Grid Optimization Competition
Challenge 3 Problem Formulation

Jesse Holzer Carleton Coffrin Christopher DeMarco Ray Duthu
Stephen Elbert Brent Eldridge Tarek Elgindy Scott Greene
Nongchao Guo Elaine Hale Bernard Lesieutre Terrence Mak
Colin McMillan Hans Mittelmann Richard O'Neill
Thomas Overbye Bryan Palmintier Farnaz Safdarian
Ahmad Tbaileh Pascal Van Hentenryck Arun Veeramany
Jessica Wert

February 16, 2022

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1 Formulation notation reference

This section gives a complete reference to the symbols used in the model formulation, in tabular form.

Units of measurement are given in Table 1, symbol main letters in Table 2, symbol superscripts in Table 3, indices and index sets in Table 4, subsets in Table 5, special set elements in Table 6, real-valued parameters in Table 7, and variables in Table 8.

Table 1: Units of measurement

Symbol	Description
binary	Binary quantities, i.e. those taking values in $\{0, 1\}$.
dimensionless	Dimensionless real number quantities.
h	hour. Time quantities are expressed in h.
integer	Integer quantities, i.e. those taking values in $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

Table 1: Continued

Symbol	Description
pu	per unit. Certain physical quantities, including voltage magnitude, real power, reactive power, apparent power, resistance, reactance, conductance, and susceptance, are expressed in a per unit convention with a specified or implied base value, indicated by a unit of pu.
rad	radian. Voltage angles and differences are expressed in rad.
$	US dollar. Cost, benefit, penalty, and objective values are expressed in $.

Table 2: Main letters

Symbol	Description
I	Set of buses
J	Set of devices
K	Set of contingencies
M	Set of cost blocks, i.e. constant marginal cost blocks of convex piecewise linear cost functions
N	Set of reserve zones
T	Set of time steps
W	Set of miscellaneous constraints
Y	Set of configurations of devices consisting of multiple units or modes of operation, e.g. combined cycle, pumped storage, battery storage
b	Susceptance
c	Cost coefficient
d	Duration of time
e	Energy, e.g. stored or total produced
g	Conductance
i	Bus
j	Device
k	Contingency
m	Cost or benefit block
n	Reserve zone
p	Real power
q	Reactive power
r	Resistance
s	Apparent power
t	Index of time points and intervals

Table 2: Continued

Symbol	Description
u	Integer variables
v	Voltage magnitude
w	Index on miscellaneous constraints
x	Reactance
y	Configuration
z	Cost, benefit, penalty, or objective
α	Participation factor, as in a distributed slack
β	Sensitivity of reactive power to real power
θ	Voltage angle
σ	Reserve requirement coefficient
τ	Winding ratio of a transformer
ϕ	Phase difference of a transformer

Table 3: Superscripts

Symbol	Description
ac	Alternating current (AC) branch
br	Branch
md	Mode, operating mode, a generalization of combined cycle configuration
nsp	Non-spinning reserve
ctg	Contingency
ch	Charging susceptance in an AC branch
cs	Consuming
dc	Direct current (DC) line
dn	down, as in ramping down or reserve down
ds	dispatchable
en	energy, convex or concave energy cost or benefit function
fr	from, side of a branch
max	maximum
min	minimum
ms	market surplus
on	online
out	out of service

Table 3: Continued

Symbol	Description
p	real power
pqe	equality constraints linking real and reactive power
pqmax	inequality constraints modeling upper bounds on reactive power depending on real power
pqmin	inequality constraints modeling lower bounds on reactive power depending on real power
pr	producing
p0	Indicates a value taken by a quantity depending on real power when real power is 0
q	reactive power
qrd	Reactive power reserve down
qru	Reactive power reserve up
rgd	Regulation down
rgu	Regulation up
rrd	ramping reserve reserve down
rru	ramping reserve reserve up
spr	Spinning reserve
ru	ramp up
rd	ramp down
s	apparent power
sr	series element in AC branch model
sd	shutdown, transition from online to offline or from closed to open
su	startup, transition from offline to online or from open to closed
to	to, side of a branch
tr	Transition between modes
up	up, as in ramping up or reserve up
v	voltage (magnitude)
0	initial value, prevailing in some period immediately prior to the model horizon
$+$	slack variable on an inequality constraints, or largest slack among a set of inequalities
δ	incremental change, as in a linear approximation of a nonlinear equation

Table 4: Index sets

Symbol	Description
$i \in I$	Buses.
$j \in J$	Bus-connected grid devices, e.g. loads, generators, lines.
$k \in K$	Contingencies.
$m \in M$	Offer or bid blocks of piecewise linear convex cost or benefit functions.
$n \in N$	Reserve zones.
$t \in T$	Time intervals.
$w \in W$	Miscellaneous constraints.
$y \in Y$	Modes.
$(y, y') \in Y \times Y$	Mode transitions.

