

Grid Optimization Competition Challenge 3 Problem Formulation

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1 Nomenclature

This section gives a complete reference to the symbols used in the model formulation, in tabular form.

Units of measurement are given in Table 1, symbol main letters in Table 2, symbol superscripts in Table 3, indices and index sets in Table 4, subsets in Table 5, special set elements in Table 6, real-valued parameters in Table 7, and variables in Table 8.

Table 1: Units of measurement

Symbol	Description
binary	Binary quantities, i.e. those taking values in $\{0, 1\}$.
dimensionless	Dimensionless real number quantities.
h	hour. Time quantities are expressed in h.
integer	Integer quantities, i.e. those taking values in $\{\dots, -2, -1, 0, 1, 2, \dots\}$.
pu	per unit. Certain physical quantities, including voltage magnitude, real power, reactive power, apparent power, resistance, reactance, conductance, and susceptance, are expressed in a per unit convention with a specified or implied base value, indicated by a unit of pu.
rad	radian. Voltage angles and differences are expressed in rad.
\$	US dollar. Cost, value, penalty, and objective values are expressed in \$.

Table 2: Main letters

Symbol	Description
F	Set of downtime-dependent startup states
I	Set of buses

Table 2 continued

Symbol	Description
J	Set of devices
K	Set of contingencies
M	Set of cost blocks, i.e. constant marginal cost blocks of convex piecewise linear cost functions
N	Set of reserve zones
T	Set of time steps
W	Set of miscellaneous constraints
a	Point in time
b	Susceptance
c	Cost coefficient
d	Duration of time
e	Energy, e.g. stored or total produced
f	Downtime-dependent startup state
g	Conductance
i	Bus
j	Device
k	Contingency
m	Cost or value block
n	Reserve zone
p	Real power
q	Reactive power
r	Resistance
s	Apparent power
t	Index of time points and intervals
u	Integer quantities
v	Voltage magnitude
w	Index on miscellaneous constraints
x	Reactance
z	Cost, value, penalty, or objective
α	Slack distribution factor
β	Sensitivity of reactive power to real power
θ	Voltage angle
σ	Reserve requirement coefficient
τ	Winding ratio of a transformer
ϕ	Phase difference of a transformer

Table 3: Superscripts

Symbol	Description
ac	Alternating current (AC) branch
br	Branch
ctg	Contingency
ch	Charging susceptance in an AC branch
cs	Consuming
dc	Direct current (DC) line
dn	down, as in ramping down or reserve down
en	energy, convex or concave energy cost or value function
end	end time of a time interval or last interval of the time horizon
fpd	fixed phase difference transformer
fr	from, side of a branch
fwr	fixed winding ratio transformer
ln	AC line
max	maximum
min	minimum
mr	must run
ms	market surplus
nsc	Non-synchronized reserve
off	Offline
on	Online, dispatchable
out	Out of service
p	Real power
pqe	Equality constraints linking real and reactive power
pqmax	Inequality constraints modeling upper bounds on reactive power depending on real power
pqmin	Inequality constraints modeling lower bounds on reactive power depending on real power
pr	Producing
p0	Indicates a value taken by a quantity depending on real power when real power is 0
q	reactive power
qrd	Reactive power reserve down
qru	Reactive power reserve up
rgd	Regulation down
rgu	Regulation up
rrd	Ramping reserve reserve down

Table 3 continued

Symbol	Description
rru	Ramping reserve reserve up
ru	Ramp up
rd	Ramp down
s	Apparent power
scr	Synchronized reserve
sd	shutdown, transition from online to offline or from closed to open
sdpc	Shutdown power curve
sh	shunt
sr	series element in AC branch model
start	Start time of a time interval or first interval of the time horizon
su	Startup, transition from offline to online or from open to closed
sus	Downtime-dependent startup state
supc	Startup power curve
to	To, side of a branch
tr	Transition between modes
up	Up, as in ramping up or reserve up
v	Voltage (magnitude)
vpd	variable phase difference transformer
vwr	variable winding ratio transformer
xf	transformer
0	Initial value, referring some period immediately prior to the model horizon
+	Slack variable on an inequality constraints, or largest slack among a set of inequalities

Table 4: Index sets

Symbol	Description
$f \in F$	Downtime-dependent startup states.
$i \in I$	Buses.
$j \in J$	Bus-connected grid devices, e.g. loads, generators, lines.
$k \in K$	Contingencies.
$m \in M$	Offer or bid blocks of piecewise linear convex cost or value functions.
$n \in N$	Reserve zones.
$t \in T$	Time intervals.

Table 4 continued

Symbol	Description
$w \in W$	Miscellaneous constraints.

Table 5: Subsets

Symbol	Description
$F_j \subset F$	Downtime-dependent startup states of device j
$I_n \subset I$	Buses contained in reserve zone n
$J_i \subset J$	Devices connected to bus i
$J_t \subset J$	Devices in service in time interval t
$J_k \subset J$	Devices that are in service in contingency k
$J^{\text{ac}} \subset J$	AC branches
$J_k^{\text{ac}} \subset J$	AC branches that are in service in contingency k
$J^{\text{br}} \subset J$	branches, i.e. devices connecting to two buses
$J_k^{\text{br}} \subset J$	Branches that are in service in contingency k
$J^{\text{cs}} \subset J$	Consuming devices (e.g. loads)
$J^{\text{dc}} \subset J$	DC lines
$J_k^{\text{dc}} \subset J$	DC lines in service in contingency k
$J^{\text{fpd}} \subset J$	transformers having fixed phase difference
$J^{\text{fwr}} \subset J$	transformers having fixed winding ratio
$J_k^{\text{out}} \subset J$	Devices that are outaged by contingency k
$J^{\text{pqe}} \subset J$	Producing or consuming devices having linear equality constraints linking real and reactive power
$J^{\text{pqmax}} \subset J$	Producing or consuming devices having linear inequality constraints modeling upper bounds on reactive power depending on real power
$J^{\text{pqmin}} \subset J$	Producing or consuming devices having linear inequality constraints modeling lower bounds on reactive power depending on real power
$J^{\text{pr}} \subset J$	Producing devices (e.g. generators)
$J_n^{\text{pr}} \subset J$	Producing devices contained in reserve zone n
$J^{\text{ln}} \subset J$	AC lines
$J^{\text{sh}} \subset J$	shunts
$J_i^{\text{to}} \subset J$	branches with to bus equal to bus i
$J^{\text{vpd}} \subset J$	transformers having variable phase difference

Table 5 continued

Symbol	Description
$J^{\text{vwr}} \subset J$	transformers having variable winding ratio
$J^{\text{xf}} \subset J$	transformers
$J^{\text{pr,cs}} \subset J$	Devices that are either producing devices or consuming devices
$J^{\text{pr,cs,ac}} \subset J$	Devices that are either producing devices, consuming devices, or AC branch devices
$J_i^{\text{fr}} \subset J$	Branches with from bus equal to i .
$J_i^{\text{to}} \subset J$	Branches with to bus equal to i .
$J_i^{\text{br}} \subset J$	Branches incident to bus i .
$J_i^{\text{pr}} \subset J$	Producing devices at bus i .
$J_i^{\text{cs}} \subset J$	Consuming devices at bus i .
$J_i^{\text{sh}} \subset J$	Shunts at bus i .
$J_i \subset J$	Devices connected to bus i .
$J_n^{\text{pr}} \subset J$	Producing devices in reserve zone n .
$J_n^{\text{cs}} \subset J$	Consuming devices in reserve zone n .
$M_{jt} \subset M$	Energy cost or value function offer or bid blocks for producing or consuming device j in interval t
$N^{\text{p}} \subset N$	Reserve zones for products associated with real power
$N^{\text{q}} \subset N$	Reserve zones for products associated with reactive power
$N_i \subset N$	Reserve zones containing bus i
$T_j^{\text{out}} \subset T$	Time intervals in which device j is out of service
$T_j^{\text{mr}} \subset T$	Time intervals in which device j is must-run
$T_{jt}^{\text{dn,min}} \subset T$	Time indices $t' < t$ such that if device j shuts down in interval t' then the minimum downtime precludes starting up in interval t
$T_{jt}^{\text{up,min}} \subset T$	Time indices $t' < t$ such that if device j starts up in interval t' then the minimum uptime precludes shutting down in interval t
$T_{jt}^{\text{supc}} \subset T$	Time indices $t' > t$ such that if device j starts up in interval t' then the startup power curve has nonzero power in interval t
$T_{jt}^{\text{sdpc}} \subset T$	Time indices $t' \leq t$ such that if device j shuts down in interval t' then the shutdown power curve has nonzero power in interval t
$T_{jf}^{\text{sus}} \subset T$	Time indices t such that a constraint is needed to ensure that if device j is starting up in state f in time interval t then it was online in some sufficiently recent prior interval
$T_{jft}^{\text{sus}} \subset T$	Time indices such that startup by device j in startup state f in time interval t implies online in some interval in T_{jft}^{sus}
$T_w^{\text{su,max}} \subset T$	Time indices incident to maximum startups constraint w

Table 5 continued

Symbol	Description
$T_w^{\text{en,max}} \subset T$	Time indices incident to maximum energy constraint w
$T_w^{\text{en,min}} \subset T$	Time indices incident to minimum energy constraint w
$W_j^{\text{en,max}} \subset W$	Multi-interval maximum energy constraints for device j
$W_j^{\text{en,min}} \subset W$	Multi-interval minimum energy constraints for device j
$W_j^{\text{su,max}} \subset W$	Multi-interval maximum startups constraints for device j

Table 6: Special set elements

Symbol	Description
$i_j \in I$	Connection bus of non-branch device $j \in J \setminus J^{\text{br}}$
$i_j^{\text{fr}} \in I$	From bus of branch $j \in J^{\text{br}}$.
$i_j^{\text{to}} \in I$	To bus of branch $j \in J^{\text{br}}$.
$j_k^{\text{out}} \in J$	The unique branch device outaged by contingency k
$t^{\text{end}} \in T$	Last time interval t .
$t^{\text{start}} \in T$	First time interval t .

Table 7: Parameters

Symbol	Description
a_t^{end}	End time of time interval t (h)
a_t^{mid}	Midpoint of time interval t (h)
a_t^{start}	Start time of time interval t (h)
$a_w^{\text{en,max,start}}$	Start time of multi-interval maximum energy constraint w (h)
$a_w^{\text{en,max,end}}$	End time of multi-interval maximum energy constraint w (h)
$a_w^{\text{en,min,start}}$	Start time of multi-interval minimum energy constraint w (h)
$a_w^{\text{en,min,end}}$	End time of multi-interval minimum energy constraint w (h)
$a_w^{\text{su,max,start}}$	Start time of multi-interval maximum startups constraint w (h)
$a_w^{\text{su,max,end}}$	End time of multi-interval maximum startups constraint w (h)

Table 7 continued

Symbol	Description
b_j^{ch}	Charging susceptance of AC branch j (pu)
b_j^{fr}	Shunt susceptance to ground of AC branch j at from bus (pu)
b_j^{sh}	Susceptance of one step of shunt j (pu)
b_j^{sr}	Series susceptance of AC branch j (pu)
b_j^{to}	Shunt susceptance to ground of AC branch j at to bus (pu)
c_{jtm}^{en}	Energy marginal cost or value of offer or bid block m of producing or consuming device j in interval t (\$/pu-h)
c^{p}	Real power bus imbalance marginal cost (\$/pu-h)
c^{q}	Reactive power bus imbalance marginal cost (\$/pu-h)
c_{jt}^{rgd}	Marginal cost of regulation down provided by device j in interval t (\$/pu-h)
c_{jt}^{rgu}	Marginal cost of regulation up provided by device j in interval t (\$/pu-h)
c_{jt}^{scr}	Marginal cost of synchronized reserve provided by device j in interval t (\$/pu-h)
c_{jt}^{nsc}	Marginal cost of non-synchronized reserve provided by device j (\$/pu-h) in interval t
$c_{jt}^{\text{rru,off}}$	Marginal cost of ramping reserve up provided by device j when offline in interval t (\$/pu-h)
$c_{jt}^{\text{rru,on}}$	Marginal cost of ramping reserve up provided by device j when online in interval t (\$/pu-h)
$c_{jt}^{\text{rrd,off}}$	Marginal cost of ramping reserve down provided by device j when offline in interval t (\$/pu-h)
$c_{jt}^{\text{rrd,on}}$	Marginal cost of ramping reserve down provided by device j when online in interval t (\$/pu-h)
c_{jt}^{qru}	Marginal cost of reactive power up reserve provided by device j in interval t (\$/pu-h)
c_{jt}^{qrd}	Marginal cost of reactive power down reserve provided by device j in interval t (\$/pu-h)
c_n^{rgu}	Marginal cost of shortfall of regulation up in zone n (\$/pu-h)
c_n^{rgd}	Marginal cost of shortfall of regulation down in zone n (\$/pu-h)
c_n^{scr}	Marginal cost of shortfall of synchronized reserve in zone n (\$/pu-h)
c_n^{nsc}	Marginal cost of shortfall of non-synchronized reserve in zone n (\$/pu-h)
c_n^{rru}	Marginal cost of shortfall of ramping reserve up in zone n (\$/pu-h)
c_n^{rrd}	Marginal cost of shortfall of ramping reserve down in zone n (\$/pu-h)

