Grid Optimization Competition
Challenge 2 Problem Formulation

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November 24, 2020

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1 Introduction

1.1 Background

This document contains the official formulation that will be used for evaluation in Challenge 2 of the Grid Optimization (GO) Competition. Minor changes may occur within the formulation. Entrants will be notified when a new version is released. Changes are not expected to be of a significance that would cause a change in approach for the Entrants.

This formulation builds upon the Challenge 1 formulation published in ARPA-E DE-FOA-0001952. Entrants will be judged based on the current official Challenge 2 formulation posted on the GO Competition website (this document, which is subject to change), not the formulation posted in DE-FOA-0001952. Entrants are permitted and encouraged to use any alternative problem formulation and modeling convention within their own software (such as convex relaxation, decoupled power flow formulations, current-voltage formulations, etc.) in an attempt to produce an exact or approximate solution to this particular mathematical program. However, the judging of all submitted approaches must conform to the official formulation presented here.

The following mathematical programming problem is a type of a security-constrained (AC based) optimal power flow, or SCOPF. There are many ways to formulate the SCOPF problem; this document may present multiple equivalent options for specified constraints. Entrants are strongly encouraged to study this formulation precisely and to engage with the broader community if anything is not clear (please see the FAQs and forum on the GO Competition website, https://gocompetition.energy.gov/).

This SCOPF problem is defined to be an alternating current (AC) formulation, which is based on a bus-branch power system network model with security constraints. In general, Entrants are tasked with determining the optimal dispatch and control settings for power generation and grid control equipment in order to maximize the market surplus associated with the operation of the grid, subject to pre- and post-contingency constraints. Feasible solutions must conform to operating standards including, but not limited to: minimum and maximum bus voltage magnitude limits, minimum and maximum real and reactive power generation from each generator, thermal transmission constraints, and constraints to ensure the reliability of the system while responding to unexpected events (i.e., a contingency). Feasible solutions must also be able to respond to contingencies of generators and transmission elements. This formulation allows for bus real and reactive power imbalance as well as branch (transmission line and transformer) rating exceedance, both at a cost included in the objective function.

Features added to this formulation since the Challenge 1 Competition include transformer tap settings, phase angle regulators, switchable shunts, transmission branch and transformer switching, generator ramp rate response to contingencies, start up and shut down of qualified generators, and price responsive demand. Please note that shunts are no longer modeled as using continuous variables within this formulation.

Challenge 2 will include power system network models that vary in size and complexity. The size of each network flow problem (number of nodes and branches) as well as the number of contingencies will vary across datasets. The largest models will reach to at least the size of the largest independent system operator in the United States. The problem presented here is a two stage single period problem with a given operating point prior to a base case state and then a post-contingency state. The modeling of the pre-contingency base case is a reflection of the first stage of a two-stage mathematical program whereas the post-contingency state represents the second stage. Limited unit commitment (the commitment/decommitment of generators) is included within the formulation only for generators designated as "fast-start". Other generators may not change their commitment in either the pre-contingency base case or the post-contingency state. Generator response between states (from the given prior operating point to the pre-contingency base case state, and from the pre-contingency base case to the post-contingency state) is limited to the available ramp rate response within each generator's operational limit given the length of time between each state. The first priority of post-contingency generator response should be to ensure a feasible post-contingency state, but this problem will also consider the market surplus of both the pre- and the post-contingency states.

1.2 Equivalent Formulations

This document describes an optimization problem in terms of a specific collection of sets, parameters, variables, constraints, and an objective, constituting the formulation of the problem. This optimization problem can be described by other formulations, different from the formulation that we have used, but algebraically equivalent to our formulation. We refer to the formulation presented in this document as the reference formulation. For example, the reference formulation describes the complex voltage of a bus in polar coordinates, with a voltage magnitude variable and a voltage angle variable, but there is an algebraically equivalent formulation in which the complex voltage is described in rectangular coordinates, with a real part and an imaginary part. And impedance correction constraints are formulated using a geometric description of a piecewise linear function defined by a sequence of vertices, but this constraint can be formulated using additional binary variables and linear constraints. We do not assert that the reference formulation is in any way better or worse than any of the algebraically equivalent formulations. We do not require or recommend that competitors use the reference formulation in any computer code that they write. We specify only the required format in which the solution should be expressed and the procedure used to evaluate the solution.

2 Symbol Reference

Units, notation, and the general nomenclature are given in Tables 1 to 10. These tables list sets, indices, subsets and special set elements, data parameters, and variables. As much as possible, the notation follows a system, which we explain here. First, a symbol consists of a main letter with attached notation such as subscripts, superscripts, oversets, and undersets. Two symbols with the same main letter but different attached notation are different symbols. Finally, the main letters of symbols generally follow conventions common in the optimal power flow literature and the optimization literature, though other letters are used where there is no established convention or to avoid an especially confusing clash of notations.

Units of measurement are listed in Table 1. Attached notation convention is given in Table 2. Main letter convention is given in Table 3. Sets are given in Table 4. Indices are given in Table 5. Subsets are given in Table 6. Distinguished set elements are given in Table 7. Strings are given in Table 8. Numeric parameters are given in Table 9. Variables are given in Table 10.

Table 1: Units of measurement

Unit	Description
1	dimensionless. Dimensionless real number quantities are indicated by a unit of 1.
bin	binary. Binary quantities, i.e. taking values in $\{0, 1\}$, are indicated by a unit of bin.
h	hour. Time is expressed in h.
deg	degree. In the input data files, angles are expressed in deg.
USD	US dollar. Cost, benefit, and objective values are expressed in USD.
kV	kilovolt. In the input data files, the voltage magnitude base values are expressed in kV.
MVAR	megavolt-ampere-reactive. In the input data files, reactive power is expressed in MVAR.
MVAR at 1 pu voltage	megavolt-ampere-reactive at unit voltage. In the input data files, susceptance is expressed in MVAR at 1 pu voltage, i.e. the indicated susceptance yields a reactive power flow (in MVARs) equal to the indicated amount when the voltage is equal to 1 pu
MW	megawatt. In the input data files, real power is expressed in MW.
MW at 1 pu voltage	megawatt at unit voltage. In the input data files, conductance is expressed in MW at 1 pu voltage, i.e. the indicated conductance yields a real power flow (in MW) equal to the indicated amount when the voltage is equal to 1 pu

Table 1: Continued

Unit	Description
pu	per unit. Real and reactive power, voltage magnitude, conductance, susceptance can be expressed in a per unit system under given base values, and the unit is denoted by pu This per unit system is used throughout the model parameters, variables, and equations, and solution output files. In the input data files, voltage magnitude is expressed in the per unit system.
rad	radian. In the model parameters, variables, and equations, and solution output files, angles are expressed in rad.

Table 2: Attached notation

example	description
$w_{\square}$	a subscript is used for an index in a set.
$w^{\square}$	a superscript is used for description of a symbol.
$\overline{w}$	overline is used for an upper bound.
$\underline{w}$	underline is used for a lower bound.
$\check{w}$	overset $\check{}$ indicates a midpoint value, e.g. midpoint between a lower bound and an upper bound.
$\tilde{w}$	overset $\sim$ indicates a base value for use in a per unit convention.
w^0	superscript 0 indicates a value in a given operating point.
w^o	superscript o indicates the origin (from, sending) bus of a branch.
w^d	superscript d indicates the destination (to, receiving) bus of a branch.
w^+	superscript + indicates an upper bound exceedance or the positive part of an equality constraint imbalance.
w^-	superscript $-$ indicates a lower bound exceedance.
W	sets are denoted by a capital letter.
$W_{\square}^{\square} \subset W$	subsets of a given set are denoted by the same letter as the given set, with distinguishing subscripts and superscripts.
$w \in W$	set elements of a given set are denoted by the same letter as the given set, in lower case.
w'	set elements are primed to denote different elements of the same set.

Table 3: Main letter convention

main letter	description
a	switched shunt block
b	susceptance
c	marginal cost or benefit
e	transmission line (arc in a transmission network)
f	transformer (arc in a transmission network)
g	conductance. g can also be generator. When g appears as a main letter with a subscript, as in g_e, g_f, it is a conductance value.
g	generator. g can also be a conductance value. When g appears as a set or an element of a set as in $g \in G$, or as a subscript on a main letter as in p_{gk}, q_{gk}, it is a generator.
h	switched shunt
i	bus (node in a transmission network)
j	load
k	case, i.e. base case or any of a set of contingency cases
m	vertex on the graph of a piecewise linear function
n	constant marginal cost segment of a convex piecewise linear cost function
p	real power
q	reactive power
r	apparent current (magnitude of complex current)
s	apparent power (magnitude of complex power)
t	interpolation coefficient
v	voltage magnitude
x	integer variable, e.g. commitment status, closed/open status, position number, or number of steps
z	market surplus, i.e. benefits minus costs
δ	length of time
ϵ	numerical tolerance
η	impedance correction factor
γ	line segment operator, $\gamma(w, w')$ denotes the line segment spanned by two points w and w'
τ	tap ratio
θ	bus voltage angle or transformer phase shift

Table 4: Primitive index sets

Symbol	Description
A	set of switched shunt blocks
E	set of lines (referred to as non-transformer branches in the RAW input file)
F	set of transformers (2-winding only)
G	set of generators
H	set of switched shunts
I	set of buses
J	set of loads
K	set of cases, i.e. the base case and contingency cases
M	set of vertices of graphs of piecewise linear functions
N	set of constant marginal cost segments of convex piecewise linear cost functions

Table 5: Indices

Symbol	Description
$a \in A$	indices of switched shunt blocks
$e \in E$	line indices
$f \in F$	transformer indices
$g \in G$	generator indices
$h \in H$	indices of switched shunts
$i \in I$	bus indices
$j \in J$	load indices
$k \in K$	case indices
$m \in M$	indices of vertices of graphs of piecewise linear functions
$n \in N$	indices of constant marginal cost segments of convex piecewise linear cost functions

Table 6: Subsets

Symbol	Description
$A_h \subset A$	switched shunt blocks for switched shunt h
$E_i^d \subset E$	lines with destination bus i, $E_i^d = \{e \in E : i_e^d = i\}$

Table 6: Continued

Symbol	Description
$E_i^o \subset E$	lines with origin bus i, $E_i^o = \{e \in E : i_e^o = i\}$
$E_k \subset E$	lines in service in case k
$E^{sw} \subset E$	lines for which switching is permitted pre- or post-contingency
$F_i^d \subset F$	transformers with destination bus i, $F_i^d = \{f \in F : i_f^d = i\}$
$F_i^o \subset F$	transformers with origin bus i, $F_i^o = \{f \in F : i_f^o = i\}$
$F_k \subset F$	transformers in service in case k
$F^{sw} \subset F$	transformers for which switching is permitted pre- or post-contingency
$F^\tau \subset F$	transformers with variable tap ratio
$F^\theta \subset F$	transformers with variable phase shift
$F^\eta \subset F$	transformers with impedance correction
$G_i \subset G$	generators connected to bus i, $G_i = \{g \in G : i_g = i\}$
$G_k \subset G$	generators in service in case k
$G^{su} \subset G$	generators that are able to start up pre-contingency
$G^{sd} \subset G$	generators that are able to shut down pre-contingency
$G^{su,ct} \subset G$	generators that are able to start up post-contingency
$G^{sd,ct} \subset G$	generators that are able to shut down post-contingency
$H_i \subset H$	switched shunts at bus i
$H_k \subset H$	switched shunts in service in case k
$J_i \subset J$	loads at bus i
$J_k \subset J$	loads in service in case k
$M_f \subset M$	vertices of the piecewise linear impedance correction function of transformer f
$N_g \subset N$	constant marginal cost segments of convex piecewise linear cost function for generator g real power output
$N_j \subset N$	constant marginal benefit segments of convex piecewise linear benefit function for load j real power consumption
$N^p \subset N$	constant marginal cost segments of convex piecewise linear cost functions for real power constraints
$N^q \subset N$	constant marginal cost segments of convex piecewise linear cost functions for reactive power constraints
$N^s \subset N$	constant marginal cost segments of convex piecewise linear cost functions for apparent power constraints

Table 7: Distinguished set elements

Symbol	Description
$i_e^d \in I$	destination bus of line e
$i_e^o \in I$	origin bus of line e
$i_f^d \in I$	destination bus of transformer f
$i_f^o \in I$	origin bus of transformer f
$i_g \in I$	bus that generator g is connected to
$i_h \in I$	bus that switched shunt h is connected to
$i_j \in I$	bus that load j is connected to
$k_0 \in K$	base case - all other cases are contingency cases

Table 8: Strings

Symbol	Description
id_e	ID of line e (1- or 2-character string, unique over $e \in E_i$ for a given $i \in I$)
id_f	ID of transformer f (1- or 2-character string, unique over $f \in F_i$ for a given $i \in I$)
id_g	ID of generator g (1- or 2-character string, unique over $g \in G_i$ for a given $i \in I$)
id_j	ID of load j (1- or 2-character string, unique over $j \in J_i$ for a given $i \in I$)
$label_k$	label of case k (string, unique over $k \in K$)

Table 9: Data parameters

Symbol	Description
b_f^0	transformer f series susceptance reference value from given operating point prior to the base case (pu)
b_e	line e series susceptance (pu)
b_e^{ch}	line e total charging susceptance (pu)
$b_h^{cs,0}$	switched shunt h susceptance in the given operating point prior to the base case (pu)
b_f^m	transformer f magnetizing susceptance (pu)

Table 9: Continued

Symbol	Description
b_i^{fs}	bus i fixed shunt susceptance (pu)
b_{ha}^{st}	susceptance step size of block a of switched shunt h (pu)
c_e^{sw}	line e switching cost (USD)
c_f^{sw}	transformer f switching cost (USD)
c_g^{on}	fixed operating cost of generator g (USD/h)
c_g^{su}	start up cost of generator g (USD)
c_g^{sd}	shut down cost of generator g (USD)
c_{gn}	marginal cost of real power for generator g in block n (USD/pu-h)
c_{jn}	marginal benefit of real power consumption for load j in block n (USD/pu-h)
c_n^p	marginal cost of real power imbalance in block n of the piecewise linear cost function (USD/pu-h)
c_n^q	marginal cost of reactive power imbalance in block n of the piecewise linear cost function (USD/pu-h)
c_n^s	marginal cost of apparent power (or current) rating exceedance in block n of the piecewise linear cost function (USD/pu-h)
g_f^0	transformer f series conductance reference value from given operating point prior to the base case (pu)
g_e	line e series conductance (pu)
g_f^m	transformer f magnetizing conductance (pu)
g_i^{fs}	bus i fixed shunt conductance (pu)
$\overline{p}_g$	maximum real power output for generator g in base case and contingency cases (pu)
$\underline{p}_g$	minimum real power output for generator g in base case and contingency cases (pu)
$\overline{p}_g^{ru}$	maximum ramp up rate of generator g in the base case (pu/h)
$\overline{p}_g^{rd}$	maximum ramp down rate of generator g in the base case (pu/h)
$\overline{p}_g^{ru,ct}$	maximum ramp up rate of generator g in contingency cases (pu/h)
$\overline{p}_g^{rd,ct}$	maximum ramp down rate of generator g in contingency cases (pu/h)
$\overline{p}_{gn}$	generator g real power maximum in constant marginal cost block n (pu)
p_g^0	real power output of generator g in given operating point prior to the base case (pu)
p_j^0	load j real power consumption in the given operating point prior to the base case (pu)

Table 9: Continued

Symbol	Description
$\overline{p}_j^{ru}$	maximum ramp up rate of load j in the base case (pu/h)
$\overline{p}_j^{rd}$	maximum ramp down rate of load j in the base case (pu/h)
$\overline{p}_j^{ru,ct}$	maximum ramp up rate of load j in contingency cases (pu/h)
$\overline{p}_j^{rd,ct}$	maximum ramp down rate of load j in contingency cases (pu/h)
$\overline{p}_{jn}$	maximum power consumption of load j in block n (pu)
$\overline{p}_n$	real power upper bound of block n of convex piecewise linear cost function for bus real power imbalance (pu)
$\overline{q}_g$	generator g reactive power maximum (pu)
$\underline{q}_g$	generator g reactive power minimum (pu)
q_j^0	load j reactive power consumption in the given operating point prior to the base case (pu)
$\overline{q}_n$	reactive power upper bound of block n of convex piecewise linear cost function for bus reactive power imbalance (pu)
$\overline{r}_e$	line e apparent current maximum in base case (pu)
$\overline{r}_e^{ct}$	line e apparent current maximum in contingencies (pu)
$\tilde{s}$	system power base (MVA)
$\overline{s}_f$	transformer f apparent power maximum in base case (pu)
$\overline{s}_f^{ct}$	transformer f apparent power maximum in contingencies (pu)
$\overline{t}_j$	load j maximum cleared fraction (1)
$\underline{t}_j$	load j minimum cleared fraction (1)
$\overline{t}_n^s$	multiple by which the current rating of a line or the power rating of a transformer is scaled to obtain the upper bound of block n of the cost function for exceedance of its rating (1)
$\overline{v}_i$	bus i voltage magnitude maximum in the base case (pu)
$\underline{v}_i$	bus i voltage magnitude minimum in the base case (pu)
$\overline{v}_i^{ct}$	bus i voltage magnitude maximum in contingencies (pu)
$\underline{v}_i^{ct}$	bus i voltage magnitude minimum in contingencies (pu)
$x_e^{sw,0}$	line e status in given operating point prior to the base case (bin)
$x_f^{sw,0}$	transformer f status in given operating point prior to the base case (bin)
$x_g^{on,0}$	commitment status of generator g (1 indicates on, 0 otherwise) in given operating point prior to the base case (bin)
$\overline{x}_{ha}^{st}$	number of steps in block a of switched shunt h (int)