Table 5: Subsets

Symbol	Description
$I_n \subset I$	Buses contained in reserve zone n
$J_i \subset J$	Devices connected to bus i
$J_t \subset J$	Devices in service in time interval t
$J_{tk} \subset J$	Devices that are in service in time interval t, contingency k
$J^{\text{ac}} \subset J$	AC branches
$J^{\text{ac}}_{tk} \subset J$	AC branches in service in time interval t, contingency k
$J^{\text{br}} \subset J$	branches, i.e. devices connecting to two buses
$J^{\text{br}}_{tk} \subset J$	branches in service in time interval t, contingency k
$J^{\text{cs}} \subset J$	Consuming devices (e.g. loads)
$J^{\text{dc}} \subset J$	DC lines
$J^{\text{dc}}_{tk} \subset J$	DC lines in service in time interval t, contingency k
$J^{\text{ds}} \subset J$	dispatchable non-branch devices
$J^{\text{ds}}_{i} \subset J$	dispatchable non-branch devices connected to bus i
$J^{\text{ds}}_{n} \subset J$	Dispatchable devices contained in reserve zone n
$J^{\text{ds}}_{tk} \subset J$	Dispatchable devices in service in time interval t, contingency k
$J^{\text{fr}}_{i} \subset J$	Branches with from bus equal to bus i
$J^{\text{md}} \subset J$	Multi-mode devices
$J^{\text{out}}_{t} \subset J$	Devices out of service in time interval t
$J^{\text{out}}_{tk} \subset J$	Devices that are out of service in time interval t, contingency k

Table 5: Continued

Symbol	Description
$J^{\mathrm{pqe}} \subset J$	Dispatchable devices having linear equality constraints linking real and reactive power
$J^{\mathrm{pqmax}} \subset J$	Dispatchable devices having linear inequality constraints modeling upper bounds on reactive power depending on real power
$J^{\mathrm{pqmin}} \subset J$	Dispatchable devices having linear inequality constraints modeling lower bounds on reactive power depending on real power
$J^{\mathrm{pr}} \subset J$	Producing devices (e.g. generators)
$J^{\mathrm{pr}}_n \subset J$	Producing devices contained in reserve zone n
$J^{\mathrm{sh}} \subset J$	shunts
$J^{\mathrm{sub}}_j \subset J$	Sub-devices of multi-mode device j
$J^{\mathrm{sub,on}}_{jy} \subset J$	Sub-devices of multi-mode device j that are online when device j is in mode y
$J^{\mathrm{to}}_i \subset J$	branches with to bus equal to bus i
$M_{jt} \subset M$	Energy cost or benefit function offer or bid blocks for producing or consuming device j in interval t
$M_{jyt} \subset M$	Energy cost or benefit function offer or bid blocks for mode y of multi-mode producing or consuming device j in interval t
$N^{\mathrm{p}} \subset N$	Reserve zones for products associated with real power
$N^{\mathrm{q}} \subset N$	Reserve zones for products associated with reactive power
$W^{\mathrm{en,max}}_j \subset W$	Multi-interval maximum energy constraints for device j
$W^{\mathrm{en,min}}_j \subset W$	Multi-interval minimum energy constraints for device j
$W^{\mathrm{su,max}}_j \subset W$	Multi-interval maximum startups constraints for device j
$Y_j \subset Y$	Modes for multi-mode device j
$Y^{\mathrm{sub,on}}_{jj'} \subset Y$	Modes of multi-mode device j with sub-device j' in which device j' is online
$Y^{\mathrm{tr}}_j \subset Y \times Y$	Mode transitions for multi-mode device j

Table 6: Special set elements

Symbol	Description
$i_j \in I$	Connection bus of non-branch device $j \in J \setminus J^{\mathrm{br}}$
$i^{\mathrm{fr}}_j \in I$	From bus of branch $j \in J^{\mathrm{br}}$.

