# PSCOPF problem formulation - phase 0

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**Abstract**

Problem specification for the PSCOPF problem in the ARPA-E competition.

## 1 Introductory description

This document presents the formulation of the AC Optimal Power Flow Problem with Preventive Security Constraints (PSCOPF) model to be solved in Phase 0 of the ARPA-E Grid Optimization Competition. This section gives an intuitive introductory description of the problem. The authoritative specification of the problem is given in the reference section.

The overall goal of the problem is to choose generator real power outputs for the near term, say the next 5 to 10 minutes, so that power flow constraints are satisfied, engineering bounds are observed, and in any realistic contingency involving the loss of a piece of power grid equipment, it is possible to recover while following certain rules of generator reaction behavior.

Broadly the features of problem include:

- Electrical buses.
- Bus-connected elements: generators, shunts, loads, lines, transformers.
- Contingencies: loss of a line or a generator or a transformer.
- Complex bus voltage and element current and power flows
- AC power flow equations.
- Engineering bounds on voltages and flows.
- PV/PQ switching: In the base case generator bus voltage magnitude and reactive power output are decision variables variable. In a contingency case the voltage magnitude from the base case acts as a set point, with generator reactive power output adjusting to maintain that set point.
- Stylized AGC: In the base case generator real power output is an independent variable. In each contingency case, generator real power outputs may deviate from base case value according to a prescribed constant of proportionality.
- The objective is to minimize the base case generation cost.

## 1.1 AC Power Flow

Given a set of electrical buses and bus-connected elements, including loads, shunts, generators, lines, 2-winding transformenrs, and 3-winding transformers, the AC power flow problem is to determine values of bus voltages and current flows into each element at each connection bus satisfying physical loaws. The relevant laws are Kirchoffs current law, Ohm's law, and the definition of complex power.

Voltages, currents, and power flows are represented numerically by complex numbers, which may be expressed in rectangular or polar coordinates.

Given current $I$ through a point with voltage $V$, the power is $S = V * \text{conj}(I)$ where conj() denotes the complex conjugate.

Kirchoff's current law states that the sum of all current flows into a bus is 0. This law is found in the PSCOPF formulation in equations (). These equations actually state that the sum of all power flows into a bus is 0, but this is equivalent in view of the definition of complex power.

Ohm's law states that the current from one bus to another equals the admittance $Y$ times the voltage drop:

$$Iorig = Y * (Vorig - Vdest)$$

## 1.2 Optimal power flow

Control variables, such as generator real power outputs and voltage magnitude set points, and state variables, such as generator reactive power outputs and branch power flows, are subject to bounds determined by engineering considerations.

The values of the decision variables should be chosen so as to minimize a measure of system cost. Costs that are considered by this model include generator real power output costs.

## 1.3 Preventive security constraints

The values of the decision variables should be chosen not only so as to meet constraints from the configuration of the electric grid as it is know to be but also as it may be in the event of the loss of a piece of equipment, or a contingency. Together with the base case, i.e. the actual currrent system configuration, these security contingencies for a set of cases.

In each contingency case, bus voltages, branch flows, generator outputs are all defined and must satisfy the power flow equations and engineering bounds as in the base case. Generator real power output costs in the contingency cases do not count toward the cost function that is to be minimized. Only the base case cost is considered.

Real power output of a generator active in a contingency case may differ from base case output, but the differences across all generators must be proportional to prescribed generator participation factors:

$$PGen[g, k] = PGen[g, k0] + PartFactGen[g] * PDelta[k]$$

for all contingencies $k$ and generators $g$ active in $k$.

| | |
|---|---|
| $i \in I$ | branches |
| $j \in J$ | buses |
| $k \in K$ | cases |
| $l \in L$ | generators |
| $m \in M$ | polynomial function exponents |

Table 1: Primitive sets

| | |
|---|---|
| $k_0$ | base case |

Table 2: Single elements

Reactive power output of a generator $g$ active in a contingency case $k$ is determined by the need to maintain the voltage of its connection bus $i$ at the level set in the base case, as much as possible:

$$QGenMin[g] \leq QGen[g,k] \leq QGenMax[g]$$

$$VMagMin[i] \leq VMag[i,k] \leq VMagMax[i]$$

$$\text{if } VMag[i,k] < VMag[i,k0] \text{ then } QGen[g,k] = QGenMax[g]$$

$$\text{if } VMag[i,k] > VMag[i,k0] \text{ then } QGen[g,k] = QGenMin[g]$$

# 2 Additional notes

- Competitors may assume that all lines have nonzero impedance.
- Some problem instances (scenarios) may be infeasible. Some may be feasible even though no solution is known. Some may have a known solution.
- There are no 3-winding transformers.