Table 9: Continued

Symbol	Description
$\overline{x}^{st}_f$	maximum position number, in each of two directions, for transformer f (int)
$x^{st,0}_{ha}$	number of steps switched on in block a of switched shunt h in the given operating point prior to the base case (int)
$x^{st,0}_f$	selected position number of transformer f in the given operating point prior to the base case (int)
z^{inf}	the objective value of a certain easily constructed feasible solution (USD)
δ	duration of the base case (h)
δ^{ct}	duration of each contingency case (h)
δ^{r}	duration (for ramping) from the given operating point prior to the base case to base case operating point (h)
$\delta^{r,ct}$	duration (for ramping) from observation of a contingency to post-contingency operating point (h)
ϵ	numerical tolerance on inequality constraints in solution evaluation procedure
η_{fm}	transformer f impedance correction factor at vertex m of piecewise linear impedance correction function (1)
τ^0_f	transformer f tap ratio in given operating point prior to the base case (1)
τ_{fm}	transformer f tap ratio at vertex m of piecewise linear impedance correction function (1)
τ^{st}_f	tap ratio step size for transformer f (1)
$\check{\tau}_f$	midpoint value of tap ratio of transformer f (1)
$\overline{\tau}_f$	tap ratio maximum value for transformer f (1)
$\underline{\tau}_f$	tap ratio minimum value for transformer f (1)
θ^0_f	transformer f phase shift in given operating point prior to the base case (rad)
θ_{fm}	transformer f phase shift at vertex m of piecewise linear impedance correction function (rad)
θ^{st}_f	phase shift step size for transformer f (rad)
$\check{\theta}_f$	midpoint value of phase shift of transformer f (rad)
$\overline{\theta}_f$	phase shift maximum value for transformer f (rad)
$\underline{\theta}_f$	phase shift minimum value for transformer f (rad)

Table 10: Variables

Symbol	Description
b_{fk}	transformer f case k series susceptance (pu)
b^{cs}_{hk}	susceptance of switched shunt h in case k (pu)
g_{fk}	transformer f case k series conductance (pu)
p^d_{ek}	real power on line e in case k from destination bus into line (pu)
p^o_{ek}	real power on line e in case k from origin bus into line (pu)
p^d_{fk}	real power on transformer f in case k from destination bus into transformer (pu)
p^o_{fk}	real power on transformer f in case k from origin bus into transformer (pu)
p_{gk}	real power output of generator g in case k (pu)
p_{gnk}	real power output of generator g in case k in constant marginal cost block n (pu)
p^+_{ik}	real power imbalance at bus i in case k, positive part, i.e. excess real power flowing into the bus from incident components (pu)
p^+_{ikn}	real power imbalance at bus i in case k, positive part, i.e. excess real power flowing into the bus from incident components, block n (pu)
p^-_{ik}	real power imbalance at bus i in case k, negative part, i.e. excess real power flowing out of the bus to incident components (pu)
p^-_{ikn}	real power imbalance at bus i in case k, negative part, i.e. excess real power flowing out of the bus to incident components, block n (pu)
p_{jk}	real power consumption of load j in case k (pu)
p_{jkn}	real power consumption of load j in case k, block n (pu)
q^d_{ek}	reactive power on line e in case k from destination bus into line (pu)
q^o_{ek}	reactive power on line e in case k from origin bus into line (pu)
q^d_{fk}	reactive power on transformer f in case k from destination bus into transformer (pu)
q^o_{fk}	reactive power on transformer f in case k from origin bus into transformer (pu)
q_{gk}	reactive power output of generator g in case k (pu)
q^+_{ik}	reactive power imbalance at bus i in case k, positive part, i.e. excess reactive power flowing into the bus from incident components (pu)
q^+_{ikn}	reactive power imbalance at bus i in case k, positive part, i.e. excess reactive power flowing into the bus from incident components, block n (pu)
q^-_{ik}	reactive power imbalance at bus i in case k, negative part, i.e. excess reactive power flowing out of the bus to incident components (pu)

Table 10: Continued

Symbol	Description
q^-_{ikn}	reactive power imbalance at bus i in case k, negative part, i.e. excess reactive power flowing out of the bus to incident components, block n (pu)
q_{jk}	reactive power consumption of load j in case k (pu)
s^+_{ek}	apparent current rating exceedance of line e in case k (pu)
s^+_{ekn}	apparent current rating exceedance of line e in case k, block n (pu)
s^+_{fk}	apparent power rating exceedance of transformer f in case k (pu)
s^+_{fkn}	apparent power rating exceedance of transformer f in case k, block n (pu)
t_{jk}	fraction of load j cleared in case k (1)
v_{ik}	voltage magnitude of bus i in case k (pu)
x^{sw}_{ek}	closed/open status of line e in case k, 1 indicates closed, 0 indicates open (bin)
x^{sw}_{fk}	closed/open status of transformer f in case k, 1 indicates closed, 0 indicates open (bin)
x^{st}_{fk}	selected position number of transformer f in case k (int)
x^{st}_{hak}	number of steps activated in block a of switched shunt h in case k (int)
x^{on}_{gk}	commitment status (1 indicates on, 0 otherwise) of generator g in case k (bin)
x^{su}_{gk}	post-contingency start up indicator (1 indicates starting up, 0 otherwise) of generator g in case k (bin)
x^{sd}_{gk}	post-contingency shut down indicator (1 indicates shutting down, 0 otherwise) of generator g in case k (bin)
z_{ek}	objective value for line e in case k (USD)
z_{fk}	objective for transformer f in case k (USD)
z_{gk}	objective for generator g in case k (USD)
z_{ik}	objective for bus i in case k (USD)
z_{jk}	objective for load j in case k (USD)
z_k	objective rate in case k (USD)
z	total market surplus objective (for maximization) (USD)
z^{sc}	the scoring objective of a solution, passed to the scoring procedure, equal to the better (maximum) of z and z^{inf}
η_{fk}	impedance correction factor of transformer f in case k (1)
τ_{fk}	tap ratio of transformer f in case k (1)
θ_{fk}	phase shift of transformer f in case k (rad)
θ_{ik}	voltage angle of bus i in case k (rad)

3 Model Formulation

3.1 Objective Definition

The objective z (for maximization) is the total market surplus defined as a weighted sum of the market surplus objectives z_k over all cases k. The base case k_0 carries weight 1, while the weights on the contingency cases $k \in K \setminus \{k_0\}$ are $1/|K \setminus \{k_0\}|$, so that the total objective is the sum of the base case and the simple average of contingency cases:

$$z = z_{k_0} + (1/|K \setminus \{k_0\}|) \sum_{k \in K \setminus \{k_0\}} z_k \tag{1}$$

The market surplus objective in each case k is defined as a sum of market surpluses corresponding to individual power grid elements, including buses, loads, lines, transformers, and generators:

$$z_k = \sum_{i \in I} z_{ik} + \sum_{j \in J_k} z_{jk} + \sum_{e \in E_k} z_{ek} + \sum_{f \in F_k} z_{fk} + \sum_{g \in G_k} z_{gk} \ \forall k \in K \tag{2}$$

The market surplus objective for each individual grid element in each case is equal to benefits minus costs, as we now specify in detail for each type of grid element.

3.1.1 Bus objective

The base case market surplus objective z_{ik} for a bus i in case k contains no benefit terms and has cost terms for real and reactive power imbalance. The costs on real and reactive power imbalance are given by piecewise linear cost functions, where a small marginal cost is applied to minor imbalances, followed by a larger marginal cost for larger imbalances. The cost function segments are indexed by $n \in N^p$ in the case of real power and by $n \in N^q$ in the case of reactive power.

To formulate this cost function, consider first the real power component. The real power imbalance at bus i is decomposed into a positive part p^+_{ik} and a negative part p^-_{ik}, which then appear in bus power balance constraints. Considering first the positive part, the imbalance is further decomposed into blocks over which the marginal cost is a constant value:

$$p^+_{ik} = \sum_{n \in N^p} p^+_{ikn} \ \forall k \in K, i \in I \tag{3}$$

The imbalance blocks p^+_{ikn} are bounded:

$$0 \le p^+_{ikn} \le \overline{p}_n \ \forall k \in K, i \in I, n \in N^p \tag{4}$$

The imbalance blocks p^+_{ikn} appear in the bus i objective with cost coefficients c^p_n.

The negative part of the real power imbalance is treated similarly:

$$p^-_{ik} = \sum_{n \in N^p} p^-_{ikn} \ \forall k \in K, i \in I \tag{5}$$

$$0 \le p^-_{ikn} \le \overline{p}_n \ \forall k \in K, i \in I, n \in N^p \tag{6}$$

The imbalance blocks p^-_{ikn} appear in the bus i objective with cost coefficients c^p_n.

The positive and negative parts of reactive power imbalance are treated similarly to the real power imbalance:

$$q^+_{ik} = \sum_{n \in N^q} q^+_{ikn} \ \forall i \in I, K \tag{7}$$

$$q^-_{ik} = \sum_{n \in N^q} q^-_{ikn} \ \forall i \in I, K \tag{8}$$

$$0 \leq q^+_{ikn} \leq \overline{q}_n \ \forall i \in I, n \in N^q, k \in K \tag{9}$$

$$0 \leq q^-_{ikn} \leq \overline{q}_n \ \forall i \in I, n \in N^q, k \in K \tag{10}$$

The imbalance blocks q^+_{ikn} and q^-_{ikn} appear in the bus i objective with cost coefficients c^q_n.

Therefore, the market surplus objective z_{ik} for bus i in case k, integrated over time, is defined by:

$$z_{ik} = -\sum_{n \in N^q} (c^p_n(p^+_{ikn} + p^-_{ikn}) + c^q_n(q^+_{ikn} + q^-_{ikn}))\delta \ \forall i \in I, k = k_0 \tag{11}$$

$$z_{ik} = -\sum_{n \in N^q} (c^p_n(p^+_{ikn} + p^-_{ikn}) + c^q_n(q^+_{ikn} + q^-_{ikn}))\delta^{ct} \ \forall i \in I, k \in K \setminus \{k_0\} \tag{12}$$

Equation (11) covering the base case uses the time constant δ, while Equation (12) covering the contingency cases uses the time constant δ^{ct}.

3.1.2 Load objective

The market surplus objective z_{jk} for a load j in case k contains a benefit term representing the benefit of consumption of a specified quantity of real power and has no cost terms. The benefit is represented by a piecewise linear benefit function defined by constant marginal benefit blocks. For this, the real power consumption p_{jk} is decomposed into blocks p_{jkn}:

$$p_{jk} = \sum_{n \in N_j} p_{jkn} \ \forall k \in K, j \in J_k \tag{13}$$

The blocks are bounded by $\overline{p}_{jn}$:

$$0 \leq p_{jkn} \leq \overline{p}_{jn} \ \forall k \in K, j \in J_k, n \in N_j \tag{14}$$

The power consumption blocks appear in the objective with coefficient c_{jn}, integrated over time:

$$z_{jk} = \sum_{n \in N_j} c_{jn} p_{jkn} \delta \ \forall k = k_0, j \in J_k \tag{15}$$

$$z_{jk} = \sum_{n \in N_j} c_{jn} p_{jkn} \delta^{ct} \ \forall k \in K \setminus \{k_0\}, j \in J_k \tag{16}$$

Equation (15) covering the base case uses the time constant δ, while Equation (16) covering contingency cases uses the time constant δ^{ct}.

3.1.3 Line objective

The market surplus objective z_{ek} for a line e in case k contains no benefit terms, and has cost terms assessing the cost of switching the line status from its prior value and the cost of exceeding the apparent current flow limit at either the origin or the destination bus:

$$z_{ek} = -c^{sw}_e |x^{sw}_{ek} - x^{sw,0}_e| - \sum_{n \in N^s} c^s_n s^+_{enk} \delta \;\forall k = k_0, e \in E_k \tag{17}$$

$$z_{ek} = -c^{sw}_e |x^{sw}_{ek} - x^{sw}_{ek_0}| - \sum_{n \in N^s} c^s_n s^+_{enk} \delta^{ct} \;\forall k \in K \setminus \{k_0\}, e \in E_k \tag{18}$$

In Equations (17) and (18), c^{sw}_e is the cost of switching the line status, x^{sw}_{ek} is the line closed/open status, and $\sum_{n \in N^s} c^s_n s^+_{enk}$ is the cost of line apparent current flow limit exceedance s^+_{ek}. Equation (17) covering the base case uses the time constant δ and prior closed/open status $x^{sw,0}_e$, while Equation (18) covering the contingency cases uses the time constant δ^{ct} and prior closed/open status $x^{sw}_{ek_0}$.

The exceedance blocks s^+_{enk} are defined, as with bus power imbalance costs, by:

$$s^+_{ek} = \sum_{n \in N^s} s^+_{enk} \;\forall k \in K, e \in E_k \tag{19}$$

$$0 \leq s^+_{enk} \leq \bar{t}^s_n \bar{r}_e \;\forall k = k_0, e \in E_k, n \in N^s \tag{20}$$

$$0 \leq s^+_{enk} \leq \bar{t}^s_n \bar{r}^{ct}_e \;\forall k \in K \setminus \{k_0\}, e \in E_k, n \in N^s \tag{21}$$

Equation (20) covering the base case uses flow ratings $\bar{s}_f$, while Equation (21) covering the contingency cases uses flow ratings $\bar{s}^{ct}_f$.

3.1.4 Transformer objective

The market surplus objective z_{fk} for a transformer f in case k, similar to that for a line, contains no benefit terms, and has cost terms assessing the cost of switching the transformer closed/open status from its prior value and the cost of exceedance of the apparent power flow limit at either the origin or the destination bus:

$$z_{fk} = -c^{sw}_f |x^{sw}_{fk} - x^{sw,0}_f| - \sum_{n \in N^s} c^s_n s^+_{fnk} \delta \;\forall k = k_0, f \in F_k \tag{22}$$

$$z_{fk} = -c^{sw}_f |x^{sw}_{fk} - x^{sw}_{fk_0}| - \sum_{n \in N^s} c^s_n s^+_{fnk} \delta^{ct} \;\forall k \in K \setminus \{k_0\}, f \in F_k \tag{23}$$

$$s^+_{fk} = \sum_{n \in N^s} s^+_{fnk} \;\forall k \in K, f \in F_k \tag{24}$$

$$0 \leq s^+_{fnk} \leq \bar{t}^s_n \bar{s}_f \;\forall k = k_0, f \in F_k, n \in N^s \tag{25}$$

$$0 \leq s^+_{fnk} \leq \bar{t}^s_n \bar{s}^{ct}_f \;\forall k \in K \setminus \{k_0\}, f \in F_k, n \in N^s \tag{26}$$

Equations (22) and (25) covering the base case use the time constant δ, flow ratings $\bar{s}_f$, and prior closed/open status $x^{sw,0}_f$, while Equations (23) and (26) covering the contingency cases use the time constant δ^{ct}, flow ratings $\bar{s}^{ct}_f$, and and prior closed/open status $x^{sw}_{fk_0}$.

3.1.5 Generator objective

The market surplus objective z_{gk} for a generator g in case k contains no benefit terms, and has cost terms assessing the cost of unit commitment, start up, and shut down, as well as the energy cost, i.e. the cost of generating the chosen amount p_{gk} of real power. The generator energy cost is defined by a convex piecewise linear cost function represented by a set N_g of constant marginal cost blocks. Each constant marginal cost block $n \in N_g$ is defined by a real power upper bound $\overline{p}_{gn}$ and a marginal cost c_{gn}. The total real power output p_{gk} is decomposed into quantities p_{gnk} from the blocks n:

$$p_{gk} = \sum_{n \in N_g} p_{gnk} \; \forall k \in K, g \in G_k \tag{27}$$

The block real power outputs are bounded:

$$0 \leq p_{gnk} \leq \overline{p}_{gn} \; \forall k \in K, g \in G_k, n \in N_g \tag{28}$$

The block real power outputs appear in the generator energy cost definition with their marginal costs along with cost terms for commitment, start up, and shut down:

$$z_{gk} = (-\sum_{n \in N_g} c_{gn} p_{gnk} - c_g^{on} x_{gk}^{on})\delta - c_g^{su} x_{gk}^{su} - c_g^{sd} x_{gk}^{sd} \; \forall k = k_0, g \in G_k \tag{29}$$

$$z_{gk} = (-\sum_{n \in N_g} c_{gn} p_{gnk} - c_g^{on} x_{gk}^{on})\delta^{ct} - c_g^{su} x_{gk}^{su} - c_g^{sd} x_{gk}^{sd} \; \forall k \in K \setminus \{k_0\}, g \in G_k \tag{30}$$

Equation (29) covering the base case uses the time constant δ, while Equation (30) covering the contingency cases uses the time constant δ^{ct}.

