \mathcal{K}, e \in E_k \end{aligned} \tag{64}$$

$$q^o_{ek} = -(b_e + b^{CH}_e/2)v^2_{i^o_e k} + (b_e \cos(\theta_{i^o_e k} - \theta_{i^d_e k}) - g_e \sin(\theta_{i^o_e k} - \theta_{i^d_e k}))v_{i^o_e k}v_{i^d_e k} \quad \forall k \in \mathcal{K}, e \in E_k \tag{65}$$

Real and reactive power flows into a line at the destination bus in each contingency are defined by:

$$p^d_{ek} = g_e v^2_{i^d_e k} + (-g_e \cos(\theta_{i^d_e k} - \theta_{i^o_e k}) - b_e \sin(\theta_{i^d_e k} - \theta_{i^o_e k}))v_{i^o_e k}v_{i^d_e k} \quad \forall k \in \mathcal{K}, e \in E_k \tag{66}$$

$$q^d_{ek} = -(b_e + b^{CH}_e/2)v^2_{i^d_e k} + (b_e \cos(\theta_{i^d_e k} - \theta_{i^o_e k}) - g_e \sin(\theta_{i^d_e k} - \theta_{i^o_e k}))v_{i^o_e k}v_{i^d_e k} \quad \forall k \in \mathcal{K}, e \in E_k \tag{67}$$

3.10 Transformer flow definitions in contingencies

Real and reactive power flows into a transformer at the origin bus in each contingency are defined by:

$$p^o_{fk} = (g_f/\tau^2_f + g^M_f)v^2_{i^o_f k} + (-g_f/\tau_f \cos(\theta_{i^o_f k} - \theta_{i^d_f k} - \theta_f) - b_f/\tau_f \sin(\theta_{i^o_f k} - \theta_{i^d_f k} - \theta_f))v_{i^o_f k}v_{i^d_f k} \quad \forall k \in \mathcal{K}, f \in F_k \tag{68}$$

$$q^o_{fk} = -(b_f/\tau^2_f + b^M_f)v^2_{i^o_f k} + (b_f/\tau_f \cos(\theta_{i^o_f k} - \theta_{i^d_f k} - \theta_f) - g_f/\tau_f \sin(\theta_{i^o_f k} - \theta_{i^d_f k} - \theta_f))v_{i^o_f k}v_{i^d_f k} \quad \forall k \in \mathcal{K}, f \in F_k \tag{69}$$

Real and reactive power flows into a transformer at the destination bus in each contingency are defined by:

$$p^d_{fk} = g_f v^2_{i^d_f k} + (-g_f/\tau_f \cos(\theta_{i^d_f k} - \theta_{i^o_f k} + \theta_f) - b_f/\tau_f \sin(\theta_{i^d_f k} - \theta_{i^o_f k} + \theta_f))v_{i^o_f k}v_{i^d_f k} \quad \forall k \in \mathcal{K}, f \in F_k \tag{70}$$

$$q^d_{fk} = -b_f v^2_{i^d_f k} + (b_f/\tau_f \cos(\theta_{i^d_f k} - \theta_{i^o_f k} + \theta_f) - g_f/\tau_f \sin(\theta_{i^d_f k} - \theta_{i^o_f k} + \theta_f))v_{i^o_f k}v_{i^d_f k} \quad \forall k \in \mathcal{K}, f \in F_k \tag{71}$$

3.11 Bus power balance constraints in contingencies

Bus real power balance constraints in each contingency with soft constraint violation variables σ_{ik}^{P+} and σ_{ik}^{P-} are given by:

$$\begin{aligned}&\sum_{g\in G_{ik}} p_{gk} - p_i^L - g_i^{FS} v_{ik}^2 \\ &\quad - \sum_{e\in E_{ik}^o} p_{ek}^o - \sum_{e\in E_{ik}^d} p_{ek}^d - \sum_{f\in F_{ik}^o} p_{fk}^o - \sum_{f\in F_{ik}^d} p_{fk}^d = \sigma_{ik}^{P+} - \sigma_{ik}^{P-} \quad \forall k\in\mathcal{K}, i\in\mathcal{I}\end{aligned} \tag{72}$$

$$\sigma_{ik}^{P+} \geq 0 \quad \forall k\in\mathcal{K}, i\in\mathcal{I} \tag{73}$$

$$\sigma_{ik}^{P-} \geq 0 \quad \forall k\in\mathcal{K}, i\in\mathcal{I} \tag{74}$$