# 3 Formulation reference

## 3.1 Symbol reference

See tables.

| | |
|---|---|
| $I^0$ | zero-impedance branches |
| $I^*$ | nonzero-impedance branches |
| $K^*$ | contingency cases |

Table 3: Subsets

| | |
|---|---|
| $o(i)$ | origin bus of branch $i$ |
| $d(i)$ | destination bus of branch $i$ |
| $c(l)$ | connection bus of generator $l$ |

Table 4: Element-valued maps

| | |
|---|---|
| $I_j^{\mathrm{orig}}$ | branches with origin bus $j$ |
| $I_j^{\mathrm{dest}}$ | branches with destination bus $j$ |
| $L_j$ | generators connected to bus $j$ |
| $I_k$ | branches active in case $k$ |
| $J_k$ | buses active in case $k$ |
| $L_k$ | generators active in case $k$ |

Table 5: Subset-valued maps

| | |
|---|---|
| $b_i^{\mathrm{c}}$ | charging susceptance on branch $i$ |
| $r_i^{\mathrm{s}}$ | series resistance on branch $i$ |
| $x_i^{\mathrm{s}}$ | series reactance on branch $i$ |
| $g_i^{\mathrm{s}}$ | series conductance on branch $i$ |
| $b_i^{\mathrm{s}}$ | series susceptance on branch $i$ |
| $\tau_i^{\mathrm{tr}}$ | transformer tap ratio on branch $i$ |
| $\theta_i^{\mathrm{tr}}$ | transformer phase shift on branch $i$ |
| $s_i^{\max}$ | maximum apparent power flow on branch $i$ |
| $g_j^{\mathrm{sh}}$ | shunt conductance at bus $j$ |
| $b_j^{\mathrm{sh}}$ | shunt susceptance at bus $j$ |
| $v_j^{\min}$ | minimum voltage magnitude at bus $j$ |
| $v_j^{\max}$ | maximum voltage magnitude at bus $j$ |
| $p_j^{\mathrm{dem}}$ | real power demand from bus $j$ |
| $q_j^{\mathrm{dem}}$ | reactive power demand from bus $j$ |
| $p_l^{\mathrm{gen,min}}$ | minimum real power from generator $l$ |
| $p_l^{\mathrm{gen,max}}$ | maximum real power from generator $l$ |
| $q_l^{\mathrm{gen,min}}$ | minimum reactive power from generator $l$ |
| $q_l^{\mathrm{gen,max}}$ | maximum reactive power from generator $l$ |
| $a_l$ | participation share of generator $l$ |
| $f_{lm}$ | cost coefficient of order $m$ for generator $l$ |

Table 6: Real-valued maps

| | |
|---|---|
| $p^{\text{orig}}_{ik}$ | real power flow at the origin into branch $i$ in case $k$ |
| $q^{\text{orig}}_{ik}$ | reactive power flow at the origin into branch $i$ in case $k$ |
| $p^{\text{dest}}_{ik}$ | real power flow at the destination into branch $i$ in case $k$ |
| $q^{\text{dest}}_{ik}$ | reactive power flow at the destination into branch $i$ in case $k$ |
| $p^{\text{sh}}_{jk}$ | real power flow to the shunt from bus $j$ in case $k$ |
| $q^{\text{sh}}_{jk}$ | reactive power flow to the shunt from bus $j$ in case $k$ |
| $v_{jk}$ | voltage magnitude at bus $j$ in case $k$ |
| $\theta_{jk}$ | voltage angle at bus $j$ in case $k$ |
| $p^{\text{gen}}_{lk}$ | real power generation from generator $l$ in case $k$ |
| $q^{\text{gen}}_{lk}$ | reactive power generation from generator $l$ in case $k$ |
| $z^{\text{cost}}_{lk}$ | generation cost of generator $l$ in case $k$ |
| $p^{\Delta}_{k}$ | pre-recovery real power shortfall in case $k$ |

Table 7: Variables

## 3.2 Relations satisfied by data

The data can be assumed to satisfy certain relations. These are documented here.