3.2 Buses

3.2.1 Voltage bounds

Bus voltages v_{ik} are bounded by:

$$\underline{v}_i \leq v_{ik} \leq \overline{v}_i \; \forall k = k_0, i \in I \tag{31}$$

$$\underline{v}_i^{ct} \leq v_{ik} \leq \overline{v}_i^{ct} \; \forall k \in K \setminus \{k_0\}, i \in I \tag{32}$$

Equation (31) covering the base case uses normal limits $\underline{v}_i$, $\overline{v}_i$, while Equation (32) covering the contingency cases uses emergency limits $\underline{v}_i^{ct}$, $\overline{v}_i^{ct}$.

3.2.2 Bus power balance

Bus real power balance constraints require that the sum of real power outputs from all generators at a given bus i in a case k is equal to the sum of all real power flows into other grid components at the bus. Any power imbalance is assessed a cost in the objective.

Variables p_{ik}^+ and p_{ik}^- are introduced to represent the positive and negative parts of the net imbalance:

$$p_{ik}^+ \geq 0 \ \forall k \in K, i \in I \tag{33}$$
$$p_{ik}^- \geq 0 \ \forall k \in K, i \in I \tag{34}$$

These power imbalance variables then appear in the objective with cost coefficients. The real power balance constraints are then formulated as

$$\sum_{g \in G_i \cap G_k} p_{gk} - \sum_{j \in J_i \cap J_k} p_{jk} - g_i^{fs} v_{ik}^2 - \sum_{e \in E_i^o \cap E_k} p_{ek}^o - \sum_{e \in E_i^d \cap E_k} p_{ek}^d - \sum_{f \in F_i^o \cap F_k} p_{fk}^o - \sum_{f \in F_i^d \cap F_k} p_{fk}^d = p_{ik}^+ - p_{ik}^- \ \forall k \in K, i \in I \tag{35}$$

Here the fixed shunt term $g_i^{fs} v_{ik}^2$ is the real power consumption of all fixed shunts at bus i, and g_i^{fs} is the total conductance the the fixed shunts at bus i. Bus reactive power balance constraints are formulated similarly, with variables q_{ik}^+ and q_{ik}^- representing the positive and negative parts of reactive power imbalance:

$$q_{ik}^+ \geq 0 \ \forall k \in K, i \in I \tag{36}$$
$$q_{ik}^- \geq 0 \ \forall k \in K, i \in I \tag{37}$$

$$\sum_{g \in G_i \cap G_k} q_{gk} - \sum_{j \in J_i \cap J_k} q_{jk} + b_i^{fs} v_{ik}^2 + \sum_{h \in H_i \cap H_k} b_{hk}^{cs} v_{ik}^2 - \sum_{e \in E_i^o \cap E_k} q_{ek}^o - \sum_{e \in E_i^d \cap E_k} q_{ek}^d - \sum_{f \in F_i^o \cap F_k} q_{fk}^o - \sum_{f \in F_i^d \cap F_k} q_{fk}^d = q_{ik}^+ - q_{ik}^- \ \forall k \in K, i \in I \tag{38}$$

Here the fixed shunt term $-b_i^{fs} v_{ik}^2$ is the real power consumption of all fixed shunts at bus i, and b_i^{fs} is the total susceptance the the fixed shunts at bus i.

3.3 Loads

3.3.1 Bounds

For a load j in a case k, the fraction t_{jk} of load that is cleared is subject to bounds:

$$\underline{t}_j \leq t_{jk} \leq \overline{t}_j \ \forall k \in K, j \in J_k \tag{39}$$

Then the resulting load is given by:

$$p_{jk} = p_j^0 t_{jk} \ \forall k \in K, j \in J_k \tag{40}$$
$$q_{jk} = q_j^0 t_{jk} \ \forall k \in K, j \in J_k \tag{41}$$

3.3.2 Ramping

Each load is subject to ramping limits:

$$p_{jk} \leq p_j^0 + \overline{p}_j^{ru} \delta^r \; \forall k = k_0, j \in J_k \tag{42}$$

$$p_{jk} \geq p_j^0 - \overline{p}_j^{rd} \delta^r \; \forall k = k_0, j \in J_k \tag{43}$$

$$p_{jk} \leq p_{jk_0} + \overline{p}_j^{ru,ct} \delta^{r,ct} \; \forall k \in K \setminus \{k_0\}, j \in J_k \tag{44}$$

$$p_{jk} \geq p_{jk_0} - \overline{p}_j^{rd,ct} \delta^{r,ct} \; \forall k \in K \setminus \{k_0\}, j \in J_k \tag{45}$$

Equations (42) and (43) covering the base case use the ramping time constant δ^r, ramp rate limits $\overline{p}_j^{ru}$ and $\overline{p}_j^{rd}$, and prior real power consumption p_j^0, while Equations (44) and (45) covering the contingency cases use the ramping time constant $\delta^{r,ct}$, ramp rate limits $\overline{p}_j^{ru,ct}$ and $\overline{p}_j^{rd,ct}$, and prior real power consumption p_{jk_0}.

3.4 Switched Shunts

3.4.1 Steps selection

For each switched shunt h, in each block a, in each case k, the number of activated steps is an integer bounded by a given maximum number of steps:

$$x_{hak}^{st} \in \{0, 1, 2, \ldots, \overline{x}_{ha}^{st}\} \; \forall k \in K, h \in H_k, a \in A_h \tag{46}$$

3.4.2 Resulting susceptance

The resulting susceptance of a switched shunt is the total susceptance over all blocks and all activated steps:

$$b_{hk}^{cs} = \sum_{a \in A_h} b_{ha}^{st} x_{hak}^{st} \; \forall k \in K, h \in H_k \tag{47}$$

3.5 Lines

3.5.1 Closed-open status indicator

Each line may be closed or open, and the closed-open status is indicated by a binary variable:

$$x_{ek}^{sw} \in \{0, 1\} \; \forall k \in K, e \in E_k \tag{48}$$

The closed-open status for lines that are not qualified to switch must remain at the value in the given operating point prior to the base case:

$$x_{ek}^{sw} = x_e^{sw,0} \; \forall k \in K, e \in E_k \setminus E^{sw} \tag{49}$$

3.5.2 Line real and reactive power flow definitions

Real and reactive power flows into a line e at the origin and destination buses in a case k are defined by:

$$p^o_{ek} = x^{sw}_{ek}(g_e v^2_{ik} - (g_e \cos(\theta_{ik} - \theta_{i'k}) + b_e \sin(\theta_{ik} - \theta_{i'k}))v_{ik}v_{i'k}) \; \forall k \in K, e \in E_k, i = i^o_e, i' = i^d_e \quad (50)$$

$$q^o_{ek} = x^{sw}_{ek}(-(b_e + b^{ch}_e/2)v^2_{ik} + (b_e \cos(\theta_{ik} - \theta_{i'k}) - g_e \sin(\theta_{ik} - \theta_{i'k}))v_{ik}v_{i'k}) \; \forall k \in K, e \in E_k, i = i^o_e, i' = i^d_e \quad (51)$$

$$p^d_{ek} = x^{sw}_{ek}(g_e v^2_{i'k} - (g_e \cos(\theta_{i'k} - \theta_{ik}) + b_e \sin(\theta_{i'k} - \theta_{ik}))v_{ik}v_{i'k}) \; \forall k \in K, e \in E_k, i = i^o_e, i' = i^d_e \quad (52)$$

$$q^d_{ek} = x^{sw}_{ek}(-(b_e + b^{ch}_e/2)v^2_{i'k} + (b_e \cos(\theta_{i'k} - \theta_{ik}) - g_e \sin(\theta_{i'k} - \theta_{ik}))v_{ik}v_{i'k}) \; \forall k \in K, e \in E_k, i = i^o_e, i' = i^d_e \quad (53)$$

3.5.3 Apparent current ratings

Real and reactive power flows into a line e at the origin and destination buses in a case k are subject to apparent current rating constraints. Any exceedance of these current rating constraints is expressed as a quantity s^+_{ek} of apparent power:

$$s^+_{ek} \geq 0 \; \forall k \in K, e \in E_k \quad (54)$$

Current exceedance appears in the objective with a cost coefficient. The current rating constraints are formulated as:

$$\sqrt{(p^o_{ek})^2 + (q^o_{ek})^2} \leq \overline{r}_e v_{ik} + s^+_{ek} \; \forall k = k_0, e \in E_k, i = i^o_e \quad (55)$$

$$\sqrt{(p^d_{ek})^2 + (q^d_{ek})^2} \leq \overline{r}_e v_{ik} + s^+_{ek} \; \forall k = k_0, e \in E_k, i = i^d_e \quad (56)$$

$$\sqrt{(p^o_{ek})^2 + (q^o_{ek})^2} \leq \overline{r}^{ct}_e v_{ik} + s^+_{ek} \; \forall k \in K \setminus \{k_0\}, e \in E_k, i = i^o_e \quad (57)$$

$$\sqrt{(p^d_{ek})^2 + (q^d_{ek})^2} \leq \overline{r}^{ct}_e v_{ik} + s^+_{ek} \; \forall k \in K \setminus \{k_0\}, e \in E_k, i = i^d_e \quad (58)$$

Equations (55) and (56) covering the base case use the normal line ratings $\overline{r}_e$, while Equations (57) and (58) covering the contingency cases use the emergency line ratings $\overline{r}^{ct}_e$, while

3.6 Transformers

3.6.1 Switching status indicator

Each transformer may be closed or open, and the closed-open status is indicated by a binary variable:

$$x^{sw}_{fk} \in \{0, 1\} \; \forall k \in K, f \in F_k \quad (59)$$

The closed-open status of a transformer that is not qualified to switch must remain fixed to the value in the given operating point prior to the base case:

$$x^{sw}_{fk} = x^{sw,0}_{f} \ \forall k \in K, f \in F_k \setminus F^{sw} \tag{60}$$

3.6.2 Tap ratio or phase shift position selection

Each transformer has a set of positions, and the selected position is indicated by an integer variable bounded by a minimum position number and a maximum position number:

$$x^{st}_{fk} \in \{-\overline{x}^{st}_{f}, -(\overline{x}^{st}_{f} - 1), \ldots, -1, 0, 1, \ldots, \overline{x}^{st}_{f} - 1, \overline{x}^{st}_{f}\} \ \forall k \in K, f \in F_k \tag{61}$$

3.6.3 Resulting tap ratio or phase shift

The tap ratio of a variable tap ratio transformer depends on the position selected:

$$\tau_{fk} = \check{\tau}_f + \tau^{st}_{f} x^{st}_{fk} \ \forall k \in K, f \in F_k \cap F^{\tau} \tag{62}$$

The tap ratio of a fixed tap ratio transformer is fixed to its value in the given operating point prior to the base case:

$$\tau_{fk} = \tau^0_f \ \forall k \in K, f \in F_k \setminus F^{\tau} \tag{63}$$

The phase shift of a variable phase shift transformer depends on the position selected:

$$\theta_{fk} = \check{\theta}_f + \theta^{st}_{f} x^{st}_{fk} \ \forall k \in K, f \in F_k \cap F^{\theta} \tag{64}$$

The phase shift of a fixed phase shift transformer is fixed to its value in the given operating point prior to the base case:

$$\theta_{fk} = \theta^0_f \ \forall k \in K, f \in F_k \setminus F^{\theta} \tag{65}$$

3.6.4 Impedance correction

The impedance correction factor η_{fk} of a transformer f with impedance correction in case k is used to modify the conductance g_f and susceptance b_f by the constraints

$$g_{fk} = g^0_f / \eta_{fk} \ \forall k \in K, f \in F^{\eta} \cap F_k \tag{66}$$

$$b_{fk} = b^0_f / \eta_{fk} \ \forall k \in K, f \in F^{\eta} \cap F_k \tag{67}$$

Transformers without impedance correction have conductance and susceptance fixed to reference values from the given operating point prior to the base case:

$$g_{fk} = g^0_f \ \forall k \in K, f \in F_k \setminus F^{\eta} \tag{68}$$

$$b_{fk} = b^0_f \ \forall k \in K, f \in F_k \setminus F^{\eta} \tag{69}$$

The value of the impedance correction factor η_{fk} for $f \in F^{\eta}$ is a piecewise linear function of τ_{fk} (for $f \in F^{\tau}$) or θ_{fk} (for $f \in F^{\theta}$). Specifically, for $f \in F^{\eta}$, the sequence of vertices on the graph of this function, indexed by an ordered set M_f with $M_f = \{1, 2, \ldots, |M_f|\}$, is given. The given vertices are (τ_{fm}, η_{fm}) for $f \in F^\tau$ and (θ_{fm}, η_{fm}) for $f \in F^\theta$. Then we formulate the functional dependence of η_{fk} on τ_{fk} or θ_{fk} using geometric constraints

$$(\tau_{fk}, \eta_{fk}) \in \bigcup_{m=1}^{|M_f|-1} \gamma((\tau_{fm}, \eta_{fm}), (\tau_{f,m+1}, \eta_{f,m+1})) \ \forall k \in K, f \in F_k \cap F^\eta \cap F^\tau \tag{70}$$

$$(\theta_{fk}, \eta_{fk}) \in \bigcup_{m=1}^{|M_f|-1} \gamma((\theta_{fm}, \eta_{fm}), (\theta_{f,m+1}, \eta_{f,m+1})) \ \forall k \in K, f \in F_k \cap F^\eta \cap F^\theta \tag{71}$$

where $\gamma(w, w')$ is the line segment with vertices w and w'.

3.6.5 Real and reactive power flow definitions

Real and reactive power flows into a transformer f at the origin and destination buses in a case k are defined by:

$$\begin{aligned} p^o_{fk} = x^{sw}_{fk}((g_{fk}/\tau^2_{fk} + g^m_f)v^2_{ik} - (g_{fk}\cos(\theta_{ik} - \theta_{i'k} - \theta_{fk}) \\ + b_{fk}\sin(\theta_{ik} - \theta_{i'k} - \theta_{fk}))v_{ik}v_{i'k}/\tau_{fk}) \ \forall k \in K, f \in F_k, i = i^o_f, i' = i^d_f \end{aligned} \tag{72}$$

$$\begin{aligned} q^o_{fk} = x^{sw}_{fk}(-(b_{fk}/\tau^2_{fk} + b^m_f)v^2_{ik} + (b_{fk}\cos(\theta_{ik} - \theta_{i'k} - \theta_{fk}) \\ - g_{fk}\sin(\theta_{ik} - \theta_{i'k} - \theta_{fk}))v_{ik}v_{i'k}/\tau_{fk}) \ \forall k \in K, f \in F_k, i = i^o_f, i' = i^d_f \end{aligned} \tag{73}$$

$$\begin{aligned} p^d_{fk} = x^{sw}_{fk}(g_{fk}v^2_{i'k} - (g_{fk}\cos(\theta_{i'k} - \theta_{ik} + \theta_{fk}) \\ + b_{fk}\sin(\theta_{i'k} - \theta_{ik} + \theta_{fk}))v_{ik}v_{i'k}/\tau_{fk}) \ \forall k \in K, f \in F_k, i = i^o_f, i' = i^d_f \end{aligned} \tag{74}$$

$$\begin{aligned} q^d_{fk} = x^{sw}_{fk}(-b_{fk}v^2_{i'k} + (b_{fk}\cos(\theta_{i'k} - \theta_{ik} + \theta_{fk}) \\ - g_{fk}\sin(\theta_{i'k} - \theta_{ik} + \theta_{fk}))v_{ik}v_{i'k}/\tau_{fk}) \ \forall k \in K, f \in F_k, i = i^o_f, i' = i^d_f \end{aligned} \tag{75}$$

3.6.6 Apparent power ratings

Real and reactive power flows into a transformer f at the origin and destination buses in a case k are subject to power rating constraints. Power rating exceedance is represented by a variable s^+_{fk}:

$$s^+_{fk} \geq 0 \ \forall k \in K, f \in F_k \tag{76}$$

Power rating exceedance then appears in the objective with a cost coefficient. The power rating constraints are formulated as:

$$\sqrt{(p^o_{fk})^2 + (q^o_{fk})^2} \leq \overline{s}_f + s^+_{fk} \ \forall k = k_0, f \in F_k \tag{77}$$

$$\sqrt{(p^d_{fk})^2 + (q^d_{fk})^2} \leq \overline{s}_f + s^+_{fk} \ \forall k = k_0, f \in F_k \tag{78}$$

$$\sqrt{(p^o_{fk})^2 + (q^o_{fk})^2} \leq \overline{s}^{ct}_f + s^+_{fk} \ \forall k \in K \setminus \{k_0\}, f \in F_k \tag{79}$$

$$\sqrt{(p^d_{fk})^2 + (q^d_{fk})^2} \leq \overline{s}^{ct}_f + s^+_{fk} \ \forall k \in K \setminus \{k_0\}, f \in F_k \tag{80}$$

Equations (77) and (78) covering the base case use the normal transformer ratings $\overline{s}_e$, while Equations (79) and (80) covering the contingency cases use the emergency transformer ratings $\overline{s}^{ct}_f$.