Bus reactive power balance constraints in each contingency with soft constraint violation variables σ_{ik}^{Q+} and σ_{ik}^{Q-} are given by:

$$\begin{aligned}&\sum_{g\in G_{ik}} q_{gk} - q_i^L - (-b_i^{FS} - b_{ik}^{CS}) v_{ik}^2 \\ &\quad - \sum_{e\in E_{ik}^o} q_{ek}^o - \sum_{e\in E_{ik}^d} q_{ek}^d - \sum_{f\in F_{ik}^o} q_{fk}^o - \sum_{f\in F_{ik}^d} q_{fk}^d = \sigma_{ik}^{Q+} - \sigma_{ik}^{Q-} \quad \forall k\in\mathcal{K}, i\in\mathcal{I}\end{aligned} \tag{75}$$

$$\sigma_{ik}^{Q+} \geq 0 \quad \forall k\in\mathcal{K}, i\in\mathcal{I} \tag{76}$$

$$\sigma_{ik}^{Q-} \geq 0 \quad \forall k\in\mathcal{K}, i\in\mathcal{I} \tag{77}$$

3.12 Line current ratings in contingencies

Line current ratings at the origin bus in each contingency with soft constraint violation variables σ_{ek}^S are modeled by:

$$\sqrt{(p_{ek}^o)^2 + (q_{ek}^o)^2} \leq \overline{R}_e^K v_{i_e^o k} + \sigma_{ek}^S \quad \forall k\in\mathcal{K}, e\in E_k \tag{78}$$

$$\sigma_{ek}^S \geq 0 \quad \forall k\in\mathcal{K}, e\in E_k \tag{79}$$

Line current ratings at the destination bus in each contingency with soft constraint violation variables σ_{ek}^S are modeled by:

$$\sqrt{(p_{ek}^d)^2 + (q_{ek}^d)^2} \leq \overline{R}_e^K v_{i_e^d k} + \sigma_{ek}^S \quad \forall k\in\mathcal{K}, e\in E_k \tag{80}$$

3.13 Transformer power ratings in contingencies

Transformer power ratings in each contingency at the origin bus with soft constraint violation variables $\sigma_f^S k$, are given by:

$$\sqrt{(p_{fk}^o)^2 + (q_{fk}^o)^2} \leq \overline{s}_f^K + \sigma_{fk}^S \quad \forall k \in \mathcal{K}, f \in F_k \tag{81}$$

$$\sigma_{fk}^S \geq 0 \quad \forall k \in \mathcal{K}, f \in F_k \tag{82}$$

Transformer power ratings in each contingency at the destination bus with soft constraint violation variables $\sigma_f^S k$, are given by:

$$\sqrt{(p_{fk}^d)^2 + (q_{fk}^d)^2} \leq \overline{s}_f^K + \sigma_{fk}^S \quad \forall k \in \mathcal{K}, f \in F_k \tag{83}$$

3.14 Generator real power contingency response

The real power output p_{gk} of a generator g in a contingency k is subject to constraints linking it to the base case value p_g.

A generator that is online in a contingency but is not selected to respond to that contingency maintains its real power output from the base case:

$$p_{gk} = p_g \quad \forall k \in \mathcal{K}, g \in G_k \setminus G_k^P \tag{84}$$

A generator that does respond to a given contingency adjusts its real power output according to its predefined (offline) participation factor until it hits an operational bound (min or max capacity). The real power output of a responding generator g in contingency k is $p_g + \alpha_g \Delta_k$, if the generator is following its required participation factor. The actual real power output p_{gk} must be equal to this value unless the generator were to violate its min production or maximum capacity. The generator must operate at its lower bound when the predefined participation factor response would force it to violate its min production level and the generator must operate at its maximum capacity when the predefined participation factor response would force it to violate its upper bound.

Given this conceptual definition of generator real power contingency response, we give several mathematical formulations, including a formulation using logical constraints, a formulation using the projection operator, and a big-M mixed integer programming formulation.