Table 7 continued

Symbol	Description
c_n^{gru}	Marginal cost of shortfall of reactive power reserve up in zone n (\$/pu-h)
c_n^{grd}	Marginal cost of shortfall of reactive power reserve down in zone n (\$/pu-h)
c^s	Branch overload marginal cost (\$/pu-h)
c_j^{on}	Fixed cost of online status of device j (\$/h)
c_j^{su}	Startup cost of device j (\$)
c_{jf}^{sus}	Downtime-dependent startup state cost of device j in startup state f (\$)
c_j^{sd}	Shutdown cost of device j (\$)
d_t	Duration of interval t (h)
$d_{jf}^{\text{dn,max}}$	Maximum prior downtime of device j if starting up in startup state f (h)
$d_j^{\text{dn,min}}$	Minimum downtime of device j (h)
$d_j^{\text{up,min}}$	Minimum uptime of device j (h)
e_w^{max}	Maximum energy for multi-interval maximum energy constraint w (pu-h)
e_w^{min}	Minimum energy for multi-interval minimum energy constraint w (pu-h)
g_j^{fr}	Shunt conductance to ground of AC branch j at from bus (pu)
g_j^{sr}	Series conductance of AC branch j (pu)
g_j^{sh}	Conductance of one step of shunt j (pu)
g_j^{to}	Shunt conductance to ground of AC branch j at to bus (pu)
$p_j^{\text{dc,max}}$	Maximum real power flow of DC line j (pu)
p_{jtm}^{max}	Maximum real power of offer or bid block m of producing or consuming device j in interval t (pu)
p_{jt}^{max}	Maximum real power of producing or consuming device j when online in interval t (pu)
p_{jt}^{min}	Minimum real power of producing or consuming device j when online in interval t (pu)
p_j^{rd}	Maximum ramp down rate of producing or consuming device j when online (pu/h)
$p_j^{\text{rd,sd}}$	Maximum ramp down rate of producing or consuming device j when shutting down (pu/h)
p_j^{ru}	Maximum ramp up rate of producing or consuming device j when online and not starting up (pu/h)
$p_j^{\text{ru,su}}$	Maximum ramp up rate of producing or consuming device j when starting up (pu/h)

Table 7 continued

Symbol	Description
$p_{jtt'}^{\text{supc}}$	Startup power of producing or consuming device j in interval t if starting up in interval $t' > t$ in its startup power curve (pu)
$p_{jtt'}^{\text{sdpC}}$	Shutdown power of producing or consuming device j in interval t if shutting down in interval $t' \leq t$ in its shutdown power curve (pu)
p_j^0	Initial real power of producing or consuming device j (pu)
$p_j^{\text{rgu,max}}$	Maximum regulation up for producing or consuming device j (pu)
$p_j^{\text{rgd,max}}$	Maximum regulation down for producing or consuming device j (pu)
$p_j^{\text{scr,max}}$	Maximum synchronized reserve for producing or consuming device j (pu)
$p_j^{\text{nsc,max}}$	Maximum non-synchronized reserve for producing or consuming device j (pu)
$p_j^{\text{rru,on,max}}$	Maximum ramping reserve up for producing or consuming device j when online (pu)
$p_j^{\text{rru,off,max}}$	Maximum ramping reserve up for producing or consuming device j when offline (pu)
$p_j^{\text{rrd,on,max}}$	Maximum ramping reserve down for producing or consuming device j when online (pu)
$p_j^{\text{rrd,off,max}}$	Maximum ramping reserve down for producing or consuming device j when offline (pu)
$p_{nt}^{\text{rru,min}}$	Exogenous ramping reserve up requirement for zone n in interval t (pu)
$p_{nt}^{\text{rrd,min}}$	Exogenous ramping reserve down requirement for zone n in interval t (pu)
q_j^{q0}	Prior value of reactive power of producing or consuming device j (pu)
q_j^{p0}	Reactive power at 0 real power of producing or consuming device j if the device has reactive power dependent on real power (pu)
$q_j^{\text{max,p0}}$	Upper bound on reactive power at 0 real power of producing or consuming device j if the device has maximum reactive power dependent on real power (pu)
$q_j^{\text{min,p0}}$	Lower bound on reactive power at 0 real power of producing or consuming device j if the device has minimum reactive power dependent on real power (pu)
q_{jt}^{max}	Maximum reactive power of producing or consuming device j when online in interval t (pu)
q_{jt}^{min}	Minimum reactive power of producing or consuming device j when online in interval t (pu)

Table 7 continued

Symbol	Description
$q_j^{\text{dc,fr,max}}$	Maximum reactive power at from bus of DC line j (pu)
$q_j^{\text{dc,to,max}}$	Maximum reactive power at to bus of DC line j (pu)
$q_j^{\text{dc,fr,min}}$	Minimum reactive power at from bus of DC line j (pu)
$q_j^{\text{dc,to,min}}$	Minimum reactive power at to bus of DC line j (pu)
$q_{nt}^{\text{qr},\text{min}}$	Exogenous reactive power reserve up requirement for zone n in interval t (pu)
$q_{nt}^{\text{qrd},\text{min}}$	Exogenous reactive power reserve down requirement for zone n in interval t (pu)
r_j^{sr}	Series resistance of AC branch j (pu)
s_j^{max}	Maximum apparent power flow of branch j (pu)
$s_j^{\text{max,ctg}}$	Maximum apparent power flow of branch j in contingencies (pu)
$u_{jt}^{\text{on,max}}$	Upper bound on online status indicator for device j in interval t (binary)
$u_{jt}^{\text{on,min}}$	Lower bound on online status indicator for device j in interval t (binary)
$u_j^{\text{on},0}$	Initial on-off status of device j (binary)
$u_w^{\text{su,max}}$	Maximum startups for multi-interval maximum startups constraint w (integer)
$u_j^{\text{sh},0}$	Prior number of activated steps of shunt j (integer)
$u_j^{\text{sh,max}}$	Maximum number of activated steps of shunt j (integer)
$u_j^{\text{sh,min}}$	Minimum number of activated steps of shunt j (integer)
v_i^0	Prior value of voltage magnitude at bus i (pu)
v_i^{max}	Maximum voltage magnitude at bus i (pu)
v_i^{min}	Minimum voltage magnitude at bus i (pu)
x_j^{sr}	Series reactance of AC branch j (pu)
α_i	Participation factor of bus i in resolving system real power imbalance in contingencies (dimensionless)
β_j	Sensitivity of reactive power with respect to real power of producing or consuming device j with equality constraint linking real and reactive power (pu/pu)
β_j^{max}	Sensitivity of upper bound on reactive power with respect to real power of producing or consuming device j with inequality constraint modeling upper bounds on reactive power depending on real power (pu/pu)

Table 7 continued

Symbol	Description
β_j^{\min}	Sensitivity of lower bound on reactive power with respect to real power of producing or consuming device j with inequality constraint modeling upper bounds on reactive power depending on real power (pu/pu)
σ_n^{rgd}	Fraction of total cleared power consumption in zone n forming regulation down requirement for zone n (dimensionless)
σ_n^{rgu}	Fraction of total cleared power consumption in zone n forming regulation up requirement for zone n (dimensionless)
σ_n^{scr}	Fraction of largest cleared power production in zone n forming synchronized reserve requirement for zone n (dimensionless)
σ_n^{nsc}	Fraction of largest cleared power production in zone n forming non-synchronized reserve requirement for zone n (dimensionless)
τ_j^0	Prior value of winding ratio of AC branch j (dimensionless)
τ_j^{\max}	Maximum winding ratio of AC branch j (dimensionless)
τ_j^{\min}	Minimum winding ratio of AC branch j (dimensionless)
θ_i^0	Prior value of voltage angle at bus i . (rad)
ϕ_j^0	Prior value of phase difference of AC branch j (rad)
ϕ_j^{\max}	Maximum phase difference of AC branch j (rad)
ϕ_j^{\min}	Minimum phase difference of AC branch j (rad)

Table 8: Variables

Symbol	Description
b_{jt}^{sh}	Susceptance of shunt j in interval t . (pu)
g_{jt}^{sh}	Conductance of shunt j in interval t . (pu)
p_{it}	Real power signed mismatch, i.e. load and other real power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t . (pu)
p_{it}^+	Real power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t . (pu)
p_{jt}	Real power of device j in interval t . Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)

Table 8 continued

Symbol	Description
p_{jtk}	Real power of device j in interval t contingency k . Oriented from the device into the connection bus for dispatchable devices, from the device into the connection bus for non-dispatchable devices, and from the from bus to the to bus for branches. (pu)
p_{jt}^{on}	Dispatchable real power of producing or consuming device j in interval t . (pu)
p_{jt}^{su}	Startup real power of producing or consuming device j in interval t . (pu)
p_{jt}^{sd}	Shutdown real power of producing or consuming device j in interval t . (pu)
p_{jtm}	Real power of producing or consuming device j in interval t in energy cost or value block m . (pu)
p_{jt}^{fr}	Real power at from bus of branch j in interval t . Oriented from the bus into the device. (pu)
p_{jt}^{rgu}	Regulation up provided by device j in interval t
p_{jt}^{rgd}	Regulation down provided by device j in interval t
p_{jt}^{scr}	Synchronized reserve provided by device j in interval t
p_{jt}^{nsc}	Non-synchronized reserve provided by device j in interval t
$p_{jt}^{\text{rru,on}}$	Ramping reserve up provided by device j in interval t when online
$p_{jt}^{\text{rru,off}}$	Ramping reserve up provided by device j in interval t when offline
$p_{jt}^{\text{rrd,on}}$	Ramping reserve down provided by device j in interval t when online
$p_{jt}^{\text{rrd,off}}$	Ramping reserve down provided by device j in interval t when offline
$p_{nt}^{\text{rgu,req}}$	Regulation up requirement for reserve zone n in interval t
$p_{nt}^{\text{rgd,req}}$	Regulation down requirement for reserve zone n in interval t
$p_{nt}^{\text{scr,req}}$	Synchronized reserve requirement for reserve zone n in interval t
$p_{nt}^{\text{nsc,req}}$	Non-synchronized reserve requirement for reserve zone n in interval t
$p_{nt}^{\text{rgu,+}}$	Regulation up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rgd,+}}$	Regulation down shortfall for reserve zone n in interval t
$p_{nt}^{\text{scr,+}}$	Synchronized reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{nsc,+}}$	Non-synchronized reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{rru,+}}$	Ramping reserve up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rrd,+}}$	Ramping reserve down shortfall for reserve zone n in interval t
p_{jt}^{to}	Real power at to bus of branch j in interval t . Oriented from the bus into the device. (pu)

Table 8 continued

Symbol	Description
p_t^{sl}	Real power system slack in interval t , positive corresponds to net positive losses in the branches (pu)
q_{it}	Reactive power signed mismatch, i.e. load and other reactive power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t . (pu)
q_{it}^+	Reactive power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t . (pu)
q_{jt}	Reactive power of non-branch device j in interval t . Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)
q_{jt}^{fr}	Reactive power at from bus of branch j in interval t . Oriented from the bus into the device. (pu)
q_{jt}^{to}	Reactive power at to bus of branch j in interval t . Oriented from the bus into the device. (pu)
q_{jt}^{gru}	Reactive power up reserve provided by device j in interval t
q_{jt}^{grd}	Reactive power down reserve provided by device j in interval t
$q_{nt}^{\text{gru},+}$	Reactive power up reserve shortfall for reserve zone n in interval t
$q_{nt}^{\text{grd},+}$	Reactive power down reserve shortfall for reserve zone n in interval t
s_{jt}^+	Branch j apparent power overload in interval t . (pu)
s_{jtk}^+	Branch j (approximate) apparent overload in interval t contingency k . (pu)
u_{jt}^{on}	On indicator of device j in interval t . The value 1 indicates the device is online or closed, 0 else. (binary)
u_{jt}^{sd}	Shutdown indicator of device j in interval t . The value 1 indicates the device is shutting down, transitioning from online to offline or from closed to open, 0 else. (binary)
u_{jt}^{sh}	Number of steps activated for shunt j in interval t . (integer)
u_{jt}^{su}	Startup indicator of device j in interval t . The value 1 indicates the device is starting up, transitioning from offline to online or from open to closed, 0 else. (binary)
u_{jft}^{sus}	Downtime-dependent startup state indicator of device j in interval t state f . (binary)
v_{it}	Voltage magnitude at bus i in interval t . (pu)
$z_{jt}^{\text{md,on}}$	Mode fixed cost incurred by multi-mode device j in interval t (\$)
z^{ms}	Total maximization objective, i.e. base case plus contingencies. Net market surplus, including values minus costs and penalties. (\$)