3.7 Generators

3.7.1 Commitment variable domains

The commitment variables x^{on}_{gk}, the start up indicators x^{su}_{gk}, and the shut down indicators x^{sd}_{gk}, are binary variables:

$$x^{on}_{gk}, x^{su}_{gk}, x^{sd}_{gk} \in \{0, 1\} \; \forall k \in K, g \in G_k \tag{81}$$

3.7.2 Start up and shut down

The start up and shut down indicators x^{su}_{gk}, x^{sd}_{gk}, are defined by changes in the commitment status x^{on}_{gk} relative to the prior commitment status:

$$x^{on}_{gk} - x^{on,0}_{g} = x^{su}_{gk} - x^{sd}_{gk} \; \forall k = k_0, g \in G_k \tag{82}$$

$$x^{on}_{gk} - x^{on}_{gk_0} = x^{su}_{gk} - x^{sd}_{gk} \; \forall k \in K \setminus \{k_0\}, g \in G_k \tag{83}$$

$$x^{su}_{gk} + x^{sd}_{gk} \leq 1 \; \forall k \in K, g \in G_k \tag{84}$$

Equation (82) covering the base case uses the prior commitment status is $x^{on,0}_{g}$, while Equation (83) covering the contingency cases uses the prior commitment status $x^{on}_{gk_0}$. Equation (84) ensures that no generator may simultaneously start up and shut down.

3.7.3 Energy bounds

The real power output p_{gk} of a committed generator in case k is subject to energy bounds, while decommitted generators have 0 real power output:

$$\underline{p}_g x^{on}_{gk} \leq p_{gk} \leq \overline{p}_g x^{on}_{gk} \; \forall k \in K, g \in G_k \tag{85}$$

Reactive power output is similarly constrained:

$$\underline{q}_g x^{on}_{gk} \leq q_{gk} \leq \overline{q}_g x^{on}_{gk} \; \forall k \in K, g \in G_k \tag{86}$$

3.7.4 Real Power Ramp Rate Constraints

Each generator g in a case k is subject to ramp rate constraints linking the real power output p_{gk} to the prior real power output:

$$p_{gk} \leq (p^0_g + \overline{p}^{ru}_g \delta^r)(x^{on}_{gk} - x^{su}_{gk}) + (\underline{p}_g + \overline{p}^{ru}_g \delta^r) x^{su}_{gk} \; \forall k = k_0, g \in G_k \tag{87}$$

$$p_{gk} \geq (p^0_g - \overline{p}^{rd}_g \delta^r)(x^{on}_{gk} - x^{su}_{gk}) \; \forall k = k_0, g \in G_k \tag{88}$$

$$p_{gk} \leq (p_{gk_0} + \overline{p}^{ru,ct}_g \delta^{r,ctg})(x^{on}_{gk} - x^{su}_{gk}) + (\underline{p}_g + \overline{p}^{ru,ct}_g \delta^{r,ctg}) x^{su}_{gk} \; \forall k \in K \setminus \{k_0\}, g \in G_k \tag{89}$$

$$p_{gk} \geq (p_{gk_0} - \overline{p}^{rd,ct}_g \delta^{r,ctg})(x^{on}_{gk} - x^{su}_{gk}) \; \forall k \in K \setminus \{k_0\}, g \in G_k \tag{90}$$

Equations (87) and (88) covering the base case use the ramping time constant δ^r, ramp rate limits $\overline{p}_g^{ru}$ and $\overline{p}_g^{rd}$, and prior real power output p_g^0, while Equations (89) and (90) covering the contingency cases use the ramping time constant $\delta^{r,ct}$, ramp rate limits $\overline{p}_g^{ru,ct}$ and $\overline{p}_g^{rd,ct}$, and prior real power output p_{gk_0}.

3.7.5 Forbidden commitment patterns

Only generators in G^{su} may start up in the base case:

$$x_{gk}^{su} = 0 \ \forall k = k_0, g \in G_k \setminus G^{su} \tag{91}$$

Only generators in $G^{su,ct}$ may start up in contingencies:

$$x_{gk}^{su} = 0 \ \forall k \in K \setminus \{k_0\}, g \in G_k \setminus G^{su,ct} \tag{92}$$

Only generators in G^{sd} may shut down in the base case:

$$x_{gk}^{sd} = 0 \ \forall k = k_0, g \in G_k \setminus G^{sd} \tag{93}$$

Only generators in $G^{sd,ct}$ may shut down in contingencies:

$$x_{gk}^{sd} = 0 \ \forall k \in K \setminus \{k_0\}, g \in G_k \setminus G^{sd,ct} \tag{94}$$

No generator may start up in the base case and then shut down in a contingency or shut down in the base case and then start up in a contingency:

$$x_{gk_0}^{su} + x_{gk}^{sd} \leq 1 \ \forall k \in K \setminus \{k_0\}, g \in G_k \tag{95}$$

$$x_{gk_0}^{sd} + x_{gk}^{su} \leq 1 \ \forall k \in K \setminus \{k_0\}, g \in G_k \tag{96}$$

3.8 Optimization Model

The objective is to maximize z. The variables are all the variables listed in Table 10. The constraints are Equations (1) to (96).

A Input Data Format

A.1 Introduction

The GO Competition Challenge 2 uses a set of three files for the input data of each problem instance. Two of these input files are formatted according to industry standard data file formats used with a popular power system software package. The third is a JSON format designed by the GO Competition for certain data not covered by the other two files. This section describes these input data formats.

The format information in this section is intended for use by GO Competition Challenge 2 Entrants. The industry standard data formats include some data elements that are not used by the GO Competition Challenge 2. These elements do not need to be understood by Entrants, and they are not described in detail. Therefore, this section cannot be viewed as a full format specification of the industry standard formats. Rather it is a format specification of the aspects of these formats that are relevant to the GO Competition Challenge 2.

A.1.1 Input Data Files

The data for each problem instance are contained in input data files described in Table 11.

Table 11: Input data files

Filename	Description
case.raw	Power Flow Raw Data File (RAW)
case.con	Contingency Description Data File (CON)
case.json	Supplementary Data File (JSON)

A.1.2 Annotation Used in Input Data Format Description

Individual data items in the RAW and CON are annotated in this document to indicate how they are to be treated. This annotation is described in Table 12. Please note that not all data items mentioned in the RAW and CON files are used by the GO Competition. Fields or sections that are not used by the GO Competition are not marked with an asterisk and can be ignored by Entrants. They are listed in this document because they are fields in the input files, and it is therefore necessary to identify them in order to parse the format correctly. The JSON file is designed specifically for the GO Competition, and all the data items in it are used.

Table 12: Annotation on data fields in the RAW and CON file format descriptions.

Symbol	Description
*	An asterisk (*) is used to indicate that a particular data item is used by the GO Competition Challenge 2.
X	A field name is underlined to indicate that the field is a key field in the table being described. Concatenating all key fields in a given table record yields a single key so each record in the table has a unique key. For example, the key fields in the Generator Data section are I and ID, so each generator has a unique value of (I, ID), though two different generators may have the same value of I or the same value of ID.
‘ ’	A field name is enclosed in matching single quotes (‘ ’) to indicate that the field contains string data.
[1]‘ ’	A field name is enclosed in matching single quotes (‘ ’) with a preceding superscript 1 if the field contains short string data, i.e. a string of one or two characters, all upper case letters or digits.
,	Field names are separated by commas (,) in the description of the table format to indicate that fields are separated by commas in the table. The Contingency Description Data File uses tokens separated by spaces, not commas, so commas are not used in the description of its format.

A.1.3 Reading the Data Files

The general structure of the data files, together with the restrictions placed by the GO Competition Challenge 2, imply that the data files can be read easily by CSV and JSON packages in a variety of programming languages. We note, for example, the CSV and JSON packages used by Python and Julia, and Java has equivalent options. The GO Competition has written a Python module, `data.py`, to read and parse these files. This code is available to entrants at the Challenge 2 Solution Evaluation page https://gocompetition.energy.gov/challenges/challenge-2/solution-evaluation/.

A.2 Power Flow Raw Data File (case.raw)

The RAW file is a text file consisting of multiple sections, which are listed in Table 13. In the RAW file, the sections appear in the order in which they are listed in the table, no section is skipped, and no section appears more than once. In the Table, sections whose content is used by the GO Competition are indicated with an asterisk (*).

Table 13: RAW file sections

Section
*Case Identification Data
*Bus Data
*Load Data
*Fixed Bus Shunt Data
*Generator Data
*Non-Transformer Branch Data
*Transformer Data
Area Interchange Data
Two-Terminal DC Transmission Line Data
Voltage Source Converter (VSC) DC Transmission Line Data
*Transformer Impedance Correction Tables
Multi-Terminal DC Transmission Line Data
Multi-Section Line Grouping Data
Zone Data
Interarea Transfer Data
Owner Data
FACTS Device Data
*Switched Shunt Data
GNE Device Data
Induction Machine Data

The Case Identification Data section consists of exactly 3 lines of text. The line immediately following the Case Identification Data section is the first line of the Bus Data section.

Each section after the Case Identification Data section is terminated by a section end line, which is a line starting with the character '0' (zero). Text following the character '0' in a section end line has no significance. The next line after a section end line is the first line of the next section, except for the last section, which does not have another section after it.

Immediately after the section end line for the last section, the file ends with a file end line, which is a line starting with the character 'Q'. Text following the character 'Q' in a file end line or in lines after a file end line has no significance.

Each section consists of a sequence of records. Each record consists of one or more lines, where the number of lines is the same for each record in any given section.

Each line in a record consists of a sequence of fields, separated by the comma character ','. For different records in a section, for the same line within each record, the number of fields in a line is always the same. I.e., the first line of one record has the same number of fields as the first line of another record, and the second lines have the same number of fields, and so on. The fields appear in consistent order, and no field can be skipped. A field that is not used by the GO Competition may be empty, but it will still be separated from the fields before it and after it by commas. Fields that are used by the GO Competition cannot be empty.

The requirement that no field used by the GO Competition be empty has two exceptions. Switched shunts have up to 8 (N,B) pairs, and transformer impedance tables have up to 11 (T,F) pairs. Unneeded fields in these two specific cases will not be included, either as empty fields marked by commas, or as nonempty fields containing invalid data.

Each field is of a specific data type. The data type of a field is either a number, a short string, or a long string. Each number field is written with the digits `0` through `9`, possibly a decimal point indicated by the period character `.`, and possibly a negative sign indicated by the hyphen character `-`. Each short string field is written with the digits `0` through `9` and the capital letters `A` through `Z`, and contains either 1 or 2 characters surrounded by matching single quotes. Each long string field is written with the digits `0` through `9`, the capital letters `A` through `Z`, the period `.`, the underscore `_`, and the hyphen `-`, and contains an arbitrary number of characters surrounded by matching single quotes. The quote mark used around string fields is the standard single quote character `'`.

Each section can be regarded as a table in which certain fields act as key fields. Within a section, each record has a key, and no keys are repeated, so the records are in one-to-one correspondence with the unique keys.

There are no end of line comments or multi-line comments. Blank lines are not allowed.

Next we describe the individual RAW file sections. For each section we give the record format, including the lines and fields. For each field we specify the field name, the data type, whether it is used by the GO Competition or not, and certain other potentially useful information. The field name is given in Roman capital letters. Fields that are used by the GO Competition are marked with an asterisk (*). Key fields are underlined. Long string fields are surrounded by single quotes. Short string fields are surrounded by single quotes and preceded by a superscript numeral 1. Number fields are not specially marked. These annotations are summarized in Table 12.

A.2.1 Case Identification Data

The Case Identification Data section consists of a single record with exactly 3 lines, in the following format:

IC, *SBASE, REV, XFRRAT, NXFRAT, BASFRQ

‘CASELINE2’

‘CASELINE3’

The Case Identification Data fields are described in Table 14.

Table 14: Case Identification Data fields

Field	Description
IC	–
*SBASE	System MVA base, in MVA.
REV	–
XFRRAT	–
NXFRAT	–
BASFRQ	–
‘CASELINE2’	This field is a non-blank, nonempty line of text.
‘CASELINE3’	This field is a non-blank, nonempty line of text.

A.2.2 Bus Data

Each bus in the system is described by a bus record in a single line with the following format:

*I, ‘NAME’, BASKV, IDE, AREA, ZONE, OWNER, *VM, *VA, *NVHI, *NVLO, *EVHI, *EVLO

The Bus Data fields are described in Table 15.

Table 15: Bus fields

Field	Description
*I	Bus number. Allowable range integer 1 through 999997. Referenced by fields I, J, and BUS in the Load Data (Section A.2.3), Fixed Bus Shunt Data (Section A.2.4), Generator Data (Section A.2.5), Non-Transformer Branch Data (Section A.2.6), Transformer Data (Section A.2.7), Switched Shunt Data (Section A.2.9), and the Branch Out-of-Service Event (Section A.3.1) and Generator Out-of-Service Event (Section A.3.2) records.
‘NAME’	–
BASKV	–
IDE	–
AREA	–
ZONE	–
OWNER	–
*VM	bus voltage magnitude in pu.
*VA	bus voltage angle in degrees.
*NVHI	Normal voltage magnitude high limit in pu.

Table 15: Continued

Field	Description
*NVLO	Normal voltage magnitude low limit in pu.
*EVHI	Emergency voltage magnitude high limit in pu.
*EVLO	Emergency voltage magnitude low limit in pu.

A.2.3 Load Data

Each load record is a single line in the following format:

*I, *[1]'ID', *STATUS, AREA, ZONE, *PL, *QL, IP, IQ, YP, YQ, OWNER, SCALE, INTRPT

The Load Data fields are described in Table 16.

Table 16: Load fields

Field	Description
*I	Bus number. Refers to field I in the Bus Data section (Section A.2.2).
*[1]'ID'	Identifier. One- or two-character string used to distinguish among multiple loads at a single bus.
*STATUS	status, binary. 1 indicates in service, 0 out of service.
AREA	–
ZONE	–
*PL	Active power component of constant power load, in MW.
*QL	Reactive power component of constant power load, in Mvar.
IP	–
IQ	–
YP	–
YQ	–
OWNER	–
SCALE	–
INTRPT	–

A.2.4 Fixed Bus Shunt Data

Each fixed shunt is represented by a record in a single line in the following format:

*I, *[1]'ID', *STATUS, *GL, *BL

The Fixed Bus Shunt Data fields are described in Table 17.

Table 17: Fixed Shunt fields

Field	Description
*I	Bus number. Refers to field I in the Bus Data section (Section A.2.2).
*[1]'ID'	Identifier. One- or two-character string used to distinguish among multiple fixed shunts at a single bus.
*STATUS	binary status indicator, 1 indicating in service, 0 out of service.
*GL	Active component of shunt admittance to ground, in MW at 1 pu voltage.
*BL	Reactive component of shunt admittance to ground, in Mvar at 1 pu voltage.

A.2.5 Generator Data

Each generator is represented by a record in a single line in the following format:

*I, *[1]'ID', *PG, *QG, *QT, *QB, VS, IREG, MBASE, ZR, ZX, RT, XT, GTAP, *STAT, RMPCT, *PT, *PB, O1, F1, O2, F2, O3, F3, O4, F4, WMOD, WPF

The Generator Data fields are described in Table 18.

Table 18: Generator fields

Field	Description
*I	bus number. Refers to field I in the Bus Data section (Section A.2.2).
*[1]'ID'	identifier. one- or two-character string used to distinguish among multiple generators at a bus. Referenced by fields ID and GENID in the Generator Out-of-Service Event record (Section A.3.2).
*PG	generator real power output, in MW.
*QG	generator reactive power output, in MVar.
*QT	maximum generator reactive power output, in Mvar.
*QB	minimum generator reactive power output, in Mvar.

Table 18: Continued

Field	Description
VS	–
IREG	–
MBASE	–
ZR	–
ZX	–
RT	–
XT	–
GTAP	–
*STAT	generator status, binary. In the industry standard data format, this field is for the status of a generator indicating in service or out of service for the context of the whole model, i.e. 1 indicates in service, 0 out of service. In the use of this data format by the GO Competition, this field is interpreted as the commitment status of the generator in the prior time period, i.e. 1 indicates committed on, 0 indicates committed off, and the commitment status of the generator may or may not be changed in the context of the model according to the solution of the optimization problem.
RMPCT	–
*PT	maximum generator active power output, MW.
*PB	minimum generator active power output, MW.
O1	–
F1	–
O2	–
F2	–
O3	–
F3	–
O4	–
F4	–
WMOD	–
WPF	–

A.2.6 Non-Transformer Branch Data

Each non-transformer branch is represented by a single line record in the following format:

*I, *J, *[1]'CKT', *R, *X, *B, *RATEA, RATEB, *RATEC, GI, BI, GJ,
BJ, *ST, MET, LEN, O1, F1, O2, F2, O3, F3, O4, F4

The Non-Transformer Branch Data fields are described in Table 19.

Table 19: Non-transformer Branch fields

Field	Description
*I	origin bus number. Refers to field I in the Bus Data section (Section A.2.2).
*J	destination bus number. not equal to I. Refers to field I in the Bus Data section (Section A.2.2).
*[1]'CKT'	circuit identifier. one- or two-character string used to distinguish among multiple branches between I and J. Referenced by field CKT in the Branch Out-of-Service record (Section A.3.1). Among all branches (non-transformer or transformer) between buses I and J (from I to J or from J to I), the value of CKT identifies each branch uniquely.
*R	branch resistance, pu.
*X	branch reactance, pu. allowable range nonzero real numbers.
*B	total branch charging susceptance, pu.
*RATEA	line rating in the base case, current expressed as MVA at bus base voltage of origin and destination buses. Origin and destination buses of a non-transformer branch must have equal base voltages.
RATEB	–
* RATEC	line rating in contingency cases, current expressed as MVA at bus base voltage of origin and destination buses. Origin and destination buses of a non-transformer branch must have equal base voltages.
GI	–
BI	–
GJ	–
BJ	–
*ST	status, binary. In the industry standard data format, this field is for the status of a line indicating in service or out of service for the context of the whole model, i.e. 1 indicates in service, 0 out of service. In the use of this data format by the GO Competition, this field is interpreted as the closed/open status of the line in the prior time period, i.e. 1 indicates closed, 0 indicates open, and the closed open status of the line may or may not be changed in the context of the model according to the solution of the optimization problem.
MET	–
LEN	–
O1	–
F1	–
O2	–
F2	–
O3	–

Table 19: Continued

Field	Description
F3	–
O4	–
F4	–

A.2.7 Transformer Data

Any three-winding transformer can be represented as a configuration of two-winding transformers, and all three-winding transformers present in the problem instances considered by the GO Competition are represented in this way. Therefore, the RAW data file will contain only two-winding transformers, and the data format of a three-winding transformer represented in a single transformer record is not described here.