3.14.1 Logical formulation

In this section we formulate generator real power contingency response using a disjunction of linear constraints. This formulation most clearly expresses the constraints that we want to impose on the solution.

$$\left.\begin{array}{lll} \{\underline{p}_g \leq p_{gk} \leq \overline{p}_g & \text{and } p_{gk} = p_g + \alpha_g \Delta_k\} & \text{or} \\ \{p_{gk} = \overline{p}_g & \text{and } p_{gk} \leq p_g + \alpha_g \Delta_k\} & \text{or} \\ \{p_{gk} = \underline{p}_g & \text{and } p_{gk} \geq p_g + \alpha_g \Delta_k\} & \end{array}\right\} \forall k \in \mathcal{K}, g \in G_k^P \tag{85}$$

3.14.2 Projection formulation

In this section we formulate generator real power contingency response using the projection operator Π. This formulation is equivalent to the logic based presentation in the preceeding section but it may not be easy to implement in standard optimization tools.

$$p_{gk} = \Pi_{[\underline{p}_g, \overline{p}_g]}(p_g + \alpha_g \Delta_k) \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{86}$$

Equation (86) is an equivalent reformulation of (85).

3.14.3 Mixed integer programming formulation

In this section, we reformulate the generator real power contingency response using the big-M mixed integer programming (MIP) technique. This approach requires the determination of a large multiplier, the big-M value, and that value must be sufficiently large enough to ensure that the MIP formulation is equivalent to the preceeding two formulations. Such an MIP formulation is easier to implement within standard optimization modeling tools. First let M^P and M denote large positive constants, left unspecified here. Then introduce binary variables x_{gk}^{P+} and x_{gk}^{P-}:

$$x_{gk}^{P+} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{87}$$

$$x_{gk}^{P-} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{88}$$

Then, equation (89) is written such that it is inactive if $x_{gk}^{P+} = 1$. When $x_{gk}^{P+} = 0$, equations (59,89) force $p_{gk} = \overline{p}_g$.

$$\overline{p}_g - p_{gk} \leq M^P x_{gk}^{P+} \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{89}$$

Equation (90) is written such that it is inactive if $x_{gk}^{P-} = 1$. When $x_{gk}^{P-} = 0$, equations (59,90) force $p_{gk} = \underline{p}_g$.

$$p_{gk} - \underline{p}_g \leq M^P x_{gk}^{P-} \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{90}$$

Equation (91) is written such that it is inactive if $x_{gk}^{P+} = 0$. When $x_{gk}^{P+} = 1$, p_{gk} is forced to be equal to or greater than the desired real power response, $p_g + \alpha_g \Delta_k$, dictated by the predefined participation factor, α_g.

$$p_g + \alpha_g \Delta_k - p_{gk} \leq M(1 - x_{gk}^{P+}) \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{91}$$

Equation (92) is written such that it is inactive if $x_{gk}^{P-} = 0$. When $x_{gk}^{P-} = 1$, p_{gk} is forced to be equal to or less than the desired real power response, $p_g + \alpha_g \Delta_k$, dictated by the predefined participation factor, α_g.

$$p_{gk} - p_g - \alpha_g \Delta_k \leq M(1 - x_{gk}^{P-}) \quad \forall k \in \mathcal{K}, g \in G_k^P \tag{92}$$

Since there exists $\underline{M}^P$ and $\underline{M}$ such that for all $M^P \geq \underline{M}^P$ and $M \geq \underline{M}$, equations (87, 88, 89, 90, 91, 92) create an equivalent reformulation of (85). Table (9) shows simplified equations (89, 90, 91, 92) for all combinations of binary variables x_{gk}^{P+} and x_{gk}^{P-}. The gray highlighted cells are the inactive constraints under the specified solution for the binary vriables. The first row, when $x_{gk}^{P+} = 1$ and $x_{gk}^{P-} = 1$, represents the case when the generator precisely follows the predefined participation factor response, which is the first state described by (85). The second row, when $x_{gk}^{P+} = 0$ and $x_{gk}^{P-} = 1$, represents the case when the participation factor response would require the generator to violate its maximum capacity; therefore, the generator operates instead at its max output. This is the second state defined by (85). The third row, when $x_{gk}^{P+} = 1$ and $x_{gk}^{P-} = 0$, represents the case when the participation factor response would require the generator to violate its minimum capacity; therefore, the generator operates instead at its min output. This is the third and final state defined by (85). The last row represents an invalid (infeasible) solution for the binary variables; based on the defined equations, it is not possible for both binary variables to take on a value of zero. This infeasibility is directly imposed as can be seen by the resulting inequalities in the last row that simultaneously force the generator's real power production to be below its min capacity and above its max capacity. Entrants may choose to add a combinatorial cut that directly excludes this state, though such a constraint is not necessary to obtain a valid solution.