Table 8 continued

Symbol	Description
z_t^{ms}	Base case market surplus objective in interval t . (\$)
z_{tk}^{ms}	Contingency market surplus objective in interval t contingency k . (\$)
z_{jt}^{en}	Energy cost or value of producing or consuming device j in interval t . (\$)
z_{jt}^{on}	Online status cost of device j in interval t . (\$)
z_{it}^{p}	Real power mismatch cost of bus i in interval t . (\$)
z_{it}^{q}	Reactive power mismatch cost of bus i in interval t . (\$)
z_{jt}^{sd}	Shutdown cost of device j in interval t . (\$)
z_{jt}^{su}	Startup cost of device j in interval t . (\$)
z_{jt}^{sus}	Downtime-dependent startup cost of device j in interval t . (\$)
z_{jt}^{s}	Branch overload cost of branch j in interval t . (\$)
z_{jtk}^{s}	Branch overload cost of branch j in interval t contingency k . (\$)
z_{jt}^{rgu}	Cost of Regulation up provided by device j in interval t (\$)
z_{jt}^{rgd}	Cost of Regulation down provided by device j in interval t (\$)
z_{jt}^{scr}	Cost of Synchronized reserve provided by device j in interval t and not counting as regulation up (\$)
z_{jt}^{nsc}	Cost of Non-synchronized reserve provided by device j in interval t (\$)
z_{jt}^{rru}	Cost of ramping reserve up provided by device j in interval t (\$)
z_{jt}^{rrd}	Cost of ramping reserve down provided by device j in interval t (\$)
z_{jt}^{qru}	Cost of Reactive power up reserve provided by device j in interval t (\$)
z_{jt}^{qrd}	Cost of Reactive power down reserve provided by device j in interval t (\$)
z_{nt}^{rgu}	Regulation up shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rgd}	Regulation down shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{scr}	Synchronized reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{nsc}	Non-synchronized reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rru}	ramping reserve up shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rrd}	ramping reserve down shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{qru}	Reactive power up reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{qrd}	Reactive power down reserve shortfall penalty for reserve zone n in interval t (\$)
θ_{it}	Voltage angle at bus i in interval t . (rad)

Table 8 continued

Symbol	Description
θ_{itk}	Approximate post-contingency voltage angle at bus i in interval t in contingency k . (rad)
τ_{jt}	Winding ratio of AC branch j in interval t . (dimensionless)
ϕ_{jt}	Phase difference of AC branch j in interval t . (rad)

2 Optimization model formulation

This section formulates the optimization model. The model formulation consists of a set of constraints involving variables and parameters, structured by index sets, subsets, and special set elements, together with an objective specified as one of the variables, and a specification of a direction of optimization, specifically that the objective should be maximized. This section formulates these constraints as algebraic equations and introduces the variables. The variables and constraints are presented in an ordered and hierarchical structure with the goals of comprehensibility, completeness, accuracy, and precision. This model formulation is called the reference formulation.

2.1 Market surplus objective

The maximization objective z^{ms} is the total market surplus, including values minus costs and penalties, in the base case and over all contingencies. In each time interval t , the base case market surplus z_t^{ms} and the post-contingency surplus z_{tk}^{ms} for contingencies k are combined as a sum of the base case, the minimum (i.e. worst case) over contingencies, and the average over contingencies, and the total is the sum over intervals. In the event that $K = \{\}$, the minimum and average terms indexed by $k \in K$ are not included. When a distinction between base case and contingencies is not specified, the base case is implied, except if explicitly indicated otherwise.

$$z^{\text{ms}} = \sum_{t \in T} \left(z_t^{\text{ms}} + \min_{k \in K} z_{tk}^{\text{ms}} + 1/|K| \sum_{k \in K} z_{tk}^{\text{ms}} \right) \quad (1)$$

The base case market surplus z_t^{ms} is the sum of appropriately signed terms representing values accrued and costs incurred by consumers, producers, device startup, shutdown, and fixed operating cost, branch limit overload penalties, device reserve procurement, bus real

and reactive power mismatch penalties, and zonal reserve shortfall penalties.

$$\begin{aligned}
z_t^{\text{ms}} = & \sum_{j \in J^{\text{cs}}} z_{jt}^{\text{en}} - \sum_{j \in J^{\text{pr}}} z_{jt}^{\text{en}} - \sum_{j \in J^{\text{pr,cs,ac}}} (z_{jt}^{\text{su}} + z_{jt}^{\text{sd}}) - \sum_{j \in J^{\text{pr,cs}}} (z_{jt}^{\text{on}} + z_{jt}^{\text{sus}}) - \sum_{j \in J^{\text{br}}} z_{jt}^{\text{s}} \\
& - \sum_{j \in J^{\text{pr,cs}}} \left(z_{jt}^{\text{rgu}} + z_{jt}^{\text{rgd}} + z_{jt}^{\text{scr}} + z_{jt}^{\text{nsc}} + z_{jt}^{\text{rru}} + z_{jt}^{\text{rrd}} + z_{jt}^{\text{qru}} + z_{jt}^{\text{qrd}} \right) - \sum_{i \in I} (z_{it}^{\text{p}} + z_{it}^{\text{q}}) \\
& - \sum_{n \in N^{\text{p}}} \left(z_{nt}^{\text{rgu}} + z_{nt}^{\text{rgd}} + z_{nt}^{\text{scr}} + z_{nt}^{\text{nsc}} + z_{nt}^{\text{rru}} + z_{nt}^{\text{rrd}} \right) - \sum_{n \in N^{\text{q}}} \left(z_{nt}^{\text{qru}} + z_{nt}^{\text{qrd}} \right) \quad \forall t \in T \quad (2)
\end{aligned}$$

The post-contingency market surplus z_{tk}^{ms} is the sum of branch limit overload costs.

$$z_{tk}^{\text{ms}} = - \sum_{j \in J_k^{\text{br}}} z_{jtk}^{\text{s}} \quad \forall t \in T, k \in K \quad (3)$$

2.2 Bus real and reactive power balance and voltage

2.2.1 Bus power mismatch and penalized mismatch definitions

For each bus i and each interval t , the penalized real and reactive power mismatches p_{it}^+ and q_{it}^+ are the absolute values of the signed mismatches p_{it} and q_{it} :

$$p_{it}^+ \geq p_{it} \quad \forall t \in T, i \in I \quad (4)$$

$$p_{it}^+ \geq -p_{it} \quad \forall t \in T, i \in I \quad (5)$$

$$q_{it}^+ \geq q_{it} \quad \forall t \in T, i \in I \quad (6)$$

$$q_{it}^+ \geq -q_{it} \quad \forall t \in T, i \in I \quad (7)$$

2.2.2 Bus power mismatch penalty

For each bus i and interval t the bus real and reactive power mismatch penalties z_{it}^{p} and z_{it}^{q} are defined by linear penalty functions applied to the penalized mismatches:

$$z_{it}^{\text{p}} = d_t c^{\text{p}} p_{it}^+ \quad \forall t \in T, i \in I \quad (8)$$

$$z_{it}^{\text{q}} = d_t c^{\text{q}} q_{it}^+ \quad \forall t \in T, i \in I \quad (9)$$

2.2.3 Bus real and reactive power balance

For each bus i and each interval t , real and reactive power balance are enforced by:

$$\sum_{j \in J_i^{\text{cs}}} p_{jt} + \sum_{j \in J_i^{\text{sh}}} p_{jt} + \sum_{j \in J_i^{\text{fr}}} p_{jt}^{\text{fr}} + \sum_{j \in J_i^{\text{to}}} p_{jt}^{\text{to}} = \sum_{j \in J_i^{\text{pr}}} p_{jt} + p_{it} \quad \forall t \in T, i \in I \quad (10)$$

$$\sum_{j \in J_i^{\text{cs}}} q_{jt} + \sum_{j \in J_i^{\text{sh}}} q_{jt} + \sum_{j \in J_i^{\text{fr}}} q_{jt}^{\text{fr}} + \sum_{j \in J_i^{\text{to}}} q_{jt}^{\text{to}} = \sum_{j \in J_i^{\text{pr}}} q_{jt} + q_{it} \quad \forall t \in T, i \in I \quad (11)$$

In Eqs. (10) and (11) power withdrawals at bus i in interval t appear on the left, and injections appear on the right. Withdrawals include consumption by consuming devices, signed absorption by shunts, signed flow directed into branches at the from and to buses. Injections include production by producing devices and signed power mismatch.

2.2.4 Bus voltage

For each bus i and each interval t , the voltage angle and magnitude are represented by variables θ_{it} and v_{it} . Limits on voltage magnitude are modeled by hard constraints:

$$v_i^{\min} \leq v_{it} \leq v_i^{\max} \quad \forall t \in T, i \in I \quad (12)$$

2.3 Zonal reserve requirements

2.3.1 Reserve shortfall domains

For each reserve product, in each reserve zone n and time interval t , a nonnegative variable represents the shortfall of procured reserve relative to the reserve requirement:

$$p_{nt}^{\text{rgu},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (13)$$

$$p_{nt}^{\text{rgd},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (14)$$

$$p_{nt}^{\text{scr},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (15)$$

$$p_{nt}^{\text{nsc},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (16)$$

$$p_{nt}^{\text{rru},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (17)$$

$$p_{nt}^{\text{rrd},+} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (18)$$

$$p_{nt}^{\text{qru},+} \geq 0 \quad \forall t \in T, n \in N^{\text{Q}} \quad (19)$$

$$p_{nt}^{\text{qrd},+} \geq 0 \quad \forall t \in T, n \in N^{\text{Q}} \quad (20)$$

2.3.2 Reserve shortfall penalties

Reserve shortfall penalties are defined by a linear penalty function applied the reserve shortfall, expressed as the interval duration times the penalty coefficient times the shortfall:

$$z_{nt}^{\text{rgu}} = d_t c_n^{\text{rgu}} p_{nt}^{\text{rgu},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (21)$$

$$z_{nt}^{\text{rgd}} = d_t c_n^{\text{rgd}} p_{nt}^{\text{rgd},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (22)$$

$$z_{nt}^{\text{scr}} = d_t c_n^{\text{scr}} p_{nt}^{\text{scr},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (23)$$

$$z_{nt}^{\text{nsc}} = d_t c_n^{\text{nsc}} p_{nt}^{\text{nsc},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (24)$$

$$z_{nt}^{\text{rru}} = d_t c_n^{\text{rru}} p_{nt}^{\text{rru},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (25)$$

$$z_{nt}^{\text{rrd}} = d_t c_n^{\text{rrd}} p_{nt}^{\text{rrd},+} \quad \forall t \in T, n \in N^{\text{P}} \quad (26)$$

$$z_{nt}^{\text{qru}} = d_t c_n^{\text{qru}} p_{nt}^{\text{qru},+} \quad \forall t \in T, n \in N^{\text{Q}} \quad (27)$$

$$z_{nt}^{\text{qrd}} = d_t c_n^{\text{qrd}} p_{nt}^{\text{qrd},+} \quad \forall t \in T, n \in N^{\text{Q}} \quad (28)$$

2.3.3 Reserve requirements

For certain products, specifically ramping reserve up and down and reactive power reserve up and down, the reserve requirement is an exogenous value given by input data, and this value is used directly in the reserve balance constraints. For the other products, the requirement is an

endogenous value formed in a prescribed fashion from the power dispatch. The endogenous reserve requirements are given by:

$$p_{nt}^{\text{rgu,req}} = \sigma_n^{\text{rgu}} \sum_{j \in J_n^{\text{CS}}} p_{jt} \quad \forall t \in T, n \in N^{\text{P}} \quad (29)$$

$$p_{nt}^{\text{rgd,req}} = \sigma_n^{\text{rgd}} \sum_{j \in J_n^{\text{CS}}} p_{jt} \quad \forall t \in T, n \in N^{\text{P}} \quad (30)$$

$$p_{nt}^{\text{scr,req}} = \sigma_n^{\text{scr}} \max_{j \in J_n^{\text{Pr}}} p_{jt} \quad \forall t \in T, n \in N^{\text{P}} \quad (31)$$

$$p_{nt}^{\text{nsc,req}} = \sigma_n^{\text{nsc}} \max_{j \in J_n^{\text{Pr}}} p_{jt} \quad \forall t \in T, n \in N^{\text{P}} \quad (32)$$

The requirements for regulation up and down are modeled endogenously in Eqs. (29) and (30) as prescribed factors of the total real power over all consuming devices. The requirements for synchronized and non-synchronized reserves are modeled endogenously in Eqs. (31) and (32) as a prescribed factors of the largest real power over all producing devices. In the event that $J_n^{\text{Pr}} = \{\}$ for some $n \in N^{\text{P}}$, the max operator in Eqs. (31) and (32) is interpreted as taking the value 0.