Each (two-winding) transformer is represented by a record consisting of four lines of data in the following format:

*I, *J, K, *[1]‘CKT’, CW, CZ, CM, *MAG1, *MAG2, NMETR, ‘NAME’, *STAT, O1, F1, O2, F2, O3, F3, O4, F4, ‘VECGRP’

*R12, *X12, SBASE12

*WINDV1, NOMV1, *ANG1, *RATA1, RATB1, *RATC1, *COD1, CONT1, *RMA1, *RMI1, VMA1, VMI1, *NTP1, *TAB1, CR1, CX1, CNXA1

*WINDV2, NOMV2

The Transformer Data fields are described in Table 20.

Table 20: Transformer fields

Field	Description
*I	origin (winding 1) bus number. References field I in the Bus Data section (Section A.2.2).
*J	destination (winding 2) bus number. References field I in the Bus Data section (Section A.2.2).
K	–
*[1]‘CKT’	circuit identifier. one- or two-character string used to distinguish among multiple branches between I and J. Referenced by field CKT in the Branch Out-of-Service record (Section A.3.1).

Table 20: Continued

Field	Description
CW	–
CZ	–
CM	–
*MAG1	transformer magnetizing conductance connected to ground at bus I, in pu on system MVA base
*MAG2	transformer magnetizing susceptance connected to ground at bus I, in pu on system MVA base
NMETR	–
'NAME'	–
*STAT	status, binary. In the industry standard data format, this field is for the status of a transformer indicating in service or out of service for the context of the whole model, i.e. 1 indicates in service, 0 out of service. In the use of this data format by the GO Competition, this field is interpreted as the closed/open status of the transformer in the prior time period, i.e. 1 indicates closed, 0 indicates open, and the closed open status of the transformer may or may not be changed in the context of the model according to the solution of the optimization problem.
O1	–
F1	–
O2	–
F2	–
O3	–
F3	–
O4	–
F4	–
'VECGRP'	–
*R12	transformer resistance, in pu on system MVA base and winding voltage base
*X12	transformer reactance, in pu on system MVA base and winding voltage base
SBASE12	–
*WINDV1	winding 1 off-nominal turns ratio in pu of winding 1 bus voltage base.
NOMV1	–
*ANG1	winding 1 phase shift angle in degrees. allowable range $(-180.0, 180.0]$
*RATA1	winding 1 three-phase power rating, in MVA
RATB1	–
* RATC1	three-phase power rating in contingencies, in MVA

Table 20: Continued

Field	Description
*COD1	transformer control mode, integer. 0 indicates fixed tap ratio and fixed phase shift. 1 or -1 indicates variable tap ratio and fixed phase shift. 3 or -3 indicates fixed tap ratio and variable phase shift.
CONT1	–
*RMA1	The interpretation depends on the value of COD1. If COD1 = 1, then RMA1 is the upper bound on the variable tap ratio (dimensionless). If COD1 = 3, then RMA1 is the upper bound on the variable phase shift (in degrees).
*RMI1	The interpretation depends on value of COD1. If COD1 = 1, then RMI1 is the lower bound on the variable tap ratio (dimensionless). If COD1 = 3, then RMI1 is the lower bound on the variable phase shift (in degrees).
VMA1	–
VMI1	–
*NTP1	number of positions of variable tap ratio or phase shift. (odd integer ≥ 1)
*TAB1	If equal to 0, indicates that the transformer does not have an impedance correction function. If not equal to 0, then this field is the number of an impedance correction table specifying the impedance correction function for this transformer. (nonnegative integer) References field I in the Transformer Impedance Correction Tables section (Section A.2.8).
CR1	–
CX1	–
CNXA1	–
*WINDV2	winding 2 off-nominal turns ratio in pu of winding 2 bus voltage base.
NOMV2	–

A.2.8 Transformer Impedance Correction Tables

Each impedance correction table is represented by a record in the following format:

*I, *T1, *F1, *T2, *F2, ..., *T11, *F11

Up to 11 pairs (Tm, Fm) may be specified, at least 2 pairs must be specified, and the pairs should be in order of increasing Tm, with strict increase. Unneeded (T,F) pairs are not included in the record, either as empty fields or as fields containing invalid data. The Transformer Impedance Correction Tables fields are described in Table 21.

Table 21: Transformer Impedance Correction Tables fields

Field	Description
*I	transformer impedance correction table number. Referenced by field TAB1 in the Transformer Data section (Section A.2.7).
*Tm	Either off-nominal turns ratio in pu or phase shift angle in degrees
*Fm	Scaling factor by which transformer nominal impedance is to be multiplied to obtain the actual transformer impedance for the corresponding Tm

A.2.9 Switched Shunt Data

Each switched shunt is represented by a single line record in the following format:

```
*I, MODSW, ADJM, *STAT, VSWHI, VSWLO, SWREM, RMPCT,
'RMIDNT', *BINIT, *N1, *B1, *N2, *B2, ..., *N8, *B8
```

Up to 8 pairs (Na, Ba), each specifying a switched shunt block, may be given. Unneeded (N,B) pairs are not included in the record, either as empty fields marked by commas or as nonempty fields containing invalid data, The Switched Shunt Data fields are described in Table 22.

Table 22: Switched Shunt fields

Field	Description
*I	bus number of bus that this shunt is connected to. References field I in the Bus Data section (Section A.2.2).
MODSW	–
ADJM	–
*STAT	status, binary. 1 indicates in service, 0 out of service.
VSWHI	–
VSWLO	–
SWREM	–
RMPCT	–
'RMIDNT'	–
*BINIT	Initial switched shunt susceptance, in Mvar at unit voltage
*Na	number of steps in block a, nonnegative integer
*Ba	susceptance of each step in block a, in Mvar at unit voltage

A.3 Contingency Description Data File (case.con)

The contingency description data file consists of multiple lines of text. The text consists of tokens separated by blank space. The tokens do not contain blank space, and no quotation marks are needed or allowed. This file can be read as CSV with blank space as the separator.

Each token is either a keyword or a data field. The keywords valid for this data format as used by the GO Competition Challenge 2 are 'CONTINGENCY', 'END', 'REMOVE', 'UNIT', 'OPEN', 'BRANCH', 'FROM', 'BUS', 'TO', 'CIRCUIT'.

Each contingency case is defined by 3 lines of text, a Start Line, then an Event Line, then an End Line.

The line after a contingency End Line can be either: (1) another contingency Start Line indicating the start of a new contingency, or (2) another End Line indicating the end of the Contingency Case Description Data File.

The format of an End Line is a line with the single keyword 'END':

```
END
```

The format of a Start Line is a line with the keyword 'CONTINGENCY' followed by a data token containing the LABEL field:

```
CONTINGENCY *LABEL
```

The Start Line LABEL field is described in Table 23.

Table 23: Start Line fields

Field	Description
*LABEL	contingency case identifier, unquoted long string, i.e. consisting of digits, upper case letters, and underscore, hyphen, and period characters.

Event lines can take several different formats. the GO Competition Challenge 2 uses only a narrow set of formats, corresponding to two types of events, the Branch Out-of-Service Event and the Generator Out-of-Service Event.

A.3.1 Branch Out-of-Service Event

A non-transformer or two-winding transformer branch may be placed out of service with an event line in the following format:

```
OPEN BRANCH FROM BUS *I TO BUS *J CIRCUIT *1CKT
```

The branch out-of-service event fields are described in Table 24.

Table 24: Branch Out-of-Service Event fields

Field	Description
*I	origin bus number. Refers to field I in Bus Data section (Section A.2.2).
*J	destination bus number. Refers to field I in Bus Data section (Section A.2.2).
*[1]CKT	circuit identifier, short string, i.e. 1- or 2-character string with only digits and upper case letters. Refers to field CKT in Non-transformer Branch Data (Section A.2.6) and Transformer Data (Section A.2.7) sections.

The more general format of the CON file used in commercial software does allow 3-winding transformers to be specified, but the GO Competition Challenge 2 does not use 3-winding transformers, so that more general format is not described here.

A.3.2 Generator Out-of-Service Event

A generator can be placed out of service by an event line in the following format:

```
REMOVE UNIT *1ID FROM BUS *I
```

The generator out-of-service event fields are described in Table 25.

Table 25: Generator Out-of-Service Event fields)

Field	Description
*I	bus number. Refers to field I in Bus Data section (Section A.2.2).
*[1]ID	generator identifier within bus I, short string, i.e. 1- or 2-character string consisting of digits and upper case letters. Refers to field ID in Generator Data section (Section A.2.5).

A.4 supplementary data file (case.json)

Additional data that cannot be included in the industry standard data formats represented by the RAW and CON files, is placed in a JSON formatted file, with file name case.json. This section describes the JSON supplementary data file format

A.4.1 General discussion of JSON file formatting

In this section we give an overview of JSON formatting as used in the GO Competition. In the JSON format, data is structured with arrays and dictionaries. An array is an ordered sequence of elements, separated by commas `,` and surrounded by square brackets `[]`. A dictionary is an unordered sequence of key-value pairs, separated by commas and surrounded by curly braces `{}`. A key-value pair is a pair of elements, separated by a colon `:`, where the first element, a string, is the key, and the second element is the value. In a dictionary, multiple key-value pairs with the same key are not allowed. Strings are surrounded by matching double quotation marks `""`. An element can be a number or a string or an array or a dictionary. A number is represented with the decimal digits `0` through `9`, possibly with a sign `+` or `-`, possibly with a decimal point `.`, and possibly with an exponential indicator `e` or `E`. An end-of-line comment starts with a sharp `#`. White space is ignored.

A.4.2 case.json

Listing 1 shows the format of the JSON supplementary data file case.json. The notation of the model data is used to indicate how the values in the JSON file are transformed to the model parameters and sets.

```
{
  "systemparameters": { # defines the system parameters
    "delta": $\delta$,
    "deltactg": $\delta^{ct}$,
    "deltar": $\delta^{r}$,
    "deltarctg": $\delta^{r,ct}$
  },
  "loads": [ # defines the supplementary data associated with loads
    ...,
    { # defines the supplementary data associated with a typical load $j \in J$
      "bus": $i_j$,
      "id": $id_j$,
      "tmin": $\underline{t}_j$,
      "tmax": $\overline{t}_j$,
      "prumax": $\overline{p}^{ru}_j \tilde{s}$,
      "prdmax": $\overline{p}^{rd}_j \tilde{s}$,
      "prumaxctg": $\overline{p}^{ru,ct}_j \tilde{s}$,
      "prdmaxctg": $\overline{p}^{rd,ct}_j \tilde{s}$,
      "cblocks": [ # defines the set $N_j$ (i.e. the set of constant marginal benefit blocks for real power for load $j$)
        ...,
        { # defines a typical element $n$
          "pmax": $\overline{p}_{jn} \tilde{s}$,
          "c": $c_{jn} / \tilde{s}$
```

```
        },
        ...
      ]
    },
    ...
  ],
  "generators": [ # defines the supplementary data associated
  with generators
    ...,
    { # defines the supplementary data associated with a
typical generator g ∈ G
      "bus": i_g,
      "id": id_g,
      "suqual": 1 if g ∈ G^su else 0,
      "sdqual": 1 if g ∈ G^sd else 0,
      "suqualctg": 1 if g ∈ G^{su,ct} else 0,
      "sdqualctg": 1 if g ∈ G^{sd,ct} else 0,
      "prumax": \overline{p}_g^{ru} \tilde{s},
      "prdmax": \overline{p}_g^{rd} \tilde{s},
      "prumaxctg": \overline{p}_g^{ru,ct} \tilde{s},
      "prdmaxctg": \overline{p}_g^{rd,ct} \tilde{s},
      "oncost": c_g^{on},
      "sucost": c_g^{su},
      "sdcost": c_g^{sd},
      "cblocks": [ # defines the set N_g (i.e. the set of
constant marginal cost blocks for real power for generator
g)
        ...,
        { # defines a typical element n
          "pmax": \overline{p}_{gn} \tilde{s},
          "c": c_{gn}/\tilde{s}
        },
        ...
      ]
    },
    ...
  ],
  "lines": [ # defines the supplementary data associated with
  lines
    ...,
    { # defines the supplementary data associated with a
typical line e ∈ E
      "origbus": i_e^o,
      "destbus": i_e^d,
```

```
      "id": $id_e$,
      "swqual": 1 if $e \in E^{sw}$ else 0,
      "csw": $c_e^{sw}$
    },
    ...
  ],
  "transformers": [ # defines the supplementary data
associated with transformers
    ...,
    { # defines the supplementary data associated with a
typical transformer $f \in F$
      "origbus": $i_f^o$,
      "destbus": $i_f^d$,
      "id": $id_f$,
      "swqual": 1 if $f \in F^{sw}$ else 0,
      "csw": $c_f^{sw}$
    },
    ...
  ],
  "pcblocks": [ # defines the set $N^p$ (i.e. the set of
constant marginal cost blocks for real power)
    ...,
    { # defines a typical element $n$
      "pmax": $\overline{p}_n \tilde{s}$,
      "c": $c_n^p / \tilde{s}$
    },
    ...
  ],
  "qcblocks": [ # defines the set $N^q$ (i.e. the set of
constant marginal cost blocks for reactive power)
    ...,
    { # defines a typical element $n$
      "qmax": $\overline{q}_n \tilde{s}$,
      "c": $c_n^q / \tilde{s}$
    },
    ...
  ],
  "scblocks": [ # defines the set $N^s$ (i.e. the set of
constant marginal cost blocks for apparent power)
    ...,
    { # defines a typical element $n$
      "tmax": $\overline{t}_n^s$,
      "c": $c_n^s / \tilde{s}$
    },
```

```
      ...
   ],
}
```

Listing 1: JSON supplementary input data file format

B Construction of Model Data

This appendix explains how to form the data of the GO Competition Challenge 2 problem formulation from the raw data that is read from the input files.

B.1 Reading Input Data Files

This section explains how to convert the data as read from the input files into its equivalent in the model data. The format of the input files is described in Appendix A. Names of fields read from the input files are written in upper case Roman font. E.g. SBASE is the name of the field of the Case Identification Data table of the RAW file that contains power units base, in MVA, and the value read from this field is denoted by $\tilde{s}$ in the model formulation. These field names are specified in the input file description document.

To form the model data, we first specify initial values of the data in Section B.2. Then for each table contained in the input files, as described in Sections B.3 to B.11, we give instructions on modifying these values as each record is read from the table.