Table 6: Continued

Symbol	Description
$i_j^{\mathrm{to}} \in I$	To bus of branch $j \in J^{\mathrm{br}}$.
$n_i \in N$	Reserve zone of bus i
$t^{\max} \in T$	Last time interval t.
$t_w^{\max} \in T$	Latest time index incident to multi-interval constraint w
$t^{\min} \in T$	First time interval t.
$t_j^{\mathrm{min,dn}} \in T$	Earliest time index t such that a constraint is needed to enforce minimum downtime on device j if starting up in interval t
$t_j^{\mathrm{min,up}} \in T$	Earliest time index t such that a constraint is needed to enforce minimum uptime on device j if shutting down in interval t
$t_{jtf}^{\mathrm{min,sd,su}} \in T$	Earliest time index t' such that shutting down in interval t' prevents device j from starting up in interval t
$t_{jtf}^{\mathrm{min,su,sd}} \in T$	Earliest time index t' such that starting up in interval t' prevents device j from shutting down in interval t
$t_w^{\min} \in T$	Earliest time index incident to multi-interval constraint w
$t_{jy}^{\mathrm{min,md,off}} \in T$	Earliest time index t such that device j mode y maximum dwell time must be enforced by a constraint requiring that if mode y is selected in interval t then it is not selected in at least one interval in a prescribed prior duration
$t_{jyt}^{\mathrm{min,md,off}} \in T$	Earliest time index t' such that if device j is in mode y in interval t, then maximum dwell time requires that mode y is not selected in at least one interval from t' to t
$t_{jy}^{\mathrm{min,md,on}} \in T$	Earliest time index t such that device j mode y minimum dwell time must be enforced by a constraint requiring that if the device transitions from mode y in interval t then mode y is selected in all intervals in a prescibed prior duration
$t_{jyt}^{\mathrm{min,md,on}} \in T$	Earliest time index t' such that if device j transitions from mode y in interval t, then minimum dwell time requires that mode y is selected in all intervals from t' through $t-1$

Table 7: Parameters

Symbol	Description
b_j^{ch}	Charging susceptance of AC branch j (pu)

Table 7: Continued

Symbol	Description
b_j^{fr}	Shunt susceptance to ground of AC branch j at from bus (pu)
b_j^{sh}	Susceptance of one step of shunt j (pu)
b_j^{sr}	Series susceptance of AC branch j (pu)
b_j^{to}	Shunt susceptance to ground of AC branch j at to bus (pu)
c_{jtm}^{en}	Energy marginal cost or benefit of offer or bid block m of producing or consuming device j in interval t (\$/pu-h)
c_{jytm}^{en}	Energy marginal cost or benefit of offer or bid block m of multi-mode producing or consuming device j in configuration y in interval t (\$/pu-h)
c_{jyt}^{md}	Mode fixed cost of device j in mode y in interval t (\$/h)
c^{p}	Real power bus imbalance marginal cost (\$/pu-h)
c^{q}	Reactive power bus imbalance marginal cost (\$/pu-h)
c_{jt}^{rgd}	Marginal cost of regulation down provided by device j in interval t (\$/pu-h)
c_{jt}^{rgu}	Marginal cost of regulation up provided by device j in interval t (\$/pu-h)
c_{jt}^{spr}	Marginal cost of spinning reserve provided by device j in interval t and not counting as regulation up (\$/pu-h)
c_{jt}^{nsp}	Marginal cost of non-spinning reserve provided by device j in interval t (\$/pu-h)
$c_{jt}^{\mathrm{rru,off}}$	Marginal cost of ramping reserve up provided by device j in interval t when offline (\$/pu-h)
$c_{jt}^{\mathrm{rru,on}}$	Marginal cost of ramping reserve up provided by device j in interval t when online (\$/pu-h)
$c_{jt}^{\mathrm{rrd,off}}$	Marginal cost of ramping reserve down provided by device j in interval t when offline (\$/pu-h)
$c_{jt}^{\mathrm{rrd,on}}$	Marginal cost of ramping reserve down provided by device j in interval t when online (\$/pu-h)
c_{jt}^{qru}	Marginal cost of reactive power up reserve provided by device j in interval t (\$/pu-h)
c_{jt}^{qrd}	Marginal cost of reactive power down reserve provided by device j in interval t (\$/pu-h)
c_n^{rgu}	Incremental cost of shortfall of regulation up in zone n (\$/pu-h)
c_n^{rgd}	Incremental cost of shortfall of regulation down in zone n (\$/pu-h)
c_n^{spr}	Incremental cost of shortfall of spinning reserve in zone n (\$/pu-h)
c_n^{nsp}	Incremental cost of shortfall of non-spinning reserve in zone n (\$/pu-h)