2.3.4 Reserve balance

For each reserve product, in each zone n and each interval t , the total reserve procured from producing and consuming devices plus any shortfall must be greater than or equal to the requirement. Excess of higher quality reserve products can substitute for lower quality products, as modeled in Eqs. (35) and (36). Some products can be provided by online devices and by offline devices and are tracked by separate variables, as in Eqs. (37) and (38).

$$\sum_{j \in J_n^{\text{Pr,CS}}} p_{jt}^{\text{rgu}} + p_{nt}^{\text{rgu,+}} \geq p_{nt}^{\text{rgu,req}} \quad \forall t \in T, n \in N^{\text{P}} \quad (33)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} p_{jt}^{\text{rgd}} + p_{nt}^{\text{rgd,+}} \geq p_{nt}^{\text{rgd,req}} \quad \forall t \in T, n \in N^{\text{P}} \quad (34)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} (p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}}) + p_{nt}^{\text{scr,+}} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{scr,req}} \quad \forall t \in T, n \in N^{\text{P}} \quad (35)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} (p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{nsc}}) + p_{nt}^{\text{nsc,+}} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{scr,req}} + p_{nt}^{\text{nsc,req}} \quad \forall t \in T, n \in N^{\text{P}} \quad (36)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} (p_{jt}^{\text{rru,on}} + p_{jt}^{\text{rru,off}}) + p_{nt}^{\text{rru,+}} \geq p_{nt}^{\text{rru,min}} \quad \forall t \in T, n \in N^{\text{P}} \quad (37)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} (p_{jt}^{\text{rrd,on}} + p_{jt}^{\text{rrd,off}}) + p_{nt}^{\text{rrd,+}} \geq p_{nt}^{\text{rrd,min}} \quad \forall t \in T, n \in N^{\text{P}} \quad (38)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} q_{jt}^{\text{gru}} + p_{nt}^{\text{gru,+}} \geq q_{nt}^{\text{gru,min}} \quad \forall t \in T, n \in N^{\text{Q}} \quad (39)$$

$$\sum_{j \in J_n^{\text{Pr,CS}}} q_{jt}^{\text{qrd}} + p_{nt}^{\text{qrd,+}} \geq q_{nt}^{\text{qrd,min}} \quad \forall t \in T, n \in N^{\text{Q}} \quad (40)$$

2.4 Device on-off status and related constraints

2.4.1 Device on-off status

For each device j and each interval t , a binary variable u_{jt}^{on} represents the on-off status, with $u_{jt}^{\text{on}} = 1$ if device j is online in interval t and 0 otherwise. Binary variables u_{jt}^{su} and u_{jt}^{sd} are also used to indicate startup and shutdown.

$$u_{jt}^{\text{on}} \in \{0, 1\} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (41)$$

$$u_{jt}^{\text{su}} \in \{0, 1\} \quad \forall t \in T, j \in J^{\text{pr,cs,ac}} \quad (42)$$

$$u_{jt}^{\text{sd}} \in \{0, 1\} \quad \forall t \in T, j \in J^{\text{pr,cs,ac}} \quad (43)$$

Device on-off statuses are subject to planned outage and must-run conditions:

$$u_{jt}^{\text{on}} = 1 \quad \forall j \in J^{\text{pr,cs}}, t \in T_j^{\text{mr}} \quad (44)$$

$$u_{jt}^{\text{on}} = 0 \quad \forall j \in J^{\text{pr,cs}}, t \in T_j^{\text{out}} \quad (45)$$

On-off status, startup, and shutdown variables are related by evolution equations:

$$u_{jt}^{\text{on}} - u_j^{\text{on},0} = u_{jt}^{\text{su}} - u_{jt}^{\text{sd}} \quad \forall t = t^{\text{start}}, j \in J^{\text{pr,cs,ac}} \quad (46)$$

$$u_{jt}^{\text{on}} - u_{j,t-1}^{\text{on}} = u_{jt}^{\text{su}} - u_{jt}^{\text{sd}} \quad \forall t > t^{\text{start}}, j \in J^{\text{pr,cs,ac}} \quad (47)$$

In Eq. (46) for the first time interval, the prior on-off status $u_j^{\text{on},0}$ is a parameter from the input data. In Eq. (47) for intervals after the first one, the prior on-off status $u_{j,t-1}^{\text{on}}$ is the on-off status variable from the prior time interval. Simultaneous startup and shutdown is prohibited:

$$u_{jt}^{\text{su}} + u_{jt}^{\text{sd}} \leq 1 \quad \forall t \in T, j \in J^{\text{pr,cs,ac}} \quad (48)$$

2.4.2 On-off status and transition costs

Online cost, startup cost, and shutdown cost are driven by the on-off status, startup, and shutdown variables:

$$z_{jt}^{\text{on}} = d_t c_j^{\text{on}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (49)$$

$$z_{jt}^{\text{su}} = c_j^{\text{su}} u_{jt}^{\text{su}} \quad \forall t \in T, j \in J^{\text{pr,cs,ac}} \quad (50)$$

$$z_{jt}^{\text{sd}} = c_j^{\text{sd}} u_{jt}^{\text{sd}} \quad \forall t \in T, j \in J^{\text{pr,cs,ac}} \quad (51)$$

2.4.3 Downtime-dependent startup costs

For some devices the startup cost depends on the prior downtime at the time of startup. To model this, we introduce a set of startup states F_j for device j , each characterized by a maximum prior downtime and a cost. We introduce a variable u_{jft}^{sus} indicating that device j is starting up in interval t in startup state f . The startup state indicator variables are binary:

$$u_{jft}^{\text{sus}} \in \{0, 1\} \quad \forall t \in T, j \in J^{\text{pr,cs}}, f \in F_j \quad (52)$$

The downtime-dependent startup cost is

$$z_{jt}^{\text{sus}} = \sum_{f \in F_j} c_{jf}^{\text{sus}} u_{jft}^{\text{sus}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (53)$$

If a device is in any of its startup states, then it must be starting up:

$$\sum_{f \in F_j} u_{jft}^{\text{sus}} \leq u_{jt}^{\text{su}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (54)$$

If a device is in a startup state, then it must have been on in a recent prior interval, depending on the maximum downtime and prior downtime at the start of the model horizon:

$$u_{jft}^{\text{sus}} \leq \sum_{t' \in T_{jft}^{\text{sus}}} u_{jt'}^{\text{on}} \quad \forall t \in T_{jf}^{\text{sus}}, j \in J^{\text{pr,cs}}, f \in F_j \quad (55)$$

Note, if a device is starting up with prior downtime exceeding the maximum for all of its startup states, then it is not in any startup state. The startup state cost coefficients c_{jf}^{sus} are generally negative and can be viewed as a discount on the regular startup cost that is earned by starting up after only a short prior downtime.

2.4.4 Minimum downtime

After a device shuts down it must remain offline for a prescribed minimum downtime before starting up. Logical constraints enforce this algebraically by requiring that a device cannot start up if it has shut down in a prescribed set of prior intervals:

$$u_{jt}^{\text{su}} \leq 1 - \sum_{t' \in T_{jt}^{\text{dn,min}}} u_{jt'}^{\text{sd}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (56)$$

2.4.5 Minimum uptime

Similar to minimum downtime, minimum uptime is enforced by a logical constraint requiring no recent startup if shutting down:

$$u_{jt}^{\text{sd}} \leq 1 - \sum_{t' \in T_{jt}^{\text{up,min}}} u_{jt'}^{\text{su}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (57)$$

2.4.6 Maximum starts over multiple intervals

Some devices have limits on the number of startups in a prescribed set of time intervals:

$$\sum_{t \in T_w^{\text{su,max}}} u_{jt}^{\text{su}} \leq u_w^{\text{su,max}} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \quad (58)$$

2.4.7 Synchronous network connectivity constraints on branch device on-off status

In each time interval, in the base case and in each contingency, the graph consisting of all buses and online in service AC branches must be connected:

$$\text{The bus-branch graph on } (I, \{j \in J^{\text{ac}} : u_{jt}^{\text{on}} = 1\}) \text{ is connected } \forall t \in T \quad (59)$$

$$\text{The bus-branch graph on } (I, \{j \in J_k^{\text{ac}} : u_{jt}^{\text{on}} = 1\}) \text{ is connected } \forall t \in T, k \in K \quad (60)$$

2.5 Device real and reactive power flow

Some devices, specifically shunts, producing devices, consuming devices, and branches, have real and reactive power flows. Shunts, producing devices, and consuming devices are each connected to a single bus, and for each such device j and each interval t , the real and reactive power flows are represented by variables p_{jt} and q_{jt} . For shunts and consuming devices, these flows are oriented so that a positive value of the flow variable represents flow directed from the bus into the device, while for producing devices, the flow is directed from the device into the bus. Each branch device j is connected to two buses, a from bus and a to bus, and the real and reactive power flows directed from the bus into the branch at the from and to buses in interval t are represented by variables p_{jt}^{fr} , q_{jt}^{fr} , p_{jt}^{to} , and q_{jt}^{to} .

2.6 Producing and consuming devices

2.6.1 Producing and consuming device startup, shutdown, and dispatchable power

Total power p_{jt} is equal to dispatchable power p_{jt}^{on} , plus startup power p_{jt}^{su} , plus shutdown power p_{jt}^{sd} :

$$p_{jt} = p_{jt}^{\text{on}} + p_{jt}^{\text{su}} + p_{jt}^{\text{sd}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (61)$$

Startup and shutdown power are determined by linear coefficients applied to future startup and past shutdown if startup and shutdown power curves exist.

$$p^{\text{su}} = \sum_{t' \in T_{jt}^{\text{supc}}} p_{jtt'}^{\text{supc}} u_{jt'}^{\text{su}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (62)$$

$$p^{\text{sd}} = \sum_{t' \in T_{jt}^{\text{sdpc}}} p_{jtt'}^{\text{sdpc}} u_{jt'}^{\text{sd}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (63)$$

Dispatchable power and reactive power are bounded by constraints involving reserves that we formulate later.

2.6.2 Ramping limits

Real power ramping limits depend on on-off status and startup and shutdown and apply to total power including dispatchable, startup, and shutdown power:

$$p_{jt} - p_j^0 \leq d_t \left(p_j^{\text{ru}} (u_{jt}^{\text{on}} - u_{jt}^{\text{su}}) + p_j^{\text{ru,su}} (u_{jt}^{\text{su}} + 1 - u_{jt}^{\text{on}}) \right) \quad \forall t = t^{\text{start}}, j \in J^{\text{pr,cs}} \quad (64)$$

$$p_{jt} - p_{j,t-1} \leq d_t \left(p_j^{\text{ru}} (u_{jt}^{\text{on}} - u_{jt}^{\text{su}}) + p_j^{\text{ru,su}} (u_{jt}^{\text{su}} + 1 - u_{jt}^{\text{on}}) \right) \quad \forall t > t^{\text{start}}, j \in J^{\text{pr,cs}} \quad (65)$$

$$p_{jt} - p_j^0 \geq -d_t \left(p_j^{\text{rd}} u_{jt}^{\text{on}} + p_j^{\text{rd,sd}} (1 - u_{jt}^{\text{on}}) \right) \quad \forall t = t^{\text{start}}, j \in J^{\text{pr,cs}} \quad (66)$$

$$p_{jt} - p_{j,t-1} \geq -d_t \left(p_j^{\text{rd}} u_{jt}^{\text{on}} + p_j^{\text{rd,sd}} (1 - u_{jt}^{\text{on}}) \right) \quad \forall t > t^{\text{start}}, j \in J^{\text{pr,cs}} \quad (67)$$

The ramp up limit is enforced in the first time interval by Eq. (64) and in later intervals by Eq. (65). The ramp down limit is enforced in the first time interval by Eq. (66) and in later intervals by Eq. (67).