B.2 Initial Parameter Values prior to Reading Data

Prior to reading data, we initialize some parameters and sets to certain values. These initial values are updated as data is read. E.g. some sets are initialized as empty sets, and elements are added to them as they are read from the data files. This initialization is specified here:

$$k_0 = \text{'BASECASE'} \tag{97}$$

$$label_{k_0} = \text{'BASECASE'} \tag{98}$$

$$K = \{k_0\} \tag{99}$$

$$A = \{1, 2, \ldots, 8\} \tag{100}$$

$$E = \{\} \tag{101}$$

$$F = \{\} \tag{102}$$

$$F^{\tau} = \{\} \tag{103}$$

$$F^{\theta} = \{\} \tag{104}$$

$$F^{\eta} = \{\} \tag{105}$$

$$G = \{\} \tag{106}$$

$$H = \{\} \tag{107}$$

$$H_{k_0} = \{\} \tag{108}$$

$$I = \{\} \tag{109}$$

$$J = \{\} \tag{110}$$

$$J_{k_0} = \{\} \tag{111}$$

$$M = \{1, 2, \ldots, 11\} \tag{112}$$

B.3 Case Identification Data from RAW

On reading field SBASE from the first line of the Case Identification Data section of the RAW file, set:

$$\tilde{s} = \text{SBASE} \tag{113}$$

B.4 Bus Data from RAW

For each record in the Bus Data section of the RAW file, read fields I, VM, VA, NVHI, NVLO, EVHI, EVLO. Then set $i = \text{I}$, and set:

$$I := I \cup \{i\} \tag{114}$$
$$v_i^0 = \text{VM} \tag{115}$$
$$\theta_i^0 = \text{VA} * \pi/180 \tag{116}$$
$$\overline{v}_i = \text{NVHI} \tag{117}$$
$$\underline{v}_i = \text{NVLO} \tag{118}$$
$$\overline{v}_i^{ct} = \text{EVHI} \tag{119}$$
$$\underline{v}_i^{ct} = \text{EVLO} \tag{120}$$

B.5 Load Data from RAW

For each record in the Load Data section of the RAW file, read fields I, ID, STATUS, PL, QL. Then set $i = \text{I}$, $id = \text{ID}$, $j = (i, id)$, and set:

$$J := J \cup \{j\} \tag{121}$$
$$i_j = i \tag{122}$$
$$id_j = id \tag{123}$$
$$p_j^0 := \text{PL}/\tilde{s} \tag{124}$$
$$q_j^0 := \text{QL}/\tilde{s} \tag{125}$$

and if STATUS = 1 then set:

$$J_{k_0} := J_{k_0} \cup \{j\} \tag{126}$$

Having read and processed the load section, set:

$$J_i = \{j \in J : i_j = i\} \; \forall i \in I \tag{127}$$

B.6 Fixed Shunt Data from RAW

Initialize the fixed shunt parameters by

$$g_i^{fs} = 0 \; \forall i \in I \tag{128}$$
$$b_i^{fs} = 0 \; \forall i \in I \tag{129}$$

For each record in the Fixed Shunt Data section of the RAW file, read fields I, STATUS, GL, BL. Then set $i = \text{I}$, and if STATUS = 1 then set:

$$g_i^{fs} := g_i^{fs} + \text{GL}/\tilde{s} \tag{130}$$

$$b_i^{fs} := b_i^{fs} + \text{BL}/\tilde{s} \tag{131}$$

B.7 Generator Data from RAW

For each record in the Generator Data section of the RAW file, read fields I, ID, PG, QG, QT, QB, STAT, PT, PB, then set $i = \text{I}$, $id = \text{ID}$, $g = (i, id)$, and set:

$$G := G \cup \{g\} \tag{132}$$

$$i_g = i \tag{133}$$

$$id_g = id \tag{134}$$

$$p_g^0 = \text{PG}/\tilde{s} \tag{135}$$

$$q_g^0 = \text{QG}/\tilde{s} \tag{136}$$

$$\overline{q}_g = \text{QT}/\tilde{s} \tag{137}$$

$$\underline{q}_g = \text{QB}/\tilde{s} \tag{138}$$

$$\overline{p}_g = \text{PT}/\tilde{s} \tag{139}$$

$$\underline{p}_g = \text{PB}/\tilde{s} \tag{140}$$

$$x_g^{on,0} := \text{STAT} \tag{141}$$

Having read and processed the generator section, set:

$$G_{k_0} = G \tag{142}$$

$$G_i = \{g \in G : i_g = i\} \; \forall i \in I \tag{143}$$

B.8 Line Data from RAW

For each record in the Non-transformer Branch Data section of the RAW file, read fields I, J, CKT, R, X, B, RATEA, RATEC, ST, then set $i = \text{I}$, $i' = \text{J}$, $id = \text{CKT}$, and $e = (i, i', id)$, and set:

$$E := E \cup \{e\} \tag{144}$$
$$i^o_e = i \tag{145}$$
$$i^d_e = i' \tag{146}$$
$$id_e = \mathrm{CKT} \tag{147}$$
$$g_e = \mathrm{R}/(\mathrm{R}^2 + \mathrm{X}^2) \tag{148}$$
$$b_e = -\mathrm{X}/(\mathrm{R}^2 + \mathrm{X}^2) \tag{149}$$
$$b^{ch}_e = \mathrm{B} \tag{150}$$
$$\overline{r}_e = \mathrm{RATEA}/\tilde{s} \tag{151}$$
$$\overline{r}^{ct}_e = \mathrm{RATEC}/\tilde{s} \tag{152}$$
$$x^{sw,0}_e = \mathrm{ST} \tag{153}$$

Having read and processed the non-transformer branch section, set:

$$E_{k_0} = E \tag{154}$$
$$E^o_i = \{e \in E : i^o_e = i\} \ \forall i \in I \tag{155}$$
$$E^d_i = \{e \in E : i^d_e = i\} \ \forall i \in I \tag{156}$$
$$\tag{157}$$

B.9 Transformer Data from RAW

For each record in the Transformer Data section of the RAW file, read fields I, J, CKT, MAG1, MAG2, STAT, R12, X12, WINDV1, ANG1, RATA1, RATC1, COD1, RMA1, RMI1, NTP1, TAB1, WINDV2, then set $i = \text{I}$, $i' = \text{J}$, $id = \text{CKT}$, $f = (i, i', id)$, and set:

$$F := F \cup \{f\} \tag{158}$$
$$i^o_f = i \tag{159}$$
$$i^d_f = i' \tag{160}$$
$$id_f = \text{CKT} \tag{161}$$
$$g^m_f = \text{MAG1} \tag{162}$$
$$b^m_f = \text{MAG2} \tag{163}$$
$$g^0_f = (\text{R12})/((\text{R12}^2 + (\text{X12})^2) \tag{164}$$
$$b^0_f = -(\text{X12})/((\text{R12})^2 + (\text{X12})^2) \tag{165}$$
$$\tau^0_f = \text{WINDV1}/\text{WINDV2} \tag{166}$$
$$\theta^0_f = \text{ANG1} * \pi/180 \tag{167}$$
$$\overline{x}^{st}_f = (\text{NTP1} - 1)/2 \tag{168}$$
$$\overline{\tau}_f = \begin{cases} \text{RMA1} & \text{if COD1} \in \{-1, 1\} \\ \tau^0_f & \text{else} \end{cases} \tag{169}$$
$$\underline{\tau}_f = \begin{cases} \text{RMI1} & \text{if COD1} \in \{-1, 1\} \\ \tau^0_f & \text{else} \end{cases} \tag{170}$$
$$\tau^{st}_f = \begin{cases} (\overline{\tau}_f - \underline{\tau}_f)/(2 * \overline{x}^{st}_f) & \text{if NTP1} > 1 \\ 0 & \text{else} \end{cases} \tag{171}$$
$$\check{\tau}_f = (\overline{\tau}_f + \underline{\tau}_f)/2 \tag{172}$$
$$\overline{\theta}_f = \begin{cases} \text{RMA1} * \pi/180 & \text{if COD1} \in \{-3, 3\} \\ \theta^0_f & \text{else} \end{cases} \tag{173}$$
$$\underline{\theta}_f = \begin{cases} \text{RMI1} * \pi/180 & \text{if COD1} \in \{-3, 3\} \\ \theta^0_f & \text{else} \end{cases} \tag{174}$$
$$\theta^{st}_f = \begin{cases} (\overline{\theta}_f - \underline{\theta}_f)/(2 * \overline{x}^{st}_f) & \text{if NTP1} > 1 \\ 0 & \text{else} \end{cases} \tag{175}$$
$$\check{\theta}_f = (\overline{\theta}_f + \underline{\theta}_f)/2 \tag{176}$$
$$\overline{s}_f = \text{RATA1}/\tilde{s} \tag{177}$$
$$\overline{s}^{ct}_f = \text{RATC1}/\tilde{s} \tag{178}$$
$$x^{sw,0}_f = \text{STAT} \tag{179}$$

If COD1 $\in \{-1, 1\}$ then set:

$$F^\tau := F^\tau \cup \{f\} \tag{180}$$

If COD1 $\in \{-3, 3\}$ then set:

$$F^\theta := F^\theta \cup \{f\} \tag{181}$$

If TAB1 $\neq 0$ and COD1 $\in \{-3, -1, 1, 3\}$, then set:

$$F^\eta := F^\eta \cup \{f\} \tag{182}$$

and look up the record in the Transformer Impedance Correction Tables whose field I has the value of TAB1, and read the fields T1, F1, T2, F2, ..., and let NUMM be the number of pairs Tm, Fm that are found. Set:

$$M_f = \{1, 2, \ldots, \text{NUMM}\} \tag{183}$$
$$\eta_{fm} = \text{F}m \ \forall m \in M_f \tag{184}$$

If COD1 $\in \{-1, 1\}$ then set:

$$\tau_{fm} = \text{T}m \ \forall m \in M_f \tag{185}$$

If COD1 $\in \{-3, 3\}$ then set:

$$\theta_{fm} = \text{T}m * \pi/180 \ \forall m \in M_f \tag{186}$$

Having read and processed the transformer section, set:

$$F_{k_0} = F \tag{187}$$
$$F_i^o = \{f \in F : i_f^o = i\} \ \forall i \in I \tag{188}$$
$$F_i^d = \{f \in F : i_f^d = i\} \ \forall i \in I \tag{189}$$

B.10 Switched Shunt Data from RAW

For each record in the Switched Shunt Data section of the RAW file, read fields I, STAT, BINIT, N1, B1, N2, B2, ..., and let NBL be the number of pairs $(\text{N}a, \text{B}a)$ that are found before any pair with $\text{N}a = 0$ or $\text{B}a = 0$. Set $i = \text{I}$, $h = i$. Then set:

$$H := H \cup \{h\} \tag{190}$$
$$i_h = i \tag{191}$$
$$b_h^{cs0} = \text{BINIT}/\tilde{s} \tag{192}$$
$$A_h = \{1, 2, \ldots, NBL\} \tag{193}$$
$$\overline{x}_{ha}^{st} = \text{N}a \ \forall a \in A_h \tag{194}$$
$$b_{ha}^{st} = \text{B}a/\tilde{s} \ \forall a \in A_h \tag{195}$$

If STAT $= 1$ then set:

$$H_{k_0} := H_{k_0} \cup \{h\} \tag{196}$$

Having read and processed the switched shunt section, set:

$$H_i = \{h \in H : i_h = i\} \ \forall i \in I \tag{197}$$
$$\tag{198}$$

B.11 Contingency Data from CON

In the CON file, read the Contingency Case Data Description records one at a time. For each record read, do the following:

Suppose the field LABEL is read from the start line of the record. Let k = LABEL. It can be assumed that the value of LABEL is not 'BASECASE'. Set:

$$K := K \cup \{k\} \tag{199}$$

$$label_k = \text{LABEL} \tag{200}$$

The rest of the record contains exactly one contingency event Line and then a contingency record end line. The contingency event is either (1) a Branch Out-of-Service Event or (2) a Generator Out-of-Service Event.

Suppose the Contingency Event is a Branch Out-of-Service Event. Then read fields I, J, CKT and set i = I, i' = J, id = CKT. It can be assumed that either $(i, i', id) \in E$ or $(i, i', id) \in F$ but not both. If $(i, i', id) = e \in E$ then set:

$$E_k := E \setminus \{e\} \tag{201}$$

If $(i, i', id) = f \in F$ then set:

$$F_k := F \setminus \{f\} \tag{202}$$

If the Contingency Event is a Generator Out-of-Service Event read fields I, ID, set i = I, id = ID, $g = (i, id)$ and set:

$$G_k := G \setminus \{g\} \tag{203}$$

Having read and processed the contingency file, set:

$$J_k = J_{k_0} \ \forall k \in K \setminus \{k_0\} \tag{204}$$

$$H_k = H_{k_0} \ \forall k \in K \setminus \{k_0\} \tag{205}$$

(206)

B.12 Supplementary Data from JSON

The conversion from the data read from JSON to its equivalent in the model data is defined by the description of the JSON format in Listing 1.

B.13 Construction of Further Data

Set:

$$label_{k_0} = \text{`BASECASE'} \tag{207}$$

$$J_k = J_{k_0} \ \forall k \in K \setminus \{k_0\} \tag{208}$$

$$G_{k_0} = G \tag{209}$$

$$E_{k_0} = E \tag{210}$$

$$F_{k_0} = F \tag{211}$$

$$H_k = H_{k_0} \ \forall k \in K \setminus \{k_0\} \tag{212}$$

$$J_i = \{j \in J : i_j = i\} \ \forall i \in I \tag{213}$$

$$G_i = \{g \in G : i_g = i\} \ \forall i \in I \tag{214}$$

$$E_i^o = \{e \in E : i_e^o = i\} \ \forall i \in I \tag{215}$$

$$E_i^d = \{e \in E : i_e^d = i\} \ \forall i \in I \tag{216}$$

$$F_i^o = \{f \in F : i_f^o = i\} \ \forall i \in I \tag{217}$$

$$F_i^d = \{f \in F : i_f^d = i\} \ \forall i \in I \tag{218}$$

$$H_i = \{h \in H : i_h = i\} \ \forall i \in I \tag{219}$$

$$\check{\tau}_f = (\overline{\tau}_f + \underline{\tau}_f)/2 \ \forall f \in F \tag{220}$$

$$\check{\theta}_f = (\overline{\theta}_f + \underline{\theta}_f)/2 \ \forall f \in F \tag{221}$$

C Solution Output File Format

The solution should be written to a collection of case solution files, one for each case $k \in K$, including the base case k_0 and the contingency cases. A sample case solution file is shown in Figure 1.

The variable values reported in the solution files should be written in the model unit convention, i.e. the same units as those in which the model is written, so no conversion from the model variables to the values written in the solution files is needed.

The file name of the case solution file for case k is 'solution_LABEL.txt', where LABEL is the string value of k. E.g., for $k = k_0$, the base case, with $label_k$ = 'BASECASE', the case solution file name is 'solution_BASECASE.txt'. And for a case k with $label_k$ = 'GEN_1_1' the case solution file name is 'solution_GEN_1_1.txt'.

Strings needed in the case solution file, including one- or two-character ID strings, should be written without quote characters.

Case solution file sections Each case solution file has the same format, a fixed sequence of six sections each with a table of comma separated values with a header row and a sequence of data rows with fields in a fixed order. The beginning of each section is delimited by a start line starting with two hyphen characters ('--'). The six sections, in order, are:

1. bus
2. load
3. generator
4. line
5. transformer
6. switched shunt

Bus section fields Each data row of the bus section contains the following fields, in order, for a particular bus $i \in I$:

1. bus number i (positive integer)
2. voltage magnitude v_{ik}
3. voltage angle θ_{ik}

Every bus $i \in I$ should be reported in exactly one row of the bus section.

Load section fields Each data row of the load section contains the following fields, in order, for a particular load $j \in J_k$:

1. bus number i_j (positive integer)
2. ID id_j (1- or 2-character string)
3. cleared fraction t_{jk}.

Every load $j \in J_k$ should be reported in exactly one row of the load section.

Generator section fields Each data row of the generator section contains the following fields, in order, for a particular generator $g \in G_k$:

1. bus number i_g (positive integer)
2. unit ID id_g (1- or 2-character string)
3. real power output p_{gk}
4. reactive power output q_{gk}
5. commitment status x^{on}_{gk} (binary, 1 indicates on, 0 off)

Every generator $g \in G_k$ should be reported in exactly one row of the generator section.

Line section fields Each data row of the line section contains the following fields, in order, for a particular line $e \in E_k$:

1. origin bus number i^o_e (positive integer)
2. destination bus number i^d_e (positive integer)
3. circuit ID id_e (1- or 2-character string)
4. closed/open status x^{sw}_{ek} (binary, 1 indicates closed, 0 open)

Every line $e \in E_k$ should be reported in exactly one row of the line section.

Transformer section fields Each data row of the transformer section contains the following fields, in order, for a particular transformer $f \in F_k$:

1. origin bus number i^o_f (postive integer)
2. destination bus number i^d_f (positive integer)
3. circuit ID id_f (1- or 2-character string)
4. closed/open status x^{sw}_{fk} (binary, 1 indicates closed, 0 open)
5. tap position selection x^{st}_{fk} (integer)

Every transformer $f \in F_k$ should be reported in exactly one row of the transformer section.

```
--bus section
i, v, theta
1, 1.01, 0.1
2, 0.99, -0.06
3, 0.98, 0.02
4, 1.02, -0.05
5, 1.005, 0.05
--load section
i, id, t
1, 1, 0.95
--generator section
i, id, p, q, x
1, 1, 1.0, 0.1, 1
1, 2, 0.5, 0.0, 1
--line section
iorig, idest, id, x
1, 2, 1, 1
1, 2, 2, 0
3, 4, 1, 1
--transformer section
iorig, idest, id, x, xst
2, 3, 1A, 1, 16
4, 5, 1B, 1, -16
--switched shunt section
i, xst1, xst2, xst3, xst4, xst5, xst6, xst7, xst8
1, 1, 4
2, 0, 2, 3
```

Figure 1: A sample case solution file `solution_label.txt`

Switched shunt section fields Each data row of the switched shunt section contains the following fields, in order, for a particular switched shunt $h \in H_k$:

1. bus number i_h (positive integer)
2. number x^{st}_{hak} of activated steps (positive integer) for $a \in A_h$ in the order of A_h

For $a \in A \setminus A_h$, no value should be reported and no field should be indicated by commas, i.e. each row has exactly the entries i_h and x^{st}_{hak} for $a \in A_h$. Every switched shunt $h \in H_k$ should be reported in exactly one row of the switched shunt section.

D Solution Evaluation

D.1 Solution Evaluation Procedure

The GO Competition has written a Python code to evaluate solutions, and this code will be used to judge algorithms submitted by entrants. The solution evaluation code is available to entrants at the Challenge 2 Solution Evaluation page https://gocompetition.energy.gov/challenges/challenge-2/solution-evaluation/. The main procedure is contained in `evaluation.py`, and this module uses the data reading code `data.py`. The solution evaluation procedure is described in Algorithm 1. The evaluation procedure uses a tolerance $\epsilon = 10^{-4}$. We introduce a concept of feasibility for the pupose of evaluation, called *evaluation feasibility*, or *e-feasibility* for short. Of the constraints in the model, some are treated as hard constraints with 0 tolerance, some as hard constraints with tolerance ϵ, and some as soft constraints with a penalty on violations. The soft constraints are explicitly modeled as such in the formulation in Section 3. For some of the soft constraints, specifically the line and transformer flow limit constraints, a violation of the constraint is a physically real phenomenon but undesirable from an engineering point of view. For others, specifically the bus real and reactive power balance constraints, a violation is not a physically real phenomenon but is included in the model to ensure that a solution can be evaluated. For these power balance constraints, a significant violation contradicts our intuitive understanding of a feasible solution to the problem but does not preclude the solution from being e-feasible. Therefore, a solution is e-feasible if it satisfies the hard constraints up to the specified tolerance. The precise definition of e-feasibility is specified in the solution evaluation procedure Algorithm 1. There are two main outputs of the solution evaluation procedure: an infeasibility indicator, equal to 0 if the solution is deemed e-feasible and 1 otherwise, and the objective value z, which is a real value in USD.