x_{gk}^{P+}	x_{gk}^{P-}	(89)	(90)	(91)	(92)
1	1	$p_{gk} \geq -M^P + \overline{p}_g$	$p_{gk} \leq M^P + \underline{p}_g$	$p_{gk} \geq p_g + \alpha_g \Delta_k$	$p_{gk} \leq p_g + \alpha_g \Delta_k$
0	1	$p_{gk} \geq \overline{p}_g$	$p_{gk} \leq M^P + \underline{p}_g$	$p_{gk} \geq -M + p_g + \alpha_g \Delta_k$	$p_{gk} \leq p_g + \alpha_g \Delta_k$
1	0	$p_{gk} \geq -M^P + \overline{p}_g$	$p_{gk} \leq \underline{p}_g$	$p_{gk} \geq p_g + \alpha_g \Delta_k$	$p_{gk} \leq M + p_g + \alpha_g \Delta_k$
0	0	$p_{gk} \geq \overline{p}_g$	$p_{gk} \leq \underline{p}_g$	$p_{gk} \geq -M + p_g + \alpha_g \Delta_k$	$p_{gk} \leq M + p_g + \alpha_g \Delta_k$

Table 9: Equations (89, 90, 91, 92) under all combinations of binary variables x_{gk}^{P+} and x_{gk}^{P-} values. All equations in Table (9) are written $\forall k \in \mathcal{K}, g \in G_k^P$. M^P and M are sufficiently large such that when they appear in the simplified equations (89, 90, 91, 92), the constraints will not be binding in any feasible solution; these constraints have been shaded gray in Table (9).

3.15 Generator reactive power contingency response

A generator g that is responsive during a contingency k tries to maintain the base case (pre-contingency) voltage magnitude at its bus, i_g, by adjusting its reactive power output. The generator will do all that it can to maintain this voltage magnitude. If the bus voltage magnitude drops below its base case magnitude, then the generator reactive power must be at its upper bound, reflecting that it has exhausted its ability to increase voltage. Similarly if the bus voltage magnitude is higher than the base case magnitude, then the generator reactive power must be at its lower bound. In power systems, this is referred to as PV/PQ switching; the generator's bus is providing adequate voltage control then it is acting as a PV bus as there is sufficient reactive power to maintain the voltage. If there is insufficient reactive power to maintain the voltage, the bus is deemed to be a PQ bus in this case and the reactive power injection is fixed (in this special case, the reactive power is fixed to either the generator's reactive power lower or upper bound).

Given this conceptual definition of generator reactive power contingency response, we give several mathematical formulations, including a formulation using logical constraints, a formulation using min and max functions, and a big-M mixed integer programming formulation.

3.15.1 Formulation with logical constraints

In this section, we formulate generator reactive power contingency response using a disjunction of linear constraints.

$$\left.\begin{array}{lll} \{\underline{q}_g \leq q_{gk} \leq \overline{q}_g & \text{and } v_{i_g k} = v_{i_g}\} & \text{or} \\ \{q_{gk} = \overline{q}_g & \text{and } v_{i_g k} \leq v_{i_g}\} & \text{or} \\ \{q_{gk} = \underline{q}_g & \text{and } v_{i_g k} \geq v_{i_g}\} & \end{array}\right\} \forall k \in \mathcal{K}, g \in G_k \tag{93}$$

3.15.2 Formulation with min and max operators

In this section, we formulate generator reactive power contingency response using min and max functions.

$$\min\{\max\{0, v_{i_g} - v_{i_g k}\}, \max\{0, \overline{q}_g - q_{gk}\}\} = 0 \quad \forall k \in \mathcal{K}, g \in G_k \tag{94}$$

$$\min\{\max\{0, v_{i_g k} - v_{i_g}\}, \max\{0, q_{gk} - \underline{q}_g\}\} = 0 \quad \forall k \in \mathcal{K}, g \in G_k \tag{95}$$

Equations (94, 95) are an equivalent reformulation of (93).