Each branch has a unique origin bus:

$$\text{There is a unique origin bus } o(i) \in J \quad \forall i \in I \tag{1}$$

Each branch has a unique destination bus:

$$\text{There is a unique destination bus } d(i) \in J \quad \forall i \in I \tag{2}$$

Each generator has a unique connection bus:

$$\text{There is a unique connection bus } c(l) \in J \quad \forall l \in L \tag{3}$$

There is a unique case designated the base case:

$$\text{There is a unique base case } k_0 \in K \tag{4}$$

The set of branches with impedance 0 is identified:

$$I^0 = \{i \in I : r^{\text{s}}_i = 0 \text{ and } x^{\text{s}}_i = 0\} \tag{5}$$

The set of branches with nonzero impedance is identified:

$$I^* = \{i \in I : r^{\text{s}}_i \neq 0 \text{ or } x^{\text{s}}_i \neq 0\} \tag{6}$$

All cases other than the base case are designated contingency cases:

$$K^* = \{k \in K : k \neq k_0\} \tag{7}$$

The set of branches originating at a given bus is identified:

$$I^{\text{orig}}_j = \{i \in I : j = o(i)\} \quad \forall j \in J \tag{8}$$

The set of branches teminating at a given bus is identified:

$$I_j^{\text{dest}} = \{i \in I : j = d(i)\} \quad \forall j \in J \tag{9}$$

The set of generators connected to a given bus is identified:

$$L_j = \{l \in L : j = c(l)\} \quad \forall j \in J \tag{10}$$

In each case some buses, branches, and generators are active, and some are inactive. The set of branches active in a given case is identified:

$$I_k = \{i \in I : i \text{ is active in case } k\} \quad \forall k \in K \tag{11}$$

The set of buses active in a given case is identified:

$$J_k = \{j \in J : j \text{ is active in case } k\} \quad \forall k \in K \tag{12}$$

The set of generators active in a given case is identified:

$$L_k = \{l \in L : l \text{ is active in case } k\} \quad \forall k \in K \tag{13}$$

At least one generator is active in each case:

$$L_k \neq \emptyset \quad \forall k \in K \tag{14}$$

At least one generator participating in AGC is active in each case:

$$\{l \in L_k : a_l \neq 0\} \neq \emptyset \quad \forall k \in K \tag{15}$$

Any generator active in any contingency case is active in the base case:

$$L_k \subset L_{k_0} \quad \forall k \in K^* \tag{16}$$

The generator real power cost functions are quadratic:

$$M = \{0, 1, 2\} \tag{17}$$

The cost functions are convex:

$$f_{l2} \geq 0 \quad \forall l \in L \tag{18}$$

Voltage magnitude bounds are self consistent:

$$v_j^{\min} \leq v_j^{\max} \quad \forall j \in J \tag{19}$$

Generator real power output bounds are self consistent:

$$p_l^{\text{gen,min}} \leq p_l^{\text{gen,max}} \quad \forall l \in L \tag{20}$$

Generator reactive power output bounds are self consistent:

$$q_l^{\text{gen,min}} \leq q_l^{\text{gen,max}} \quad \forall l \in L \tag{21}$$

Bounds on branch apparent power flow are self consistent:

$$s_i^{\max} \geq 0 \quad \forall i \in I \tag{22}$$

The tap ratio of each branch is positive:

$$\tau_i^{\mathrm{tr}} > 0 \quad \forall i \in I \tag{23}$$

Bus voltage magnitude bounds are positive:

$$v_j^{\min} > 0 \quad \forall j \in J \tag{24}$$

Real power generation bounds are nonnegative:

$$p_l^{\mathrm{gen,min}} \geq 0 \quad \forall l \in L \tag{25}$$

Branch conductance is defined with respect to impedance:

$$g_i^{\mathrm{s}} = r_i^{\mathrm{s}}/((r_i^{\mathrm{s}})^2 + (x_i^{\mathrm{s}})^2) \quad \forall i \in I^* \tag{26}$$

Branch susceptance is defined with respect to impedance:

$$b_i^{\mathrm{s}} = -x_i^{\mathrm{s}}/((r_i^{\mathrm{s}})^2 + (x_i^{\mathrm{s}})^2) \quad \forall i \in I^* \tag{27}$$

## 3.3 Model objective

The model is posed as a minimization problem. The objective to be minimized is the total generation cost in the base case $k = k_0$, given by:

$$\sum_{l \in L_{k_0}} z_{lk_0}^{\mathrm{cost}} \tag{28}$$

## 3.4 Model constraints

The decision variables must satisfy a number of constraints. These are documented here.