2.6.3 Maximum/minimum energy over multiple intervals

Producing devices and consuming devices may have limits on total energy produced or consumed over one or more consecutive time intervals:

$$\sum_{t \in T_w^{\text{en,max}}} d_t p_{jt} \leq e_w^{\text{max}} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \quad (68)$$

$$\sum_{t \in T_w^{\text{en,min}}} d_t p_{jt} \geq e_w^{\text{min}} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \quad (69)$$

2.6.4 Device reserve variable domains

Device reserve variables are nonnegative.

$$p_{jt}^{\text{rgu}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (70)$$

$$p_{jt}^{\text{rgd}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (71)$$

$$p_{jt}^{\text{scr}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (72)$$

$$p_{jt}^{\text{nsc}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (73)$$

$$p_{jt}^{\text{rru,on}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (74)$$

$$p_{jt}^{\text{rru,off}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (75)$$

$$p_{jt}^{\text{rrd,on}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (76)$$

$$p_{jt}^{\text{rrd,off}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (77)$$

$$q_{jt}^{\text{qru}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (78)$$

$$q_{jt}^{\text{qrd}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (79)$$

2.6.5 Device reserve costs

Devices may include a cost in their reserve offers. This is modeled as a cost coefficient on each device reserve provision variable.

$$z_{jt}^{\text{rgu}} = d_t c_{jt}^{\text{rgu}} p_{jt}^{\text{rgu}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (80)$$

$$z_{jt}^{\text{rgd}} = d_t c_{jt}^{\text{rgd}} p_{jt}^{\text{rgd}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (81)$$

$$z_{jt}^{\text{scr}} = d_t c_{jt}^{\text{scr}} p_{jt}^{\text{scr}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (82)$$

$$z_{jt}^{\text{nsc}} = d_t c_{jt}^{\text{nsc}} p_{jt}^{\text{nsc}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (83)$$

$$z_{jt}^{\text{rru}} = d_t \left(c_{jt}^{\text{rru,on}} p_{jt}^{\text{rru,on}} + c_{jt}^{\text{rru,off}} p_{jt}^{\text{rru,off}} \right) \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (84)$$

$$z_{jt}^{\text{rrd}} = d_t \left(c_{jt}^{\text{rrd,on}} p_{jt}^{\text{rrd,on}} + c_{jt}^{\text{rrd,off}} p_{jt}^{\text{rrd,off}} \right) \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (85)$$

$$z_{jt}^{\text{qru}} = d_t c_{jt}^{\text{qru}} q_{jt}^{\text{qru}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (86)$$

$$z_{jt}^{\text{qrd}} = d_t c_{jt}^{\text{qrd}} q_{jt}^{\text{qrd}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (87)$$

2.6.6 Absolute reserve limits, based on ramp rates

A producing or consuming device may have limits on reserves, independent of real and reactive power dispatch, that are based on operating characteristics of the device. Since these constraints are independent of the dispatch, we refer to them as absolute reserve limits. The absolute reserve limits might typically be determined as an applicable ramp rate times the ramping duration for the lowest quality product considered. In general these limits are supplied as part of the reserve offer of a device and might differ from this simple formula for various reasons. Therefore we model these as parameters provided by the data. At each time scale, all the reserve products consuming ramping capability up to that time scale are included. Some products cannot be provided by some device types, and these are indicated here with bounds of 0.

$$p_{jt}^{\text{rgu}} \leq p_j^{\text{rgu,max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (88)$$

$$p_{jt}^{\text{rgd}} \leq p_j^{\text{rgd,max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (89)$$

$$p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} \leq p_j^{\text{scr,max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (90)$$

$$p_{jt}^{\text{nsc}} \leq p_j^{\text{nsc,max}} (1 - u_{jt}^{\text{on}}) \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (91)$$

$$p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{rru,on}} \leq p_j^{\text{rru,on,max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (92)$$

$$p_{jt}^{\text{nsc}} + p_{jt}^{\text{rru,off}} \leq p_j^{\text{rru,off,max}} (1 - u_{jt}^{\text{on}}) \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (93)$$

$$p_{jt}^{\text{rgd}} + p_{jt}^{\text{rrd,on}} \leq p_j^{\text{rrd,on,max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (94)$$

$$p_{jt}^{\text{rrd,off}} \leq p_j^{\text{rrd,off,max}} (1 - u_{jt}^{\text{on}}) \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (95)$$

$$p_{jt}^{\text{rrd,off}} = 0 \quad \forall t \in T, j \in J^{\text{pr}} \quad (96)$$

$$p_{jt}^{\text{nsc}} = 0 \quad \forall t \in T, j \in J^{\text{cs}} \quad (97)$$

$$p_{jt}^{\text{rru,off}} = 0 \quad \forall t \in T, j \in J^{\text{cs}} \quad (98)$$

2.6.7 Relative reserve limits, based on headroom to max/min, producing devices

A producing device j can provide reserves up to the space (often called headroom) between its power value p_{jt} and the appropriate operating limit p_{jt}^{\max} or p_{jt}^{\min} . Each reserve product occupies its own part of the appropriate headroom. Since these constraints on reserves depend on the dispatch, we refer to them as relative reserve limits. Up-reserve products consume headroom to upper operating limits, and down-reserve products consume headroom to lower operating limits. Online reserve provision variables are constrained relative to the dispatchable power p_{jt}^{on} , while offline reserve provision variables are constrained relative to the startup and shutdown power p_{jt}^{su} and p_{jt}^{sd} . Since the reserve provision variables are nonnegative, these constraints serve as the bounds on p_{jt}^{on} .

$$p_{jt}^{\text{on}} + p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{rru,on}} \leq p_{jt}^{\max} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr}} \quad (99)$$

$$p_{jt}^{\text{on}} - p_{jt}^{\text{rgd}} - p_{jt}^{\text{rrd,on}} \geq p_{jt}^{\min} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{pr}} \quad (100)$$

$$p_{jt}^{\text{su}} + p_{jt}^{\text{sd}} + p_{jt}^{\text{nsc}} + p_{jt}^{\text{rru,off}} \leq p_{jt}^{\max} (1 - u_{jt}^{\text{on}}) \quad \forall t \in T, j \in J^{\text{pr}} \quad (101)$$

Reactive power reserves have similar relative limits, and the appropriate upper and lower bounds are applied when the device is dispatchable or in its startup or shutdown power curve:

$$q_{jt} + q_{jt}^{\text{gru}} \leq q_{jt}^{\max} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) \quad \forall t \in T, j \in J^{\text{pr}} \quad (102)$$

$$q_{jt} - q_{jt}^{\text{grd}} \geq q_{jt}^{\min} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) \quad \forall t \in T, j \in J^{\text{pr}} \quad (103)$$

Producing devices having constraints linking real and reactive power have additional constraints on reactive power reserve depending on the real power dispatch, requiring the deployment of reactive power reserves be feasible under the dispatched real power value:

$$q_{jt} + q_{jt}^{\text{gru}} \leq q_j^{\max, \text{p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j^{\max} p_{jt} \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqmax}} \quad (104)$$

$$q_{jt} - q_{jt}^{\text{grd}} \geq q_j^{\min, \text{p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j^{\min} p_{jt} \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqmin}} \quad (105)$$

$$q_{jt} = q_j^{\text{p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j p_{jt} \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \quad (106)$$

$$q_{jt}^{\text{gru}} = 0 \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \quad (107)$$

$$q_{jt}^{\text{grd}} = 0 \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \quad (108)$$

2.6.8 Relative reserve limits, based on headroom to max/min, consuming devices

Relative reserve limits for consuming devices are symmetric to those for producing devices. The difference is that up-reserve products consume headroom to lower operating limits, and down-reserve products consume headroom to upper operating limits.

$$p_{jt}^{\text{on}} + p_{jt}^{\text{rgd}} + p_{jt}^{\text{rrd,on}} \leq p_{jt}^{\text{max}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{cs}} \quad (109)$$

$$p_{jt}^{\text{on}} - p_{jt}^{\text{rgu}} - p_{jt}^{\text{scr}} - p_{jt}^{\text{rru,on}} \geq p_{jt}^{\text{min}} u_{jt}^{\text{on}} \quad \forall t \in T, j \in J^{\text{cs}} \quad (110)$$

$$p_{jt}^{\text{su}} + p_{jt}^{\text{sd}} + p_{jt}^{\text{rrd,off}} \leq p_{jt}^{\text{max}} (1 - u_{jt}^{\text{on}}) \quad \forall t \in T, j \in J^{\text{cs}} \quad (111)$$

Relative limits on reactive power reserves, including reactive power dispatch, for consuming devices are similar to those for producing devices:

$$q_{jt} + q_{jt}^{\text{grd}} \leq q_{jt}^{\text{max}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) \quad \forall t \in T, j \in J^{\text{cs}} \quad (112)$$

$$q_{jt} - q_{jt}^{\text{gru}} \geq q_{jt}^{\text{min}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) \quad \forall t \in T, j \in J^{\text{cs}} \quad (113)$$

Consuming devices having constraints linking real and reactive power have additional constraints on reactive power reserve depending on the real power dispatch, requiring the deployment of reactive power reserves be feasible under the dispatched real power value:

$$q_{jt} + q_{jt}^{\text{grd}} \leq q_j^{\text{max,p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j^{\text{max}} p_{jt} \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqmax}} \quad (114)$$

$$q_{jt} - q_{jt}^{\text{gru}} \geq q_j^{\text{min,p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j^{\text{min}} p_{jt} \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqmin}} \quad (115)$$

$$q_{jt} = q_j^{\text{p0}} \left(u_{jt}^{\text{on}} + \sum_{t' \in T_{jt}^{\text{supc}}} u_{jt'}^{\text{su}} + \sum_{t' \in T_{jt}^{\text{sdpc}}} u_{jt'}^{\text{sd}} \right) + \beta_j p_{jt} \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \quad (116)$$

$$q_{jt}^{\text{gru}} = 0 \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \quad (117)$$

$$q_{jt}^{\text{grd}} = 0 \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \quad (118)$$

2.6.9 Energy cost and value

Producing or consuming device real power p_{jt} results in an objective term z_{jt}^{en} that is either a cost (for producing devices) or a value (for consuming devices). The value of this energy cost (or value) term is determined by a piecewise linear convex (or concave) cost (or value) function, evaluated at the power value p_{jt} . To model this relationship, we split the power into offer (or bid) blocks and apply an objective coefficient to each block:

$$0 \leq p_{jtm} \leq p_{jtm}^{\max} \quad \forall t \in T, j \in J^{\text{pr,cs}}, m \in M_{jt} \quad (119)$$

$$p_{jt} = \sum_{m \in M_{jt}} p_{jtm} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (120)$$

$$z_{jt}^{\text{en}} = d_t \sum_{m \in M_{jt}} c_{jtm}^{\text{en}} p_{jtm} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (121)$$

2.7 Shunt devices

Shunt real and reactive power p_{jt} and q_{jt} are:

$$p_{jt} = g_{jt}^{\text{sh}} v_{it}^2 \quad \forall t \in T, j \in J^{\text{sh}}, i = i_j \quad (122)$$

$$q_{jt} = -b_{jt}^{\text{sh}} v_{it}^2 \quad \forall t \in T, j \in J^{\text{sh}}, i = i_j \quad (123)$$

Shunt conductance and susceptance g_{jt}^{sh} and b_{jt}^{sh} are defined by a fixed step size times a variable number u_{jt}^{sh} of steps activated:

$$g_{jt}^{\text{sh}} = g_j^{\text{sh}} u_{jt}^{\text{sh}} \quad \forall t \in T, j \in J^{\text{sh}} \quad (124)$$

$$b_{jt}^{\text{sh}} = b_j^{\text{sh}} u_{jt}^{\text{sh}} \quad \forall t \in T, j \in J^{\text{sh}} \quad (125)$$

For each shunt, the number of steps activated is an integer that is bounded by input data:

$$u_{jt}^{\text{sh}} \in \{\dots, -1, 0, 1, \dots\} \quad \forall t \in T, j \in J^{\text{sh}} \quad (126)$$

$$u_j^{\text{sh,min}} \leq u_{jt}^{\text{sh}} \leq u_j^{\text{sh,max}} \quad \forall t \in T, j \in J^{\text{sh}} \quad (127)$$

2.8 Branch devices

2.8.1 Branch flow limits and penalties

Branch flows are subject to limits that are modeled as soft constraints, with overload incurring a penalty that appears in the objective. For each branch j and each interval t , the flow limit is applied to the apparent power at the from and to buses, the overload is represented by a nonnegative variable s_{jt}^+ , and the penalty is z_{jt}^{s} :

$$0 \leq s_{jt}^+ \quad \forall t \in T, j \in J^{\text{br}} \quad (128)$$

$$z_{jt}^{\text{s}} = d_t c^{\text{s}} s_{jt}^+ \quad \forall t \in T, j \in J^{\text{br}} \quad (129)$$

$$\left((p_{jt}^{\text{fr}})^2 + (q_{jt}^{\text{fr}})^2 \right)^{1/2} \leq s_j^{\text{max}} + s_{jt}^+ \quad \forall t \in T, j \in J^{\text{br}} \quad (130)$$

$$\left((p_{jt}^{\text{to}})^2 + (q_{jt}^{\text{to}})^2 \right)^{1/2} \leq s_j^{\text{max}} + s_{jt}^+ \quad \forall t \in T, j \in J^{\text{br}} \quad (131)$$

2.8.2 AC branch controls

AC branches j have variables ϕ_{jt} and τ_{jt} for the phase difference and winding ratio in intervals t . These variables are bounded by parameters from data. AC lines have phase difference equal to 0 and winding ratio equal to 1:

$$\phi_{jt} = 0 \quad \forall t \in T, j \in J^{\text{ln}} \quad (132)$$

$$\tau_{jt} = 1 \quad \forall t \in T, j \in J^{\text{ln}} \quad (133)$$

Fixed phase difference transformers have phase difference equal to the given initial value:

$$\phi_{jt} = \phi_j^0 \quad \forall t \in T, j \in J^{\text{fpd}} \quad (134)$$

Fixed winding ratio transformers have winding ratio equal to the given initial value:

$$\tau_{jt} = \tau_j^0 \quad \forall t \in T, j \in J^{\text{fwr}} \quad (135)$$

Variable phase difference transformers have phase difference bounds:

$$\phi_j^{\min} \leq \phi_{jt} \leq \phi_j^{\max} \quad \forall t \in T, j \in J^{\text{vpd}} \quad (136)$$