3.15.3 Formulation with binary variables

In this section, we formulate generator reactive power contingency response using the big-M mixed integer programming technique. First let M^Q and M^v be large positive constants. Then introduce binary variables x_{gk}^{Q+} and x_{gk}^{Q-}:

$$x_{gk}^{Q+} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k \tag{96}$$

$$x_{gk}^{Q-} \in \{0, 1\} \quad \forall k \in \mathcal{K}, g \in G_k \tag{97}$$

Then equation (98) is written such that it is inactive if $x_{ik}^{Q+} = 1$. When $x_{ik}^{Q+} = 0$, equations (61,98) force $q_{gk} = \overline{q}_g$.

$$\overline{q}_g - q_{gk} \leq M^Q x_{gk}^{Q+} \quad \forall k \in \mathcal{K}, g \in G_k \tag{98}$$

Equation (99) is written such that it is inactive if $x_{ik}^{Q-} = 1$. When $x_{ik}^{Q-} = 0$, equations (61,99) force $q_{gk} = \underline{q}_g$.

$$q_{gk} - \underline{q}_g \leq M^Q x_{gk}^{Q-} \quad \forall k \in \mathcal{K}, g \in G_k \tag{99}$$

The following two equations then handle the voltage.

Equation (100) is inactive if $x_{ik}^{Q+} = 0$. When $x_{ik}^{Q+} = 1$, v_{i_gk} is bounded below by v_{i_g}.

$$v_{i_g} - v_{i_gk} \leq M^v(1 - x_{gk}^{Q+}) \quad \forall k \in \mathcal{K}, g \in G_k \tag{100}$$

For equation (101), it is inactive if $x_{ik}^{Q-} = 0$. When $x_{ik}^{Q-} = 1$, v_{i_gk} is bounded above by v_{i_g}.

$$v_{i_gk} - v_{i_g} \leq M^v(1 - x_{gk}^{Q-}) \quad \forall k \in \mathcal{K}, g \in G_k \tag{101}$$

Since there exists $\underline{M}^Q$ and $\underline{M}^v$ such that for all $M^Q \geq \underline{M}^Q$ and $M^v \geq \underline{M}^v$, the equations (96, 97, 98, 99, 100, 101) are an equivalent reformulation of (93). Table (10) shows simplified equations (98, 99, 100, 101) for all combinations of binary variables x_{gk}^{P+} and x_{gk}^{P-}. The gray highlighted cells are the inactive constraints under the specified solution for the binary variables.

x_{gk}^{Q+}	x_{gk}^{Q-}	(98)	99)	(100)	(101)
1	1	$q_{gk} \geq -M^Q + \overline{q}_g$	$q_{gk} \leq M^Q + \underline{q}_g$	$v_{i_gk} \geq v_{i_g}$	$v_{i_gk} \leq v_{i_g}$
0	1	$q_{gk} \geq \overline{q}_g$	$q_{gk} \leq M^Q + \underline{q}_g$	$v_{i_gk} \geq -M^v + v_{i_g}$	$v_{i_gk} \leq v_{i_g}$
1	0	$q_{gk} \geq -M^Q + \overline{q}_g$	$q_{gk} \leq \underline{q}_g$	$v_{i_gk} \geq v_{i_g}$	$v_{i_gk} \leq M^v + v_{i_g}$
0	0	$q_{gk} \geq \overline{q}_g$	$q_{gk} \leq \underline{q}_g$	$v_{i_gk} \geq -M^v + v_{i_g}$	$v_{i_gk} \leq M^v + v_{i_g}$

Table 10: Equations (10) shows simplified equations (98, 99, 100, 101) under all combinations of binary variables x_{gk}^{Q+} and x_{gk}^{Q-} values. All equations in Table (10) are written $\forall k \in \mathcal{K}, g \in G_k^P$. M^Q and M^v are sufficiently large such that when appear in the simplified equations (10) shows simplified equations (98, 99, 100, 101), the constraints will not be binding in any feasible solution; these constraints have been shaded gray in Table (10).