Cost function definition:

$$z_{lk} = \sum_{m \in M} f_{lm} (p_{lk}^{\mathrm{gen}})^m \quad \forall k \in K, l \in L_k \tag{29}$$

Bus voltage magnitudes satisfy prescribed bounds:

$$v_j^{\min} \leq v_{jk} \leq v_j^{\max} \quad \forall k \in K, j \in J_k \tag{30}$$

Generator real power bounds:

$$p_l^{\mathrm{gen,min}} \leq p_{lk}^{\mathrm{gen}} \leq p_l^{\mathrm{gen,max}} \quad \forall k \in K, l \in L_k \tag{31}$$

Generator reactive power bounds:

$$q_l^{\mathrm{gen,min}} \leq q_{lk}^{\mathrm{gen}} \leq q_l^{\mathrm{gen,max}} \quad \forall k \in K, l \in L_k \tag{32}$$

Real power flow to the shunt at each bus is defined by the following constraint:

$$p_{jk}^{\mathrm{sh}} = g_j^{\mathrm{sh}} v_{jk}^2 \quad \forall k \in K, j \in J_k \tag{33}$$

Bus shunt reactive power flow definition:

$$q_{jk}^{\mathrm{sh}} = -b_j^{\mathrm{sh}} v_{jk}^2 \quad \forall k \in K, j \in J_k \tag{34}$$

According to Ohm's law and the definition of complex power, real power flow into a branch at the origin bus is a function of voltages at the origin and destination buses, if the branch has nonzero impedance:

$$\begin{aligned} p_{ik}^{\mathrm{orig}} =& (g_i^{\mathrm{s}}/(\tau_i^{\mathrm{tr}})^2)v_{o(i)k}^2 \\ & - (g_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\cos(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & + (b_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\sin(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & \forall k \in K, i \in I^* \cap I_k \end{aligned} \tag{35}$$

Nonzero-impedance branch reactive power origin flow definition:

$$\begin{aligned} q_{ik}^{\mathrm{orig}} =& ((-b_i^{\mathrm{s}} - b_i^{\mathrm{c}}/2)/(\tau_i^{\mathrm{tr}})^2)v_{o(i)k}^2 \\ & + (b_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\cos(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & + (g_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\sin(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & \forall k \in K, i \in I^* \cap I_k \end{aligned} \tag{36}$$

Nonzero-impedance branch real power destination flow definition:

$$\begin{aligned} p_{ik}^{\mathrm{dest}} =& g_i^{\mathrm{s}}v_{d(i)k}^2 \\ & - (g_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\cos(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & - (b_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\sin(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & \forall k \in K, i \in I^* \cap I_k \end{aligned} \tag{37}$$

Nonzero-impedance branch reactive power destination flow definition:

$$\begin{aligned} q_{ik}^{\mathrm{dest}} =& (-b_i^{\mathrm{s}} - b_i^{\mathrm{c}}/2)v_{d(i)k}^2 \\ & + (b_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\cos(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & - (g_i^{\mathrm{s}}/\tau_i^{\mathrm{tr}})v_{o(i)k}v_{d(i)k}\sin(\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}}) \\ & \forall k \in K, i \in I^* \cap I_k \end{aligned} \tag{38}$$

According to Krirchoff's voltage law, the voltage drop across a branch with impedance 0 is given by a fixed scalar multiple. Zero-impedance branch voltage magnitude constraint:

$$v_{d(i)k} - v_{o(i)k}/\tau_i^{\mathrm{tr}} = 0 \quad \forall k \in K, i \in I^0 \cap I_k \tag{39}$$

Zero-impedance branch voltage angle constraint:

$$\theta_{d(i)k} - \theta_{o(i)k} + \theta_i^{\mathrm{tr}} = 0 \quad \forall k \in K, i \in I^0 \cap I_k \tag{40}$$

According to Kirchoff's current law, the power flow into the origin and destination of a branch with impedance 0 satisfy simple conservation constraints. Zero-impedance branch real power constraint:

$$p_{ik}^{\mathrm{orig}} + p_{ik}^{\mathrm{dest}} = 0 \quad \forall k \in K, i \in I^0 \cap I_k \tag{41}$$

Zero-impedance branch reactive power constraint:

$$q_{ik}^{\mathrm{orig}} + q_{ik}^{\mathrm{dest}} + b^{\mathrm{c}}v_{d(i)k}^2 = 0 \quad \forall k \in K, i \in I^0 \cap I_k \tag{42}$$

The apparent power flow on a branch at both origin and destination must satisfy prescribed bounds. Branch origin apparent flow bound:

$$\sqrt{(p_{ik}^{\mathrm{orig}})^2 + (q_{ik}^{\mathrm{orig}})^2} \le s_i^{\max} \quad \forall k \in K, i \in I_k \tag{43}$$