Variable winding ratio transformers have winding ratio bounds:

$$\tau_j^{\min} \leq \tau_{jt} \leq \tau_j^{\max} \quad \forall t \in T, j \in J^{\text{vwr}} \quad (137)$$

2.8.3 AC branch flows

For each AC branch device j and each interval t , real and reactive power flows into the branch at the from and to buses are represented by variables p^{fr} , q^{fr} , p_{jt}^{to} , and q_{jt}^{to} . These flows are defined by nonlinear functions of branch parameters and controls and bus voltages, including bus voltage angles θ_{it} , and are equal to 0 if the device is offline:

$$\begin{aligned} p_{jt}^{\text{fr}} &= u_{jt}^{\text{on}}((g_j^{\text{sr}} + g_j^{\text{fr}})v_{it}^2/\tau_{jt}^2 + (-g_j^{\text{sr}} \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\ &\quad - b_j^{\text{sr}} \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \quad \forall t \in T, j \in J^{\text{ac}}, i = i_j^{\text{fr}}, i' = i_j^{\text{to}} \end{aligned} \quad (138)$$

$$\begin{aligned} q_{jt}^{\text{fr}} &= u_{jt}^{\text{on}}((-b_j^{\text{sr}} - b_j^{\text{fr}} - b_j^{\text{ch}}/2)v_{it}^2/\tau_{jt}^2 + (b_j^{\text{sr}} \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\ &\quad - g_j^{\text{sr}} \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \quad \forall t \in T, j \in J^{\text{ac}}, i = i_j^{\text{fr}}, i' = i_j^{\text{to}} \end{aligned} \quad (139)$$

$$\begin{aligned} p_{jt}^{\text{to}} &= u_{jt}^{\text{on}}((g_j^{\text{sr}} + g_j^{\text{to}})v_{i't}^2 + (-g_j^{\text{sr}} \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\ &\quad + b_j^{\text{sr}} \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \quad \forall t \in T, j \in J^{\text{ac}}, i = i_j^{\text{fr}}, i' = i_j^{\text{to}} \end{aligned} \quad (140)$$

$$\begin{aligned} q_{jt}^{\text{to}} &= u_{jt}^{\text{on}}((-b_j^{\text{sr}} - b_j^{\text{to}} - b_j^{\text{ch}}/2)v_{i't}^2 + (b_j^{\text{sr}} \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\ &\quad + g_j^{\text{sr}} \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \quad \forall t \in T, j \in J^{\text{ac}}, i = i_j^{\text{fr}}, i' = i_j^{\text{to}} \end{aligned} \quad (141)$$

2.8.4 DC lines

For each DC line j and each interval t , the real and reactive power flows into the line at the from and to buses are represented by variables p_{jt}^{fr} , q_{jt}^{fr} , p_{jt}^{to} , q_{jt}^{to} . Bounds on these flows are

defined by input data:

$$-p_j^{\text{dc,max}} \leq p_{jt}^{\text{fr}} \leq p_j^{\text{dc,max}} \quad \forall t \in T, j \in J^{\text{dc}} \quad (142)$$

$$q_j^{\text{dc,min,fr}} \leq q_{jt}^{\text{fr}} \leq q_j^{\text{dc,max,fr}} \quad \forall t \in T, j \in J^{\text{dc}} \quad (143)$$

$$-p_j^{\text{dc,max}} \leq p_{jt}^{\text{to}} \leq p_j^{\text{dc,max}} \quad \forall t \in T, j \in J^{\text{dc}} \quad (144)$$

$$q_j^{\text{dc,min,to}} \leq q_{jt}^{\text{to}} \leq q_j^{\text{dc,max,to}} \quad \forall t \in T, j \in J^{\text{dc}} \quad (145)$$

DC line real power losses are not modeled in this formulation:

$$p_{jt}^{\text{fr}} + p_{jt}^{\text{to}} = 0 \quad \forall t \in T, j \in J^{\text{dc}} \quad (146)$$

2.9 Post-contingency power flow, balance, limits

The state of the grid following a contingency is modeled by a DC power flow ignoring changes to reactive power, voltage, and losses. Each contingency is characterized by the loss of a set of branches. Non-branch devices are assumed to maintain their real power values. System-wide real power mismatch among the non-branch devices is distributed over the buses in proportion to a prescribed slack distribution. The ensuing DC real power flow on the remaining branches is combined with the pre-contingency reactive power branch flows to form an approximation of the post-contingency branch apparent power flows. The approximate post-contingency branch apparent power flows are subject to post-contingency branch flow limits, formulated as soft constraints with any violation incurring a penalty that appears in the objective.

2.9.1 Penalty on post-contingency branch overload

Post-contingency branch overload is modeled by a nonnegative variable s_{jtk}^+ , which incurs in a penalty z_{jtk}^s appearing in the objective:

$$s_{jtk}^+ \geq 0 \quad \forall t \in T, k \in K, j \in J_k^{\text{br}} \quad (147)$$

$$z_{jtk}^s = d_t c^s s_{jtk}^+ \quad \forall t \in T, k \in K, j \in J_k^{\text{br}} \quad (148)$$

2.9.2 Post-contingency power flow limits

Post-contingency branch real power flow is modeled by a variable p_{jtk} , representing an approximation of the real power flow on branch j , oriented from the from bus to the to bus, in interval t , under contingency k . Changes to reactive power flow caused by contingencies are not modeled, so the post-contingency apparent power flow constraints at the from and to buses are modeled using the pre-contingency reactive power flows q_{jt}^{fr} and p_{jt}^{to} , the post-contingency reactive power p_{jtk} , the post-contingency apparent power overload s_{jtk}^+ , and the post-contingency flow limit $s_j^{\text{max,ctg}}$:

$$\left((p_{jtk})^2 + (q_{jt}^{\text{fr}})^2 \right)^{1/2} \leq s_j^{\text{max,ctg}} + s_{jtk}^+ \quad \forall t \in T, k \in K, j \in J_k^{\text{br}} \quad (149)$$

$$\left((p_{jtk})^2 + (q_{jt}^{\text{to}})^2 \right)^{1/2} \leq s_j^{\text{max,ctg}} + s_{jtk}^+ \quad \forall t \in T, k \in K, j \in J_k^{\text{br}} \quad (150)$$

2.9.3 Post-contingency branch real power flows

Post-contingency branch real power flows follow a DC flow model. For this we introduce a variable θ_{itk} for the post-contingency voltage angle at bus i in interval t in contingency k . DC line flows, branch on-off status variables, and AC branch phase difference variables, are fixed to their pre-contingency values. Then the DC flow model is

$$p_{jtk} = p_{jt}^{\text{fr}} \quad \forall t \in T, k \in K, j \in J_k^{\text{dc}} \quad (151)$$

$$p_{jtk} = -b_j^{\text{sr}} u_{jt}^{\text{on}} (\theta_{itk} - \theta_{i'tk} - \phi_{jt}) \quad \forall t \in T, k \in K, j \in J_k^{\text{ac}}, i = i_j^{\text{fr}}, i' = i_j^{\text{to}} \quad (152)$$

2.9.4 Post-contingency real power balance

In the DC post-contingency model, the real power injections and withdrawals of non-branch devices, i.e. producing, consuming, and shunt devices, are fixed to their pre-contingency values, and real power balance is enforced at each bus. Since the pre-contingency real power values reflect an AC model that includes losses, and the post-contingency branch flows follow a lossless DC model, there will be a nonzero system-wide real power mismatch that is represented by a variable p_t^{sl} :

$$p_t^{\text{sl}} = \sum_{j \in J^{\text{pr}}} p_{jt} - \sum_{j \in J^{\text{cs}}} p_{jt} - \sum_{j \in J^{\text{sh}}} p_{jt} \quad \forall t \in T \quad (153)$$

The system slack is distributed across the buses i in proportion to fixed slack distribution coefficients α_i . Then the post-contingency power balance constraints are

$$\begin{aligned} & \sum_{j \in J_k^{\text{br}} \cap J_i^{\text{fr}}} p_{jtk} - \sum_{j \in J_k^{\text{br}} \cap J_i^{\text{to}}} p_{jtk} \\ & = \sum_{j \in J_i^{\text{pr}}} p_{jt} - \sum_{j \in J_i^{\text{cs}}} p_{jt} - \sum_{j \in J_i^{\text{sh}}} p_{jt} - \alpha_i p_t^{\text{sl}} \quad \forall t \in T, k \in K, i \in I \end{aligned} \quad (154)$$

3 Data Formats and Construction of Derived Data

Input (problem) and output (solution) data formats are described in [5]. The data directly provided by the input data file is referred to as elementary data. Other data required by the problem formulation is derived data, computed from elementary data. The output data file contains the data on solution variable values that is needed to completely reconstruct and evaluate the solution.

3.1 Elementary Data

Data read directly from the input data file is referred to as elementary data. Elementary data items are listed below:

- Index sets: F, I, J, K, M, N, T, W

- Subsets: $J^{dc}, J^{ln}, J^{xf}, J^{pr}, J^{cs}, J^{sh}, J^{pqe}, J^{pqmax}, J^{pqmin}, F_j, J_k^{out}, M_{jt}, N^p, N^q, N_i, W_j^{en,max}, W_j^{en,min}, W_j^{su,max}$
- Special elements of sets: i_j, i_j^{fr}, i_j^{to}
- Parameters: $a_w^{en,max,start}, a_w^{en,max,end}, a_w^{en,min,start}, a_w^{en,min,end}, a_w^{su,max,start}, a_w^{su,max,end}, b_j^{ch}, b_j^{fr}, b_j^{sh}, b_j^{to}, c_{jtm}^{en}, c^p, c^q, c_{jt}^{rgd}, c_{jt}^{rgu}, c_{jt}^{scr}, c_{jt}^{nsc}, c_{jt}^{rru,off}, c_{jt}^{rru,on}, c_{jt}^{rrd,off}, c_{jt}^{rrd,on}, c_{jt}^{qru}, c_{jt}^{qrd}, c_n^{rgu}, c_n^{rgd}, c_n^{scr}, c_n^{nsc}, c_n^{rru}, c_n^{rrd}, c_n^{qru}, c_n^{qrd}, c^s, c_j^{on}, c_j^{su}, c_{jf}^{sus}, c_j^{sd}, d_t, d_{jf}^{dn,max}, d_j^{dn,min}, d_j^{up,min}, e_w^{max}, e_w^{min}, g_j^{fr}, g_j^{sh}, g_j^{to}, p_j^{dc,max}, p_{jtm}^{max}, p_{jt}^{max}, p_{jt}^{min}, p_j^{rd}, p_j^{rd,sd}, p_j^{ru}, p_j^{ru,su}, p_j^0, p_j^{rgu,max}, p_j^{rgd,max}, p_j^{scr,max}, p_j^{nsc,max}, p_j^{rru,on,max}, p_j^{rru,off,max}, p_j^{rrd,on,max}, p_j^{rrd,off,max}, p_{nt}^{rru,min}, p_{nt}^{rrd,min}, q_j^0, q_j^{p0}, q_j^{max,p0}, q_j^{min,p0}, q_{jt}^{max}, q_{jt}^{min}, q_j^{dc,fr,max}, q_j^{dc,to,max}, q_j^{dc,fr,min}, q_j^{dc,to,min}, q_{nt}^{qru,min}, q_{nt}^{qrd,min}, r_j^{sr}, s_j^{max}, s_j^{max,ctg}, u_{jt}^{on,max}, u_{jt}^{on,min}, u_j^{on,0}, u_w^{su,max}, u_j^{sh,0}, u_j^{sh,max}, u_j^{sh,min}, v_i^0, v_i^{max}, v_i^{min}, x_j, \beta_j, \beta_j^{max}, \beta_j^{min}, \sigma_n^{rgd}, \sigma_n^{rgu}, \sigma_n^{scr}, \sigma_n^{nsc}, \tau_j^0, \tau_j^{max}, \tau_j^{min}, \theta_i^0, \phi_j^0, \phi_j^{max}, \phi_j^{min}$

3.2 Construction of Derived Data from Elementary Data

Data constructed from elementary data is referred to as derived data. This section shows how the derived data is constructed from the elementary data.

The set J_k^{out} of devices outaged in contingency k contains a single element, and this element is identified as j_k^{out} . The set T of time intervals is ordered, and the first and last elements of this set are identified as t^{end} and t^{start} .