D.2 Construction of Infeasibility Solution and Objective

In this section we give a very simple method to construct an e-feasible solution that we then use to define the worst possible objective assigned to any solution. This method extends the given operating point prior to the base case into the base case and the contingencies and relies on the bus power imbalance variables and the line and transformer rating exceedance variables to ensure e-feasibility. The solution constructed in this way is called the *infeasibility solution*, and the *infeasibility objective* z^{inf} is defined to be the objective value of this solution. A python code implementing the construction of the infeasibility solution is available at the Challenge 2 Solution Evaluation page https://gocompetition.energy.gov/challenges/challenge-2/solution-evaluation/.

Algorithm 1 Solution Evaluation

1: Check solution file format against the specification in Appendix C. If a solution file is missing or formatted incorrectly, then the solution is deemed e-infeasible.
2: Read solution input variables x^{on}_{gk}, x^{sw}_{ek}, x^{sw}_{fk}, x^{st}_{hak}, x^{st}_{fk}, v_{ik}, θ_{ik}, p_{gk}, q_{gk}, t_{jk} from solution files.
3: Round integer input variables x^{st}_{hak}, x^{sw}_{ek}, x^{sw}_{fk}, x^{st}_{fk}, x^{on}_{gk} to the nearest integer values.
4: Check domains of integer input variables, i.e. Equations (46), (48), (49), (59) to (61) and (81). If any violation > 0 is found, then the solution is deemed e-infeasible.
5: Compute generator start up and shut down variables x^{su}_{gk}, x^{sd}_{gk} from Equations (82) to (84).
6: Check constraints on generator start up and shut down variables, i.e. Equations (91) to (96). If any violation > 0 is found, then the solution is deemed e-infeasible.
7: Check simple bounds on continuous input variables v_{ik}, t_{jk}, i.e. Equations (31), (32) and (39). If any violation $> \epsilon$ is found, or any $t_{jk} < 0$, then the solution is deemed e-infeasible.
8: Compute load real and reactive power consumption variables p_{jk}, q_{jk} from Equations (40) and (41).
9: Check simple inequality constraints for load ramping, generator bounds, and generator ramping, i.e. Equations (42) to (45) and (85) to (90). If any violation $> \epsilon$ is found, or any $p_{gk} < 0$, then the solution is deemed e-infeasible.
10: Compute switched shunt susceptance variables b^{cs}_{hk} from Equation (47).
11: Compute transformer tap ratio and phase shift variables τ_{fk}, θ_{fk} from Equations (62) to (65).
12: Compute transformer impedance correction variables η_{fk} from Equations (70) and (71).
13: Compute transformer series conductance and susceptance variables g_{fk}, b_{fk} from Equations (66) to (69)
14: Compute line and transformer real and reactive power flow variables p^{o}_{ek}, p^{d}_{ek}, q^{o}_{ek}, q^{d}_{ek}, p^{o}_{fk}, p^{d}_{fk}, q^{o}_{fk}, q^{d}_{fk} from Equations (50) to (53) and (72) to (75).
15: Compute minimal bus real and reactive power imbalance variables p^{+}_{ik}, p^{-}_{ik}, q^{+}_{ik}, q^{-}_{ik} from Equations (33) to (38)
16: Compute minimal line and transformer rating exceedance variables s^{+}_{ek}, s^{+}_{fk}, from Equations (54) to (58) and (76) to (80).
17: Compute bus imbalance block variables p^{+}_{ikn}, p^{-}_{ikn}, q^{+}_{ikn}, q^{-}_{ikn} and maximal bus objective variables z_{ik} from Equations (3) to (12).
18: Compute load block variables p_{jkn} and maximal load objective variables z_{jk} from Equations (13) to (16).
19: Compute line rating exceedance block variables s^{+}_{enk} and maximal line objective variables z_{ek} from Equations (17) to (21).
20: Compute transformer rating exceedance block variables s^{+}_{fnk} and maximal transformer objective variables z_{fk} from Equations (22) to (26).
21: Compute generator real power block variables p_{gnk} and maximal generator objective variables z_{gk} from Equations (27) to (30).
22: Compute case objective variables z_k from Equation (2).
23: Compute total objective variable z from Equation (1).
24: Return infeasibility indicator 0 if e-feasible and 1 otherwise, and total objective value z.

To construct the infeasibility solution, first set:

$$x^{on}_{gk} = x^{on,0}_{g}, \ \forall k \in K, g \in G_k \tag{222}$$
$$x^{sw}_{ek} = x^{sw,0}_{e}, \ \forall k \in K, e \in E_k \tag{223}$$
$$x^{sw}_{fk} = x^{sw,0}_{f}, \ \forall k \in K, f \in F_k \tag{224}$$
$$x^{st}_{hak} = 0 \ \forall k \in K, h \in H_k, a \in A_h \tag{225}$$
$$x^{st}_{fk} = 0 \ \forall k \in K, f \in F_k \setminus (F^{\tau} \cup F^{\theta}) \tag{226}$$
$$v_{ik} = \min\{\max\{v^0_i, \underline{v}_i\}, \overline{v}_i\} \ \forall k = k_0, i \in I \tag{227}$$
$$v_{ik} = \min\{\max\{v^0_i, \underline{v}^{ct}_i\}, \overline{v}^{ct}_i\} \ \forall k \in K \setminus \{k_0\}, i \in I \tag{228}$$
$$\theta_{ik} = \theta^0_i \ \forall k \in K, i \in I \tag{229}$$
$$t_{jk} = \min\{\max\{1, \underline{t}_j\}, \overline{t}_j\} \ \forall k \in K, j \in J_k \tag{230}$$
$$p_{gk} = \min\{\max\{p^0_g, \underline{p}_g\}, \overline{p}_g\} x^{on,0}_g \ \forall k \in K, g \in G_k \tag{231}$$
$$q_{gk} = \min\{\max\{q^0_g, \underline{q}_g\}, \overline{q}_g\} x^{on,0}_g \ \forall k \in K, g \in G_k \tag{232}$$

Then for $k \in K$, $f \in F_k \cap F^{\tau}$, set $x^{st}_{fk} = x^{st,0}_f$ as defined by

$$x^{st,0}_f = \min\{\max\{\text{round}((\tau^0_f - \check{\tau}_f)/\tau^{st}_f), -\overline{x}_f\}, \overline{x}_f\} \ \forall f \in F^{\tau} \tag{233}$$

Then for $k \in K$, $f \in F_k \cap F^{\theta}$, set $x^{st}_{fk} = x^{st,0}_f$ as defined by

$$x^{st,0}_f = \min\{\max\{\text{round}((\theta^0_f - \check{\theta}_f)/\theta^{st}_f), -\overline{x}_f\}, \overline{x}_f\} \ \forall f \in F^{\theta} \tag{234}$$

Then for $h \in H$, $a \in A_h$, $k \in K$, set $x^{st}_{hak} = x^{st,0}_{ha}$, for any choice of

$$\left(x^{st,0}_{ha}\right)_{a \in A_h} \in \text{argmin}_x \left\{ \left| b^{cs,0}_h - \sum_{a \in A_h} b^{st}_{ha} x_a \right| : x_a \in \{0, 1, \dots, \overline{x}_{ha}\} \ \forall a \in A_h \right\} \ \forall h \in H \tag{235}$$

In the implementation used by the GO Competition, $x^{st,0}_{ha}$ is computed by enumeration. Then write the solution output files as directed in Appendix C. Then compute z^{inf} as the objective value of this infeasibility solution as directed in Section D.1. The GO Competition Administrator will compute z^{inf} using the infeasibility solution construction code and the solution evaluation code.

D.3 Solution Scoring Objective

Each submitted solution is assigned a scoring objective z^{sc}. If no solution is submitted within the required time limit, or the solution files are unreadable or incorrectly formatted, or the solution is e-infeasible, then $z^{sc} = z^{inf}$. If the solution is e-feasible with objective value z, then $z^{sc} = \max(z, z^{inf})$. The scoring objective values of all the submitted algorithms on all the problem scenarios are the inputs to the algorithm scoring procedure, which may include further processing of the z^{sc} values, and is described in the scoring document available at the Challenge 2 Scoring page https://gocompetition.energy.gov/challenges/challenge-2/scoring/.

E Data Properties

In this section we document the properties that we guarantee the data will have. All numeric values in the input data are finite real or integer numbers.

E.1 Power balance in the prior operating point

The given operating point prior to the base case, as represented by v_i^0, θ_i^0, p_g^0, q_g^0, p_j^0, q_j^0, b_a^0, τ_f^0, θ_f^0, $x_g^{on,0}$, $x_e^{sw,0}$, $x_f^{sw,0}$, satisfies real and reactive power balance at each bus to a reasonable numerical tolerance.

E.2 Positivity of certain scalar values

The system power base and all time constants are strictly positive:

$$\tilde{s} > 0 \tag{236}$$
$$\delta > 0 \tag{237}$$
$$\delta^{ct} > 0 \tag{238}$$
$$\delta^{r} > 0 \tag{239}$$
$$\delta^{r,ct} > 0 \tag{240}$$

E.3 Evaluation tolerance

The evaluation tolerance is a small positive number large enough to cover floating point errors:

$$\epsilon = 10^{-4} \tag{241}$$

E.4 Bus voltage bounds

Bus voltage bounds and the voltage in the given operating point prior to the base case are strictly positive, and the emergency (post-contingency) bounds are no tighter than the normal (base case) bounds:

$$0 < \underline{v}_i^{ct} \leq \underline{v}_i \leq \overline{v}_i \leq \overline{v}_i^{ct} \ \forall i \in I \tag{242}$$
$$0 < v_i^0 \ \forall i \in I \tag{243}$$

E.5 Nonnegative load and generation in prior operating point

In the prior operating point, loads have nonnegative real power consumption, and generators have nonnegative real power output:

$$0 \leq p_j^0 \ \forall j \in J \tag{244}$$
$$0 \leq p_g^0 \ \forall g \in G \tag{245}$$

E.6 Nonnegative load bounds

Load dispatch bounds are nonnegative, and the lower bound is no higher than the upper bound:

$$0 \leq \underline{t}_j \leq \overline{t}_j \ \forall j \in J \tag{246}$$

E.7 Nonnegative ramp rates

Load and generator ramp rates are nonnegative:

$$p_j^{ru} \geq 0 \ \forall j \in J \tag{247}$$
$$p_j^{rd} \geq 0 \ \forall j \in J \tag{248}$$
$$p_j^{ru,ct} \geq 0 \ \forall j \in J \tag{249}$$
$$p_j^{rd,ct} \geq 0 \ \forall j \in J \tag{250}$$
$$p_g^{pu} \geq 0 \ \forall g \in G \tag{251}$$
$$p_g^{pd} \geq 0 \ \forall g \in G \tag{252}$$
$$p_g^{pu,ct} \geq 0 \ \forall g \in G \tag{253}$$
$$p_g^{pd,ct} \geq 0 \ \forall g \in G \tag{254}$$

E.8 Binary prior status values

Generator commitment status and line and transformer closed/open status are binary values in the given operating point prior to the base case:

$$x_g^{on,0} \in \{0, 1\} \ \forall g \in G \tag{255}$$
$$x_e^{sw,0} \in \{0, 1\} \ \forall e \in E \tag{256}$$
$$x_f^{sw,0} \in \{0, 1\} \ \forall f \in F \tag{257}$$

E.9 Generator bounds

Generator real power bounds are nonnegative, and, for real and reactive power, the lower bound is no higher than the upper bound:

$$0 \leq \underline{p}_g \leq \overline{p}_g \ \forall g \in G \tag{258}$$
$$\underline{q}_g \leq \overline{q}_g \ \forall g \in G \tag{259}$$

E.10 Generator prior operation

In the given operating point prior to the base case, generator real and reactive power outputs are equal to 0 if the commitment status is 0:

$$\text{if } x_g^{on,0} = 0 \text{ then } p_g^0 = 0 \ \forall g \in G \tag{260}$$
$$\text{if } x_g^{on,0} = 0 \text{ then } q_g^0 = 0 \ \forall g \in G \tag{261}$$

E.11 Feasible operation of load relative to prior operating point

The system ramping time constant δ^r the load technical characteristics $\underline{t}_j$, $\overline{t}_j$, p^{ru}_j, p^{rd}_j and the load prior real power p^0_g ensure that a load has a nonempty operating range in the base case:

$$\max\{p^0_j - \delta^r p^{rd}_j, \underline{t}_j p^0_j\} \leq \min\{p^0_j + \delta^r p^{ru}_j, \overline{t}_j p^0_j\} \ \forall j \in J \tag{262}$$

E.12 Feasible operation of generator relative to prior operating point when remaining on

The system ramping time constant δ^r, the generator technical characteristics $\underline{p}_g$, $\overline{p}_g$, p^{ru}_g, p^{rd}_g and the generator prior operating characteristics p^0_g, $x^{on,0}_g$ ensure that a generator remaining on from the prior point to the base case has a nonempty operating range:

$$\max\{p^0_g - \delta^r p^{rd}_g, \underline{p}_g\} \leq \min\{p^0_g + \delta^r p^{ru}_g, \overline{p}_g\} \ \forall g \in G \text{ with } x^{on,0}_g = 1 \tag{263}$$

E.13 Cost and benefit function block widths are nonnegative

The widths of blocks of cost and benefit functions are nonnegative:

$$0 \leq \overline{p}_{jn} \ \forall j \in J, n \in N_j \tag{264}$$

$$0 \leq \overline{p}_{gn} \ \forall g \in G, n \in N_g \tag{265}$$

$$0 \leq \overline{p}_n \ \forall n \in N^p \tag{266}$$

$$0 \leq \overline{q}_n \ \forall n \in N^q \tag{267}$$

$$0 \leq \overline{t}^s_n \ \forall n \in N^s \tag{268}$$

E.14 Generator cost function domains

The cost function domain of each generator covers its real power bounds plus the evaluation tolerance:

$$\overline{p}_g + \epsilon \leq \sum_{n \in N_g} \overline{p}_{gn} \ \forall g \in G \tag{269}$$

E.15 Load benefit function domains

The benefit function domain of each load covers its dispatchable bounds plus the evaluation tolerance:

$$p^0_j(\overline{t}_j + \epsilon) \leq \sum_{n \in N_j} \overline{p}_{jn} \ \forall j \in J \tag{270}$$

E.16 Power imbalance and rating exceeedance cost function domains

Bus power imbalance and line and transformer rating exceedance cost functions cover any practical need:

$$\sum_{n \in N^p} \overline{p}_n \geq 10^{12} \tag{271}$$

$$\sum_{n \in N^q} \overline{q}_n \geq 10^{12} \tag{272}$$

$$\sum_{n \in N^s} \overline{t}^s_n \geq 10^{12} \tag{273}$$

E.17 Positive ratings

Line and transformer ratings are strictly postive, and the emergency rating is no lower than the normal rating:

$$0 < \overline{r}_e \leq \overline{r}^{ct}_e \ \forall e \in E \tag{274}$$

$$0 < \overline{s}_f \leq \overline{s}^{ct}_f \ \forall f \in F \tag{275}$$

E.18 Non-zero line impedance

There are no zero-impedance lines. This property is reflected in th Non-transformer Branch (i.e. line) section of the RAW file as a requirement on the fields R, X, of each record. In particular, we assert that every line will have

$$(\mathrm{R}^2 + \mathrm{X}^2)^{1/2} > 0 \tag{276}$$

E.19 Non-zero transformer impedance

There are no zero-impedance transformers. This property is reflected in the Transformer section of the RAW file as a requirement on the fields R12, X12, of each record. In particular, we assert that every transformer will have

$$(\mathrm{R12}^2 + \mathrm{X12}^2)^{1/2} > 0 \tag{277}$$

E.20 Tap ratio bounds

The tap ratio bounds of each transformer are positive, and the tap ratio in the given operating point prior to the base case is within the bounds:

$$0 < \underline{\tau}_f \leq \tau^0_f \leq \overline{\tau}_f \ \forall f \in F \tag{278}$$

E.21 Phase shift bounds

The phase shift of each transformer in the given operating point prior to the base case is within the phase shift bounds:

$$\underline{\theta}_f \leq \theta_f^0 \leq \overline{\theta}_f \ \forall f \in F \tag{279}$$

E.22 Impedance correction factor positive

The impedance correction factor of every transformer impedance correction function is always strictly positive:

$$\eta_{mf} > 0 \ \forall f \in F^\eta, m \in M_f \tag{280}$$

E.23 Impedance correction vertices in order, variable tap

The vertices of the impedance correction function for every variable tap ratio transformer with impedance correction are listed in order of increasing tap ratio, and no two vertices have the same tap ratio value:

$$\tau_{mf} < \tau_{m+1,f} \ \forall f \in F^\tau \cap F^\eta, m, m+1 \in M_f \tag{281}$$

E.24 Impedance correction vertices in order, variable phase

The vertices of the impedance correction function for every variable phase shift transformer with impedance correction are listed in order of increasing phase shift, and no two vertices have the same phase shift value:

$$\theta_{mf} < \theta_{m+1,f} \ \forall f \in F^\theta \cap F^\eta, m, m+1 \in M_f \tag{282}$$

E.25 At least two impedance correction vertices

Every transformer impedance correction function is represented by at least two vertices:

$$|M_f| \geq 2 \ \forall f \in F^\eta \tag{283}$$

E.26 Impedance correction domain, variable tap

The domain of the impedance correction function for each variable tap ratio transformer with impedance correction covers the tap ratio operating range:

$$\min\{\tau_{mf} : m \in M_f\} \leq \underline{\tau}_f \ \forall f \in F^\tau \cap F^\eta \tag{284}$$

$$\overline{\tau}_f \leq \max\{\tau_{mf} : m \in M_f\} \ \forall f \in F^\tau \cap F^\eta \tag{285}$$

E.27 Impedance correction domain, variable phase

The domain of the impedance correction function for each variable phase shift transformer with impedance correction covers the phase shift range:

$$\min\{\theta_{mf} : m \in M_f\} \leq \underline{\theta}_f \ \forall f \in F^\theta \cap F^\eta \tag{286}$$

$$\overline{\theta}_f \leq \max\{\theta_{mf} : m \in M_f\} \ \forall f \in F^\theta \cap F^\eta \tag{287}$$

E.28 Transformer COD1 field allowed values

The allowed values of the COD1 field in each transformer record in the RAW file are -3, -1, 0, 1, 3. Any other value is an error.