Branch destination apparent flow bound:

$$\sqrt{(p_{ik}^{\text{dest}})^2 + (q_{ik}^{\text{dest}})^2} \leq s_i^{\max} \quad \forall k \in K, i \in I_k \tag{44}$$

According to Kirchoff's current law the sum of all currents into a bus is 0. Taking complex conjugates and multiplying by voltage, we find that the sum of all power flows into a bus is 0. This is formulated as balance constraints fro real and reactive power. Bus real power balance:

$$\begin{aligned} \sum_{l \in L_j \cap L_k} p_{lk}^{\text{gen}} = & p_{jk}^{\text{sh}} + p_j^{\text{dem}} \\ & + \sum_{i \in I_j^{\text{orig}} \cap I_k} p_{ik}^{\text{orig}} + \sum_{i \in I_j^{\text{dest}} \cap I_k} p_{ik}^{\text{dest}} \\ & \forall k \in K, j \in J_k \end{aligned} \tag{45}$$

Bus reactive power balance:

$$\begin{aligned} \sum_{l \in L_j \cap L_k} q_{lk}^{\text{gen}} = & q_{jk}^{\text{sh}} + q_j^{\text{dem}} \\ & + \sum_{i \in I_j^{\text{dest}} \cap I_k} q_{ik}^{\text{orig}} + \sum_{i \in I_j^{\text{dest}} \cap I_k} q_{ik}^{\text{dest}} \\ & \forall k \in K, j \in J_k \end{aligned} \tag{46}$$

Automatic generation control (AGC) is modeled in a stylized fashion. Generator real power output in a contingency case may differ from that in the base case, but the differences across all generators for a fixed contingency must be proportional to prescribed participation factors. Contingency case real power generation definition in terms of base case real power and participation factors:

$$p_{lk}^{\text{gen}} = p_{lk_0}^{\text{gen}} + a_l p_k^{\Delta} \quad \forall k \in K^*, l \in L_k \tag{47}$$

Physically, generator reactive power output is not independently controllable. Rather, a voltage magnitude set point is supplied and the generator provides reactive power in order to maintain that voltage magnitude set point. If the set point cannot be maintained then it must be that the generator reactive power is at a bound. Specifically, the convention is that increasing reactive power generation tends to increase voltage magnitude, so, if the voltage is below the set point, then the generator reactive power must be at its upper bound, and if the voltage is above the set point, then the generator reactive power must be at its lower bound. This generator behavior is called PV/PQ switching, since a generator bus is a PV bus as long as the voltage can be maintained, but becomes a PQ bus when the voltage can no longer be maintained as the generator reactive power has hit a bound. In this model, the base case generator reactive power outputs are considered to be independently controllable variables, but in contingency cases, the generator reactive power outputs are subject to PV/PQ switching, with the voltage magnitude set points provided by the base case voltage magnitudes. Contingency case voltage magnitude under set point at a generator bus implies maximum reactive power output:

$$\begin{aligned}\min(\quad & \max(0, v_{jk_0} - v_{jk}),\\ & \max(0, q_l^{\text{gen,max}} - q_{lk}^{\text{gen}}) \quad ) = 0\\ & \quad \forall k \in K^*, l \in L_k, j = c(l)\end{aligned} \tag{48}$$

Contingency case voltage magnitude over set point at a generator bus implies minimum reactive power output:

$$\begin{aligned}\min(\quad & \max(0, v_{jk} - v_{jk_0}),\\ & \max(0, q_l^{\text{gen}} - q_{lk}^{\text{gen,min}}) \quad ) = 0\\ & \quad \forall k \in K^*, l \in L_k, j = c(l)\end{aligned} \tag{49}$$

Note: The constraints (48,49) enforcing complementarity between generator bus voltage magnitude reactive power output are formulated using the min() function in order to permit a natural and unambiguous definition of constraint violation. Both terms in the min() functions are expressed as per unit quantities. For greater understanding of these constraints, they can be formulated logically as

$$\begin{aligned}&\text{if} \quad && v_{jk} < v_{jk_0}\\ &\text{then} \quad && q_{lk}^{\text{gen}} = q_l^{\text{gen,max}}\\ &\forall k \in K^*, l \in L_k, j = c(l)\end{aligned} \tag{50}$$

and

$$\begin{aligned}&\text{if} \quad && v_{jk} > v_{jk_0}\\ &\text{then} \quad && q_{lk}^{\text{gen}} = q_l^{\text{gen,min}}\\ &\forall k \in K^*, l \in L_k, j = c(l)\end{aligned} \tag{51}$$

The min() formulation (48,49), not the logical formulation (50,51), is used for the evaluation of constraint violation in a given solution.

# 4 References


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