The first row, when $x_{gk}^{Q+} = 1$ and $x_{gk}^{Q-} = 1$, the generator has sufficient enough reactive power support to maintain the bus voltage magnitude at the pre-contingency voltage set point, i.e., $v_{i_gk} = v_{i_g}$; this corresponds to the first state described by (93). The second row represents the second state in (93); with $x_{gk}^{Q+} = 0$ and $x_{gk}^{Q-} = 1$, $q_{gk} = \overline{q}_g$ due to (61,98) and v_{i_gk} must be bounded above by v_{i_g}, (101), signifying that the generator ran out of reactive power support and could not maintain the post-contingency bus voltage magnitude at the pre-contingency voltage set point. The third row represents the third state in (93); with $x_{gk}^{Q+} = 1$ and $x_{gk}^{Q-} = 0$, $q_{gk} = \underline{q}_g$ due to (61,99) and v_{i_gk} must be bounded below by v_{i_g}, (100), signifying that the generator could not reduce the post-contingency bus voltage magnitude to the pre-contingency voltage set point. The last row represents an invalid (infeasible) solution for the binary variables; based on the defined equations, it is not possible for both binary variables to take on a value of zero. This infeasibility is directly imposed as can be seen by the resulting inequalities in the last row that simultaneously force the generator's reactive power production to be below its min capacity and above its max capacity. Entrants could choose to add a combinatorial cut that directly excludes this state, though such a constraint is not necessary to obtain a valid solution.

3.16 Optimization model

The objective is to minimize c.

The variables are: (c, c_g, c^{σ}, c_k^{σ}, t_{gh}, Δ_k, v_i, θ_i, b_i^{CS}, σ_i^{P+}, σ_i^{P-}, σ_i^{Q+}, σ_i^{Q-}, σ_e^S, σ_f^S, p_g, q_g, p_e^o, p_e^d, q_e^o, q_e^d, p_f^o, p_f^d, q_f^o, q_f^d, v_{ik}, θ_{ik}, b_{ik}^{CS}, σ_{ik}^{P+}, σ_{ik}^{P-}, σ_{ik}^{Q+}, σ_{ik}^{Q-}, σ_{ek}^S, σ_{fk}^S, p_{gk}, q_{gk}, p_{ek}^o, p_{ek}^d, q_{ek}^o, q_{ek}^d, p_{fk}^o, p_{fk}^d, q_{fk}^o, q_{fk}^d).

The constraints are: (1, 2, 3, 4, 5, 6, 7, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 78, 79, 81, 82, 83, 84, 85, 93).

3.17 Deviations from the focused formulation presented in ARPA-E DE-FOA-0001952

The formulation presented in ARPA-E DE-FOA-0001952 served to give applicants an idea of the structure of the full formulation presented here. In general, this official formulation expands upon the focused formulation presented in ARPA-E DE-FOA-0001952 by including transformers and shunt elements and providing detailed mathematical structure for generator real and reactive power response to contingencies. However, this formulation incorporates some notational deviations from the focused formulation presented in ARPA-E DE-FOA-0001952, which are noted below.

1. In the focused formulation presented in ARPA-E DE-FOA-0001952, slack variables representing soft constraint violations are represented by $s_{\square}^{\square}$. In this formulation, these variables are represented by $\sigma_{\square}^{\square}$.

2. In the focused formulation presented in ARPA-E DE-FOA-0001952, the objective function explicitly contains cost terms for real power generation in the base case as well as explicitly contains penalties for nodal real and reactive power violations and branch overloading in the base case and contingencies. In this formulation, the objective function includes the cost of real power generation in the base case (c_g) as well as dummy variables for the cost of real and reactive nodal violations and branch overloading in the base case (c^{σ}) and the cost of real and reactive nodal violations and branch overloading in the contingencies (c_k^{σ}). The dummy variables c^{σ} and c_k^{σ} are explicitly defined in equations (6) and (7).

3. In the focused formulation presented in ARPA-E DE-FOA-0001952, superscripts $+$ and $-$ are used to represent origin $(+)$ and destination $(-)$ buses. In this formulation, the superscripts o and d are used to denote origin (o) and destination (d) buses.