E.29 Variable tap or phase only

No transformer has both variable tap ratio and variable phase shift:

$$F^\tau \cap F^\theta = \{\} \tag{288}$$

E.30 Impedance correction applies to variable tap or variable phase

Every transformer having impedance correction is either variable tap ratio or variable phase shift:

$$F^\eta \subset F^\tau \cup F^\theta \tag{289}$$

E.31 Contingencies

There is at least one contingency case:

$$|K \setminus \{k_0\}| \geq 1 \tag{290}$$

E.32 Bus indices

There are no repeated index values among the buses. The set I of buses is in one-to-one correspondence with the set of bus indices.

E.33 Load IDs

There are no repeated ID values among the loads at a given bus. The set J_i of loads at any bus $i \in I$ is in one-to-one correspondence with the set $\{id_j : j \in J_i\}$ of load IDs of such loads.

E.34 Generator IDs

There are no repeated ID values among the generators at a given bus. The set G_i of generators at any bus $i \in I$ is in one-to-one correspondence with the set $\{id_g : g \in G_i\}$ of generator IDs of such generators.

E.35 Line and transformer IDs

There are no repeated ID values among the lines and transformers connected to a given pair of buses. For any two buses $i, i' \in I$ with $i \neq i'$, the set $(E_i^o \cap E_{i'}^d) \cup (E_i^d \cap E_{i'}^o) \cup (F_i^o \cap F_{i'}^d) \cup (F_i^d \cap F_{i'}^o)$ of lines and transformers connected to i and i' is in one-to-one correspondence with the set $\{id_e : e \in (E_i^o \cap E_{i'}^d) \cup (E_i^d \cap E_{i'}^o)\} \cup \{id_f : f \in (F_i^o \cap F_{i'}^d) \cup (F_i^d \cap F_{i'}^o)\}$ of line IDs and transformer IDs of such lines and transformers.

E.36 Case labels

There are no repeated label values among the cases. The set K of cases is in one-to-one correspondence with the set $\{label_k : k \in K\}$ of case labels. In particular, since the base case label is $label_{k_0} =$ 'BASECASE', this means that $label_k \neq$ 'BASECASE' for any contingency case $k \in K \setminus \{k_0\}$.

E.37 No islands

The electrical network does not have multiple islands in the prior operating point, and it remains connected after any contingency. To state this assumption precisely, we use the terminology of graph theory. For each case $k \in K$ we specify an undirected graph, called the *unswitched system graph*, and assert that this graph is connected.

For the base case the unswitched system graph is defined as follows. First, the set of vertices of the graph is I. Second, every $e \in E$ with $x_e^{sw,0} = 1$ is an edge between the vertices i_e^o and i_e^d. Third, every $f \in F$ with $x_f^{sw,0} = 1$ is an edge between the vertices i_f^o and i_f^d, Fourth, the edges specified in this way are all the edges of the graph.

For any contingency $k \in K$, the unswitched system graph is defined in a similar way, but using only branches that remain in service in contingency k. First, the set of vertices of the graph is I. Second, every $e \in E_k$ with $x_e^{sw,0} = 1$ is an edge between the vertices i_e^o and i_e^d. Third, every $f \in F_k$ with $x_f^{sw,0} = 1$ is an edge between the vertices i_f^o and i_f^d, Fourth, the edges specified in this way are all the edges of the graph.

Finally we assume:

For each $k \in K$ the unswitched system graph of case k is connected. (291)

E.38 Transformer tap and switched shunt susceptance values in prior operating point are feasible

The transformer tap ratios τ_f^0 and and phase shifts θ_f^0 and the switched shunt susceptances $b_h^{cs,0}$ in the given operating point prior to the base case are feasible under the given discrete settings, and we state this assumption precisely as follows.

First, for every $f \in F^\tau$ there exists an integer $x_f^{st,0}$ with $-\overline{x}_f^{st} \leq x_f^{st,0} \leq \overline{x}_f^{st}$ such that

$$\tau_f^0 = \check{\tau}_f + \tau_f^{st} x_f^{st,0} \tag{292}$$

to a reasonable numerical tolerance. Values of $x_f^{st,0}$ satisfying this assumption can be computed by Equation (233).

Second, for every $f \in F^\theta$ there exists an integer $x_f^{st,0}$ with $-\overline{x}_f^{st} \leq x_f^{st,0} \leq \overline{x}_f^{st}$ such that

$$\theta_f^0 = \check{\theta}_f + \theta_f^{st} x_f^{st,0} \tag{293}$$

to a reasonable numerical tolerance. Values of $x_f^{st,0}$ satisfying this assumption can be computed by Equation (234).

Third, for every $h \in H$ and $a \in A_h$ there exist integers $x_{ha}^{st,0}$ with $0 \leq x_{ha}^{st,0} \leq \overline{x}_{ha}^{st}$ such that

$$b_h^{cs,0} = \sum_{a \in A_h} b_{ha}^{st} x_{ha}^{st,0} \tag{294}$$

to a reasonable numerical tolerance. Values of $x_{ha}^{st,0}$ satisfying this assumption can be computed by Equation (235).

E.39 Bounds on number of steps in switched shunt blocks

The number of steps $\overline{x}_{ha}^{st}$ in block a of switched shunt h is an integer satisfying

$$0 \leq \overline{x}_{ha}^{st} \leq 9 \tag{295}$$

F Change Log

Changes to this document after its first posting will be listed here with references to where they appear in the document.

F.1 2020-07-21. Bus voltage in prior operating point

The bus voltage in the prior operating point can be assumed to be positive, but does not necessarily satisfy voltage bounds. The corrected assumptions on bus voltage and bounds are given in Equations (242) and (243).

F.2 2020-07-21. Power balance in prior operating point

The given operating point prior to the base case can be assumed to satisfy real and reactive power balance. This property is stated in Appendix E.

F.3 2020-07-21. Revised construction of infeasibility solution

In the procedure to construct the infeasibility solution, the values of certain variables are projected onto their bounds in order to ensure feasibility. This change is made in Section D.2.

F.4 2020-08-18. Modified generator ramp rate constraints

The generator ramp rate constraints have been modified to account for start up and shut down. See Section 3.7.4.

F.5 2020-08-19. Modified sample output file

The generator real and reactive power output values in the sample output file in Figure 1 have been modified. The values originally were written in MW and Mvar, contrary to the specification that they should be in pu. Now these values are written in pu.

F.6 2020-08-19. Modified infeasibility solution

The bus voltage magnitude values in the infeasibility solution have been modified. Originally the bus voltage in both the base case and the contingency cases were obtained by projecting the bus voltage from the prior operating point onto the base case voltage bounds. Now the contingency voltage bounds are used in the contingency cases. See Equations (227) and (228).

F.7 2020-08-21. Required nonegative generator real power output in prior operating point

The real power output p_g^0 of each generator in the given operating point prior to the base case can be assumed to be nonnegative. This assumption is made explicit in Equation (245).

F.8 2020-08-21. Multiple islands not allowed in the operating points represented by the problem data

The electrical network does not have multiple islands in the prior operating point, and it remains connected after any contingency. This assumption is made explicit and precise in Equation (291)

F.9 2020-08-21. Transformer tap and switched shunt susceptance values in prior operating point are feasible

The transformer tap ratios τ_f^0 and and phase shifts θ_f^0 and the switched shunt susceptances $b_h^{cs,0}$ in the given operating point prior to the base case are feasible under the given discrete settings. This assumption is made explicit and precise in Equations (292) to (294). Also the symbols $b_h^{cs,0}$, $x_{ha}^{st,0}$, $x_f^{st,0}$ used in formulating this assumption were added to Table 9.

F.10 2020-08-21. Inconsistent notation for values of integer variables in prior operating point

Erroneous appearances of certain notations have been corrected. The incorrect notations and the corrected versions are: x_g^0 corrected to $x_g^{on,0}$, x_e^0 corrected to $x_e^{sw,0}$, and x_f^0 corrected to $x_f^{sw,0}$.

F.11 2020-08-28. Different ramp rate limits for base case and contingencies

The problem has been modified to allow for the possibility that ramp rate limits in the contingencies might differ from those in the base case. In Table 9, new parameters $\overline{p}_j^{ru,ct}$, $\overline{p}_j^{rd,ct}$, $\overline{p}_g^{ru,ct}$, and $\overline{p}_g^{rd,ct}$ have been introduced to cover the contingency cases, while the old parameters $\overline{p}_g^{ru}$, $\overline{p}_g^{rd}$, $\overline{p}_j^{ru}$, and $\overline{p}_j^{rd}$ now apply to the base case. The base case and contingency ramp rate limits are used in the constraints in Equations (42) to (45) and (87) to (90). In the data format described in Listing 1, new fields `"prumaxctg"` and `"prdmaxctg"` have been added to comunicate the new parameters. For the contingency ramp rates, we impose nonnegativity assumptions in Equations (249), (250), (253) and (254), while the base case nonnegativity assumptions are contained in Equations (247), (248), (251) and (252). And in the base case we make certain assumptions in Equations (262) and (263) in order to guarantee feasibility of ramping constraints.

F.12 2020-08-28. Changes to branch flow limit exceedance costs

The branch flow rating exceedance cost functions were modified so that the width of each block of the cost function is a specified multiple of the rating. In Table 9, a new parameter $\overline{t}_n^s$, was added, representing the multiple by which the rating of each branch is scaled to obtain the width of block n of its rating exceedance cost function. The parameter $\overline{s}_n$ then became unnecessary and was removed. The cost equations for lines and transformers were modified in Equations (20), (21), (25) and (26). In the data format described in Listing 1, a new field `"tmax"` was added to convey $\bar{t}^s_n$, and the unnecessary field `"smax"` was removed. Assumptions on the branch cost function domains were modified in Equation (273).

F.13 2020-08-28. Construction of feasible transformer tap settings in prior operating point

Previously the feasible transformer tap settings $x^{st,0}_h$ corresponding to the tap values τ^0_f and θ^0_f in the given operating point prior to the base case were described abstractly. Now an explicit construction of $x^{st,0}_f$ has been given in Equations (233) and (234), used in Section D.2, and referred to in Equations (292) and (293).

F.14 2020-08-29. Added an assumption on cost and benefit function blocks

The widths of blocks of cost and benefit functions are nonnegative. This assumption was documented in Equations (264) to (268).

F.15 2020-09-08. Construction of feasible switched shunt setting in prior operating point

Previously the feasible switched shunt settings $x^{st,0}_{ha}$ corresponding to the susceptance values $b^{cs,0}_h$ in the given operating point prior to the base case were described abstractly. Now an explicit construction of $x^{st,0}_{ha}$ has been given in Equation (235), used in Section D.2, and referred to in Equation (294).

F.16 2020-09-08. Clarification of assumptions number of steps in switched shunt blocks

The number of steps in each block of each switched shunt is bounded, which we documnent in Section E.39.

F.17 2020-09-08. Interpretation of COD1 field in transformer table of RAW file

Previously it was required that the COD1 field of each transformer record in the transformer table of the RAW file take one of the values 0, 1, or 3. Now we allow that the value of COD1 may be any integer, but any value other than 1 or 3 should be interpreted as if it were 0, indicating that the transformer has fixed tap ratio and fixed phase shift. This is effected by a change to the description of the COD1 field in Table 20.

F.18 2020-09-13. Explanation of fixed shunt terms in bus power balance constraints

Notes explaining the fixed shunt terms in the power balance constraints, Equations (35) and (38), were added to Section 3.2.2.

F.19 2020-09-13. Rearrangement of load ramp rate constraints

The load ramp rate constraints in Section 3.3.2 were rearranged to make them more directly analogous to the generator ramp rate constraints. The new constraints are Equations (42) to (45). The new constraints are algebraically equivalent to the old constraints that they are replacing, and this change does not affect the numerical value of the objective or feasibility of any solution.

F.20 2020-09-15. Terminology used in evaluation and scoring

Some terminology used in evaluation and scoring has been introduced and defined. The solution evaluation procedure uses a specific concept of feasibility, called *evaluation feasibility*, which we define in Section D.1. The objective value assigned to a solution and used as the input to the scoring method, i.e. the better of the infeasibility objective z^{inf} and the computed objective value z of the solution, is called the *scoring objective* and denoted by z^{sc}. This definition is contained in Section D.3.

F.21 2020-09-16. Base case and contingencies have separate equations prohibiting generators from starting up or shutting down if they are not qualified to do so

The equations prohibiting generators from starting up or shutting down if they are not qualified to do so have been rewritten. The new versions, in Equations (91) to (94), handle the base case separately from contingencies.

F.22 2020-10-04. Restructure section on construction of further data

In Appendix B, the section on construction of further data formerly contained a number of equations that could logically be placed in the other sections. These equations have been moved to the sections where they logically belong, and the section on construction of further data has been removed.

F.23 2020-10-13. Correction of N to N^p, N^q, N^s, N_j, N_g

In several equations in the formulation in Section 3, the index sets of the segments of certain cost functions were incorrectly given as N rather than the specific subsets N^p, N^q, N^s, N_j, N_g. These have been corrected.

F.24 2020-10-20. Added missing keywords 'REMOVE' and 'UNIT' in CON file format description

The list of keywords in the description of the CON file format in Section A.3 was missing 'REMOVE' and 'UNIT'. These have been added to the list.

F.25 2020-10-26. Revised treatment of transformer COD1 field and of impedance correction with both fixed tap and fixed phase

Previously it was unclear whether or not an impedance correction table referenced in a transformer record should be applied if the transformer had both fixed tap and fixed phase. This has been clarified, and in the process the treatment of the transformer COD1 field has been changed. The treatment of the COD1 field and impedance correction, after this update, is:

- The values of the COD1 field are limited to 0, ± 1, and ± 3. Any other value is an error.
- If COD1 $= 0$, then:
  - The transformer has fixed tap ratio. I.e. the tap ratio in the solution remains at the value it takes in the prior operating point.
  - The transformer has fixed phase shift. I.e. the phase shift in the solution remains at the value it takes in the prior operating point.
  - The impedance correction function, if there is one, is ignored.
- If COD1 $= \pm 1$, then:
  - The transformer has variable tap ratio. I.e. the tap ratio in the solution may be changed from the value it takes in the prior operating point, according to the RMA1, RMI1, and NTP1 fields.
  - The transformer has fixed phase shift. I.e. the phase shift in the solution remains at the value it takes in the prior operating point.
  - The impedance correction function, if there is one, is applied to the tap ratio.
- If COD1 $= \pm 3$, then:
  - The transformer has fixed tap ratio. I.e. the tap ratio in the solution remains at the value it takes in the prior operating point.
  - The transformer has variable phase shift. I.e. the phase shift may be changed from the value it takes in the prior operating point, according to the RMA1, RMI1, and NTP1 fields.
  - The impedance correction function, if there is one, is applied to the phase shift.

Changes have been made in this document reflecting this update in the description of the COD1 field in Section A.2.7, the construction of transformer model data in Section B.9, and the new data properties in Sections E.28 and E.30.

F.26 2020-11-24. Correction on NTP1 allowable values

Initially it was asserted that the value of the NTP1 field of transformer should be an odd integer ≥ 3. In fact NTP1 may be equal to 1. This change is made in the description of the NTP1 field in the Transformer section of the RAW input data file in Table 20. Then, Equations (171) and (175) were modified to handle the case NTP1 $= 1$. At the same time, a copy-paste error was fixed in Equations (172) and (176).

F.27 2020-11-24. Typo in description of transformer impedance correction

The text "or θ_{fk} (for $f \in F^{\tau}$)" in Section 3.6.4 should have had F^{θ} instead of F^{τ}. This has been fixed.