The set of buses in each reserve zone is constructed from the reserve zones for each bus:

$$I_n = \{i \in I : n \in N_i\} \quad \forall n \in N \quad (155)$$

Certain union sets are constructed:

$$J^{ac} = J^{ln} \cup J^{xf} \quad (156)$$

$$J^{br} = J^{ac} \cup J^{dc} \quad (157)$$

$$J^{pr,cs} = J^{pr} \cup J^{cs} \quad (158)$$

$$J^{pr,cs,ac} = J^{pr} \cup J^{cs} \cup J^{ac} \quad (159)$$

Transformers are categorized as fixed or variable with respect to phase difference and winding ratio according to the presented bounds on controls:

$$J^{vpd} = \{j \in J^{xf} : \phi^{min} < \phi^{max}\} \quad (160)$$

$$J^{vwr} = \{j \in J^{xf} : \tau^{min} < \tau^{max}\} \quad (161)$$

$$J^{fpd} = J^{xf} \setminus J^{vpd} \quad (162)$$

$$J^{fwr} = J^{xf} \setminus J^{vwr} \quad (163)$$

Devices at each bus:

$$J_i^{\text{fr}} = \{j \in J^{\text{br}} : i_j^{\text{fr}} = i\} \quad \forall i \in I \quad (164)$$

$$J_i^{\text{to}} = \{j \in J^{\text{br}} : i_j^{\text{to}} = i\} \quad \forall i \in I \quad (165)$$

$$J_i^{\text{br}} = J_i^{\text{fr}} \cup J_i^{\text{to}} \quad \forall i \in I \quad (166)$$

$$J_i^{\text{pr}} = \{j \in J^{\text{pr}} : i_j = i\} \quad \forall i \in I \quad (167)$$

$$J_i^{\text{cs}} = \{j \in J^{\text{cs}} : i_j = i\} \quad \forall i \in I \quad (168)$$

$$J_i^{\text{sh}} = \{j \in J^{\text{sh}} : i_j = i\} \quad \forall i \in I \quad (169)$$

$$J_i = J_i^{\text{br}} \cup J_i^{\text{pr}} \cup J_i^{\text{cs}} \cup J_i^{\text{sh}} \quad \forall i \in I \quad (170)$$

Devices in each reserve zone:

$$J_n^{\text{pr}} = \cup_{i \in I_n} J_i^{\text{pr}} \quad \forall n \in N \quad (171)$$

$$J_n^{\text{cs}} = \cup_{i \in I_n} J_i^{\text{cs}} \quad \forall n \in N \quad (172)$$

A device is outaged in a given time interval if it is prevented from operating by either its on-off state bounds or its minimum downtime and prior downtime at the start of the model horizon:

$$T_j^{\text{out}} = \{t \in T : u_{jt}^{\text{max}} = 0\} \quad \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{up},0} > 0 \quad (173)$$

$$T_j^{\text{out}} = \{t \in T : u_{jt}^{\text{max}} = 0\} \cup \{t \in T : d_j^{\text{dn},0} + a_t^{\text{start}} < d_j^{\text{dn},\text{min}}\} \\ \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{dn},0} > 0 \quad (174)$$

Similarly, a device is must-run in a given interval if it is required to be operating by either its on-off state bounds or its minimum uptime and prior uptime at the start of the model horizon:

$$T_j^{\text{mr}} = \{t \in T : u_{jt}^{\text{min}} = 1\} \quad \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{dn},0} > 0 \quad (175)$$

$$T_j^{\text{mr}} = \{t \in T : u_{jt}^{\text{min}} = 1\} \cup \{t \in T : d_j^{\text{up},0} + a_t^{\text{start}} < d_j^{\text{up},\text{min}}\} \\ \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{up},0} > 0 \quad (176)$$

The devices in service in a given contingency are those that are not out of service:

$$J_k = J \setminus J_k^{\text{out}} \quad \forall k \in K \quad (177)$$

Certain useful intersection sets are constructed:

$$J_k^{\text{ac}} = J^{\text{ac}} \cap J_k \quad \forall k \in K \quad (178)$$

$$J_k^{\text{dc}} = J^{\text{dc}} \cap J_k \quad \forall k \in K \quad (179)$$

$$J_k^{\text{br}} = J^{\text{br}} \cap J_k \quad \forall k \in K \quad (180)$$

Conductance and susceptance of the series element of each AC branch is constructed:

$$g_j^{\text{sr}} = r_j^{\text{sr}} / ((r_j^{\text{sr}})^2 + (x_j^{\text{sr}})^2) \quad \forall j \in J^{\text{ac}} \quad (181)$$

$$b_j^{\text{sr}} = -x_j^{\text{sr}} / ((r_j^{\text{sr}})^2 + (x_j^{\text{sr}})^2) \quad \forall j \in J^{\text{ac}} \quad (182)$$

Time interval start, end, and midpoint times are constructed from interval durations:

$$a_t^{\text{start}} = \sum_{t' < t} d_{t'} \quad \forall t \in T \quad (183)$$

$$a_t^{\text{end}} = \sum_{t' \leq t} d_{t'} \quad \forall t \in T \quad (184)$$

$$a_t^{\text{mid}} = 1/2(a_t^{\text{start}} + a_t^{\text{end}}) \quad \forall t \in T \quad (185)$$

Sets of time intervals for minimum downtime and minimum uptime constraints are constructed by:

$$T_{jt}^{\text{dn,min}} = \{t' < t : a_t^{\text{start}} - a_{t'}^{\text{start}} < d^{\text{dn,min}}\} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (186)$$

$$T_{jt}^{\text{up,min}} = \{t' < t : a_t^{\text{start}} - a_{t'}^{\text{start}} < d^{\text{up,min}}\} \quad \forall t \in T, j \in J^{\text{pr,cs}} \quad (187)$$

Startup and shutdown power curves are derived from startup and shutdown ramp rates:

$$p_{jtt'}^{\text{supc}} = p_{jt'}^{\text{min}} - p_j^{\text{ru,su}}(a_{t'}^{\text{end}} - a_t^{\text{end}}) \quad \forall j \in J^{\text{pr,cs}}, t < t' \in T \quad (188)$$

$$T_{jt}^{\text{supc}} = \{t' > t : p_{jtt'}^{\text{supc}} > 0\} \quad \forall j \in J^{\text{pr,cs}}, t \in T \quad (189)$$

$$p_{jtt'}^{\text{sdpc}} = p_{j,t'-1}^{\text{min}} - p_j^{\text{rd,sd}}(a_t^{\text{end}} - a_{t'}^{\text{start}}) \quad \forall j \in J^{\text{pr,cs}}, t, t' \in T, t \geq t' > t^{\text{start}} \quad (190)$$

$$p_{jtt'}^{\text{sdpc}} = p_j^0 - p_j^{\text{rd,sd}}(a_t^{\text{end}} - a_{t'}^{\text{start}}) \quad \forall j \in J^{\text{pr,cs}}, t, t' \in T, t \geq t' = t^{\text{start}} \quad (191)$$

$$T_{jt}^{\text{sdpc}} = \{t' \leq t : p_{jtt'}^{\text{sdpc}} > 0\} \quad \forall j \in T^{\text{pr,cs}}, t \in T \quad (192)$$

Time intervals for modeling downtime-dependent startup costs are derived from elementary data on maximum prior downtime for each startup state:

$$T_{jft}^{\text{sus}} = \{t' \in T : t' < t, a_t^{\text{start}} - a_{t'}^{\text{start}} \leq d_{jf}^{\text{dn,max}}\} \quad \forall j \in J^{\text{pr,cs}}, f \in F_j, t \in T \quad (193)$$

$$T_{jf}^{\text{sus}} = \{t \in T : d_j^{\text{dn,0}} + a_t^{\text{start}} > d^{\text{dn,max}}\} \quad \forall j \in J^{\text{pr,cs}}, f \in F_j \quad (194)$$

Sets of time intervals for constraints modeling maximum startups in a prescribed interval are constructed by:

$$T_w^{\text{su,max}} = \{t \in T : a_w^{\text{su,max,start}} \leq a_t^{\text{start}} \text{ and } a_t^{\text{start}} < a_w^{\text{su,max,end}}\} \\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \quad (195)$$

Sets of time intervals for constraints modeling maximum or minimum energy over a prescribed interval are constructed by:

$$T_w^{\text{en,max}} = \{t \in T : a_w^{\text{en,max,start}} < a_t^{\text{mid}} \text{ and } a_t^{\text{mid}} \leq a_w^{\text{en,max,end}}\} \\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \quad (196)$$

$$T_w^{\text{en,min}} = \{t \in T : a_w^{\text{en,min,start}} < a_t^{\text{mid}} \text{ and } a_t^{\text{mid}} \leq a_w^{\text{en,min,end}}\} \\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \quad (197)$$

Bus participation factors for the post-contingency power flow and balance model are uniform and set up to sum to 1:

$$\alpha_i = 1/|I| \quad \forall i \in I \quad (198)$$

3.3 Output data

The output data file needs to contain the following output data items, so as to completely reconstruct the solution:

- Bus voltage magnitudes and angles: v_{it}, θ_{it} for $t \in T, i \in I$.
- Shunt steps activated: u_{jt}^{sh} for $t \in T, j \in J^{\text{sh}}$.
- AC line on-off status: u_{jt}^{on} for $t \in T, j \in J^{\text{ln}}$.
- Transformer on-off status and control variables: $u_{jt}^{\text{on}}, \phi_{jt}, \tau_{jt}$ for $t \in T, j \in J^{\text{xf}}$.
- DC line real and reactive power flows: $p_{jt}^{\text{fr}}, q_{jt}^{\text{fr}}, q_{jt}^{\text{to}}$ for $t \in T, j \in J^{\text{dc}}$.
- Producing and consuming device commitment, power dispatch, and reserve provision: $u_{jt}^{\text{on}}, p_{jt}^{\text{on}}, q_{jt}, p_{jt}^{\text{rgu}}, p_{jt}^{\text{rgd}}, p_{jt}^{\text{scr}}, p_{jt}^{\text{nsc}}, p_{jt}^{\text{rru,on}}, p_{jt}^{\text{rru,off}}, p_{jt}^{\text{rrd,on}}, p_{jt}^{\text{rrd,off}}, q_{jt}^{\text{gru}}, q_{jt}^{\text{grd}}$ for $t \in T, j \in J^{\text{pr}} \cup J^{\text{cs}}$.

4 Data Properties

This section describes properties that the input data of the problem should have. For an instance of the problem, these properties can be checked, and any violation should be treated as a data error. Python code checking these properties is available in [4], which relies on a data model code in [3]. We note here that many of these properties concern the mutual consistency of sets of parameters defining lower and upper bounds on variables or expressions. In such cases we typically assert only that the lower bound is less than or equal to the upper bound. In particular, the two bounds may be equal unless this is explicitly ruled out. The performance of some solution techniques may be affected by variables whose lower and upper bounds are equal.

Devices include branch devices and non-branch devices:

$$J^{\text{br}} \subset J \tag{199}$$

Branch devices include AC branches and DC lines:

$$J^{\text{br}} = J^{\text{ac}} \cup J^{\text{dc}} \tag{200}$$

$$J^{\text{ac}} \cap J^{\text{dc}} = \{\} \tag{201}$$

AC branches include AC lines and transformers:

$$J^{\text{ac}} = J^{\text{ln}} \cup J^{\text{xf}} \tag{202}$$

$$J^{\text{ln}} \cap J^{\text{xf}} = \{\} \tag{203}$$

Transformers have either fixed or variable phase difference:

$$J^{\text{xf}} = J^{\text{fpd}} \cup J^{\text{vpd}} \tag{204}$$

$$J^{\text{fpd}} \cap J^{\text{vpd}} = \{\} \tag{205}$$

Transformers have either fixed or variable winding ratio:

$$J^{xf} = J^{fwr} \cup J^{vwr} \quad (206)$$

$$J^{fwr} \cap J^{vwr} = \{\} \quad (207)$$

No transformer is both variable phase difference and variable winding ratio:

$$J^{vpd} \cap J^{vwr} = \{\} \quad (208)$$

Non-branch devices include shunts, producing devices, and consuming devices:

$$J \setminus J^{br} = J^{sh} \cup J^{pr} \cup J^{cs} \quad (209)$$

$$J^{sh} \cap J^{pr} = \{\} \quad (210)$$

$$J^{sh} \cap J^{cs} = \{\} \quad (211)$$

$$J^{pr} \cap J^{cs} = \{\} \quad (212)$$

The devices outaged in a contingency are branch devices:

$$J_k^{out} \subset J^{br} \quad \forall k \in K \quad (213)$$

Each contingency outages exactly one device:

$$J_k^{out} = \{j_k^{out}\} \quad \forall k \in K \quad (214)$$

In the base case and in each contingency, the graph consisting of all buses and all AC branches in service in the prior operating point is connected:

$$\text{The bus-branch graph on } (I, \{j \in J^{ac} : u_t^{on,0} = 1\}) \text{ is connected} \quad (215)$$

$$\text{The bus-branch graph on } (I, \{j \in J_k^{ac} : u_t^{on,0} = 1\}) \text{ is connected } \forall k \in K \quad (216)$$

For each device, the prior on-off status is a binary integer:

$$u_j^{on,0} \in \{0, 1\} \quad \forall j \in J^{pr,cs,ac} \quad (217)$$

For each device, on-off status bounds are binary integers and are mutually consistent:

$$u_{jt}^{max} \in \{0, 1\} \quad \forall t \in T, j \in J^{pr,cs} \quad (218)$$

$$u_{jt}^{min} \in \{0, 1\} \quad \forall t \in T, j \in J^{pr,cs} \quad (219)$$

$$u_{jt}^{min} \leq u_{jt}^{max} \quad \forall t \in T, j \in J^{pr,cs} \quad (220)$$

For each shunt, the bounds on the number of activated steps and the initial number of activated steps are nonnegative integers and are mutually consistent:

$$u_j^{sh,max} \in \{0, 1, \dots\} \quad \forall j \in J^{sh} \quad (221)$$

$$u_j^{sh,min} \in \{0, 1, \dots\} \quad \forall j \in J^{sh} \quad (222)$$

$$u_j^{sh,0} \in \{0, 1, \dots\} \quad \forall j \in J^{sh} \quad (223)$$

$$u_j^{sh,min} \leq u_j^{sh,0} \leq u_j^{sh,max} \quad \forall j \in J^{sh} \quad (224)$$

For each device, the prior uptime and prior downtime are both nonnegative, and exactly one of them is positive:

$$d_j^{\text{up},0} \geq 0 \quad \forall j \in J \quad (225)$$

$$d_j^{\text{dn},0} \geq 0 \quad \forall j \in J \quad (226)$$

$$d_j^{\text{up},0} = 0 \text{ or } d_j^{\text{dn},0} = 0 \quad \forall j \in J \quad (227)$$

$$d_j^{\text{up},0} > 0 \text{ or } d_j^{\text{dn},0} > 0 \quad \forall j \in J \quad (228)$$

For each bus, the prior voltage value and the voltage bounds are positive and mutually consistent:

$$0 < v_i^{\min} \leq v_i^0 \leq v_i^{\max} \quad \forall i \in I \quad (229)$$

For each AC branch, the impedance of the series element is non-zero:

$$r_j^{\text{sr}} \neq 0 \text{ or } x_j^{\text{sr}} \neq 0 \quad \forall j \in J^{\text{ac}} \quad (230)$$

For each transformer, the bounds and initial values on phase difference and winding ratio are mutually consistent, and the winding ratios are positive:

$$\phi_j^{\min} \leq \phi_j^0 \leq \phi_j^{\max} \quad \forall j \in J^{\text{xf}} \quad (231)$$

$$0 < \tau_j^{\min} \leq \tau_j^0 \leq \tau_j^{\max} \quad \forall j \in J^{\text{xf}} \quad (232)$$

For each transformer, either the bounds on phase difference are equal to each other, or the bounds on winding ratio are equal to each other:

$$\tau_j^{\min} = \tau_j^{\max} \text{ or } \phi_j^{\min} = \phi_j^{\max} \quad \forall j \in J^{\text{xf}} \quad (233)$$

For each DC line, the bounds on reactive power at each bus are mutually consistent, the bounds on real power are nonnegative, and the bounds permit 0 flow:

$$p_j^{\text{dc,max}} \geq 0 \quad \forall j \in J^{\text{dc}} \quad (234)$$

$$q_j^{\text{dc,fr,min}} \leq 0 \leq q_j^{\text{dc,fr,max}} \quad \forall j \in J^{\text{dc}} \quad (235)$$

$$q_j^{\text{dc,to,min}} \leq 0 \leq q_j^{\text{dc,to,max}} \quad \forall j \in J^{\text{dc}} \quad (236)$$

For each producing or consuming device, the p bounds are nonnegative, and p and q bounds are mutually consistent:

$$0 \leq p_{jt}^{\min} \leq p_{jt}^{\max} \quad \forall t \in T, j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (237)$$

$$q_{jt}^{\min} \leq q_{jt}^{\max} \quad \forall t \in T, j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (238)$$

For each producing or consuming device, the ramping limits and the absolute bounds on

reserve provision are nonnegative:

$$p_j^{\text{ru}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (239)$$

$$p_j^{\text{rd}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (240)$$

$$p_j^{\text{ru,su}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (241)$$

$$p_j^{\text{rd,sd}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (242)$$

$$p_j^{\text{rgu,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (243)$$

$$p_j^{\text{rgd,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (244)$$

$$p_j^{\text{scr,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (245)$$

$$p_j^{\text{nsc,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (246)$$

$$p_j^{\text{rru,on,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (247)$$

$$p_j^{\text{rru,off,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (248)$$

$$p_j^{\text{rrd,on,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (249)$$

$$p_j^{\text{rrd,off,max}} \geq 0 \quad \forall j \in J^{\text{pr}} \cup J^{\text{cs}} \quad (250)$$

For each producing or consuming device, the bid or offer blocks have nonnegative width:

$$p_{jtm}^{\text{max}} \geq 0 \quad \forall t \in T, j \in J^{\text{pr}} \cup J^{\text{cs}}, m \in M_{jt} \quad (251)$$

For each producing or consuming device with p-q constraints, the bounds are mutually consistent:

$$q_j^{\text{min,p0}} \leq q_j^{\text{max,p0}} \quad \forall j \in J^{\text{pqmax}} \cap J^{\text{pqmin}} \quad (252)$$

For each branch device, the apparent power flow limits are positive, and the contingency limits are not tighter than the base case limits:

$$s_j^{\text{max}} > 0 \quad \forall j \in J^{\text{br}} \quad (253)$$

$$s_j^{\text{max,ctg}} \geq s_j^{\text{max}} \quad \forall j \in J^{\text{br}} \quad (254)$$

For each time interval, the duration is positive:

$$d_t > 0 \quad \forall t \in T \quad (255)$$

The reserve requirements are nonnegative:

$$\sigma_n^{\text{rgu}} \geq 0 \quad \forall n \in N^{\text{P}} \quad (256)$$

$$\sigma_n^{\text{rgd}} \geq 0 \quad \forall n \in N^{\text{P}} \quad (257)$$

$$\sigma_n^{\text{scr}} \geq 0 \quad \forall n \in N^{\text{P}} \quad (258)$$

$$\sigma_n^{\text{nsc}} \geq 0 \quad \forall n \in N^{\text{P}} \quad (259)$$

$$p_{nt}^{\text{rru,min}} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (260)$$

$$p_{nt}^{\text{rrd,min}} \geq 0 \quad \forall t \in T, n \in N^{\text{P}} \quad (261)$$

$$q_{nt}^{\text{qru,min}} \geq 0 \quad \forall t \in T, n \in N^{\text{q}} \quad (262)$$

$$q_{nt}^{\text{qrd,min}} \geq 0 \quad \forall t \in T, n \in N^{\text{q}} \quad (263)$$

Constraint relaxation penalties are nonnegative:

$$c^p \geq 0 \tag{264}$$

$$c^q \geq 0 \tag{265}$$

$$c^s \geq 0 \tag{266}$$

$$c_n^{\text{rgu}} \geq 0 \forall n \in N^p \tag{267}$$

$$c_n^{\text{rgd}} \geq 0 \forall n \in N^p \tag{268}$$

$$c_n^{\text{scr}} \geq 0 \forall n \in N^p \tag{269}$$

$$c_n^{\text{nsc}} \geq 0 \forall n \in N^p \tag{270}$$

$$c_n^{\text{rru}} \geq 0 \forall n \in N^p \tag{271}$$

$$c_n^{\text{rrd}} \geq 0 \forall n \in N^p \tag{272}$$

$$c_n^{\text{qru}} \geq 0 \forall n \in N^q \tag{273}$$

$$c_n^{\text{qr}} \geq 0 \forall n \in N^q \tag{274}$$

Certain index sets are nonempty:

$$I \neq \{\} \tag{275}$$

$$T \neq \{\} \tag{276}$$

5 Solver Requirements

The solver should be invoked with a command taking certain arguments. The arguments have not been finalized but will include a run time limit and an indication of whether topology switching is allowed. If topology switching is not allowed, then the solution must have $u_{jt}^{\text{on}} = u_j^{\text{on},0}$ for all $t \in T, j \in J^{\text{ac}}$.

6 Solution Evaluation

Solution evaluation is performed by a Python code ([4]) that in turn relies on a data model code ([3]). Broadly, solution evaluation includes the following steps and general organizing principles:

- Where possible without ambiguity, use projection, constraints, and the optimality principle to determine the values of certain variables once the values of other variables are fixed, enabling the construction of a complete solution from only the variables contained in the solution data file.
- Evaluate constraints involving only discrete variables before those involving continuous variables.
- Evaluate one time interval of the solution before the next time interval.
- Project v variables onto bounds.

- Check integrality of integer variables: If any are more than 0.25 away from the nearest integer, then the solution is deemed infeasible.
- Round integer variables.
- Project u^{on} variables onto bounds and any constraints restricting startup or shutdown relative to the previous interval.
- Check base case and post-contingency connectivity constraints Eqs. (59) and (60) using the `connected_components` and `bridges` functions from the NetworkX ([2]) Python module ([1]).
- Project p^{on} variables for producing and consuming devices onto bounds determined by u^{on} , p max/min, and ramping limits.
- Compute p^{su} , p^{sd} , and p variables for producing and consuming devices.
- Project q variables for producing and consuming devices onto bounds determined by u^{on} , q max/min, and ramping limits.
- Project p^{fr} , q^{fr} , and q^{to} variables for DC lines onto bounds, and set $p^{\text{to}} = -p^{\text{fr}}$.
- Project u^{sh} onto bounds.
- Compute g^{sh} and b^{sh} .
- Compute p^{sh} and q^{sh} .
- Project ϕ and τ onto bounds.
- Compute p^{fr} , q^{fr} , p^{to} , q^{to} for AC branches.
- Compute branch limit violations and costs.
- Compute bus real and reactive power imbalance and costs.
- Project device reserve provision variables onto bounds and constraints, in the following order: regulation up, regulation down, synchronized reserve, non-synchronized reserve, ramping reserve up online, ramping reserve up offline, ramping reserve down online, ramping reserve down offline, reactive power reserve up, reactive power reserve down.
- Compute device reserve costs
- Compute zonal reserve shortages and costs.
- Compute device energy costs and values.
- Compute device on, startup, shutdown costs.
- Solve the post-contingency DC model in each contingency for the post-contingency branch flows, and compute the post-contingency branch flow limit violations and costs.

- Add up all the costs, values, and penalties to compute the market surplus objective.
- If any projection step fails due to an empty target set, then the solution is infeasible.
- Constraints not included in the formulation but passed to solver through command arguments are also checked, and if there are any violations, then the solution is infeasible.
- If the solution file cannot be read due to incorrect formatting, then the solution is infeasible.
- An evaluation report is produced, including, minimally, an indication of feasibility and, if feasible, the computed objective value.

7 Change Log

7.1 July 1, 2022

Extensive changes have been made to the formulation since the previous version (February 16, 2022). These changes are too numerous to list in detail here. In particular, the previous version had multi-mode devices, which were conceived of as a means for modeling combined cycle generators and storage devices. In the current version of the formulation, multi-mode devices have been removed. For trial event 1, multi-mode devices will not be a part of the formulation, and combined cycle generators and storage devices will not be modeled explicitly as such. Combined cycle generators, storage devices, and multi-mode devices may be added to the formulation later. Some, but not all, specific changes are listed here.

Added sections: Section 3 on data formats and construction, Section 4 on data properties, Section 5 on solver requirements, Section 6 on solution evaluation, and a change log in Section 7.

Reordered equations and added hierarchical structure and explanatory text in Section 2.

Changed network connectivity constraint in Eqs. (59) and (60), i.e requirement on solutions: In each time interval, in the base case and contingency, the bus-branch graph consisting of all buses and the AC in service online branches is connected.

Added network connectivity data property in Eqs. (215) and (216), i.e. requirement on data: In the base case and each contingency, the bus-branch graph consisting of all buses and the AC branches in service in the prior operating point is connected.

Removed all references to multi-mode devices and storage devices. Multi-mode devices and storage devices will not appear in the problem instances for Trial 1. These devices may be added to the formulation later.

Added constraints limiting reactive power reserve provision by devices based on reactive power dispatch and real power dispatch, for devices with linear constraints linking real and reactive power.

8 References

- [1] Networkx. <https://networkx.org/documentation/stable/index.html>.

- [2] Aric A. Hagberg, Daniel A. Schult, and Pieter J. Swart. Exploring network structure, dynamics, and function using networkx. In Gäel Varoquaux, Travis Vaught, and Jarrod Millman, editors, *Proceedings of the 7th Python in Science Conference (SciPy2008)*, pages 11–15, Pasadena, CA USA, August 2008.
- [3] Elaine Hale and Tarek Elgindy. Bid-ds-data-model. **TBA**.
- [4] Jesse Holzer and Arun Veeramany. C3datautilities. **TBA**.
- [5] Terrence Mak. Arpa-e go competition challenge 3 data format. **TBA**.