Table 7: Continued

Symbol	Description
c_n^{rru}	Incremental cost of shortfall of ramping reserve up in zone n (\$/pu-h)
c_n^{rrd}	Incremental cost of shortfall of ramping reserve down in zone n (\$/pu-h)
c_n^{qru}	Incremental cost of shortfall of reactive power reserve up in zone n (\$/pu-h)
c_n^{qrd}	Incremental cost of shortfall of reactive power reserve down in zone n (\$/pu-h)
c^{s}	Branch overload marginal cost (\$/pu-h)
$c_{jyy't}^{\mathrm{tr}}$	Mode transition cost of device j from mode y to mode y' in interval t (\$)
c_j^{on}	Fixed cost of online status of device j (\$/h)
c_j^{su}	Startup cost of device j (\$)
c_j^{sd}	Shutdown cost of device j (\$)
d_t	Duration of interval t (h)
$d_j^{\mathrm{dn,min}}$	Minimum downtime of device j (h)
$d_j^{\mathrm{up,min}}$	Minimum uptime of device j (h)
e_w^{max}	Maximum energy for multi-interval maximum energy constraint w (pu-h)
e_w^{min}	Minimum energy for multi-interval minimum energy constraint w (pu-h)
g_j^{fr}	Shunt conductance to ground of AC branch j at from bus (pu)
g_j^{sr}	Series conductance of AC branch j (pu)
g_j^{sh}	Conductance of one step of shunt j (pu)
g_j^{to}	Shunt conductance to ground of AC branch j at to bus (pu)
$p_j^{\mathrm{dc,max}}$	Maximum real power flow of DC line j (pu)
p_{jtm}^{max}	Maximum real power of offer or bid block m of producing or consuming device j in interval t (pu)
p_{jt}^{max}	Maximum real power of dispatchable device j when online in interval t (pu)
p_{jyt}^{max}	Maximum real power of multi-mode dispatchable device j in configuration y in interval t (pu)
p_{jytm}^{max}	Maximum real power of offer or bid block m of multi-mode producing or consuming device j in configuration y in interval t (pu)
p_{jt}^{min}	Minimum real power of dispatchable device j when online in interval t (pu)
p_{jyt}^{min}	Minimum real power of multi-mode dispatchable device j in configuration y in interval t (pu)
p_{jyt}^{rd}	Maximum ramp down rate of multi-mode dispatchable device j in mode y in interval t (pu/h)

Table 7: Continued

Symbol	Description
p_{jt}^{rd}	Maximum ramp down rate of dispatchable device j when online in interval t (pu/h)
$p_{jt}^{\mathrm{rd,sd}}$	Maximum ramp down rate of dispatchable device j when shutting down in interval t (pu/h)
p_{jyt}^{ru}	Maximum ramp up rate of multi-mode dispatchable device j in mode y in interval t (pu/h)
p_{jt}^{ru}	Maximum ramp up rate of dispatchable device j when online in interval t (pu/h)
$p_{jt}^{\mathrm{ru,su}}$	Maximum ramp up rate of dispatchable device j when starting up in interval t (pu/h)
p_{j}^{0}	Initial real power of dispatchable device j (pu)
$p_{jt}^{\mathrm{rgu,max}}$	Maximum regulation up for dispatchable device j in interval t (pu)
$p_{jt}^{\mathrm{rgd,max}}$	Maximum regulation down for dispatchable device j in interval t (pu)
$p_{jt}^{\mathrm{spr,max}}$	Maximum spinning reserve for dispatchable device j in interval t (pu)
$p_{jt}^{\mathrm{nsp,max}}$	Maximum non-spinning reserve for dispatchable device j in interval t (pu)
$p_{jt}^{\mathrm{rru,on,max}}$	Maximum ramping reserve up for dispatchable device j when online in interval t (pu)
$p_{jt}^{\mathrm{rru,off,max}}$	Maximum ramping reserve up for dispatchable device j when offline in interval t (pu)
$p_{jt}^{\mathrm{rrd,on,max}}$	Maximum ramping reserve down for dispatchable device j when online in interval t (pu)
$p_{jt}^{\mathrm{rrd,off,max}}$	Maximum ramping reserve down for dispatchable device j when offline in interval t (pu)
$p_{nt}^{\mathrm{rru,min}}$	Exogenous ramping reserve up requirement for zone n in interval t (pu)
$p_{nt}^{\mathrm{rrd,min}}$	Exogenous ramping reserve down requirement for zone n in interval t (pu)
q_{jt}^{p0}	Reactive power at 0 real power of dispatchable device j if the device has reactive power dependent on real power (pu)
$q_{jt}^{\mathrm{max,p0}}$	Upper bound on reactive power at 0 real power of dispatchable device j in interval t if the device has maximum reactive power dependent on real power (pu)
q_{jyt}^{max}	Maximum reactive power of multi-mode device j in mode y in interval t (pu)
$q_{jt}^{\mathrm{min,p0}}$	Lower bound on reactive power at 0 real power of dispatchable device j in interval t if the device has minimum reactive power dependent on real power (pu)

Table 7: Continued

Symbol	Description
$q_{jyt}^{\min}$	Minimum reactive power of multi-mode device j in mode y in interval t (pu)
$q_{jt}^{\max}$	Maximum reactive power of dispatchable device j when online in interval t (pu)
$q_{jyt}^{\max}$	Maximum reactive power of multi-mode device j in mode y in interval t (pu)
$q_{jt}^{\min}$	Minimum reactive power of dispatchable device j when online in interval t (pu)
$q_{jyt}^{\min}$	Minimum reactive power of multi-mode device j in mode y in interval t (pu)
$q_{j}^{\mathrm{dc,fr,max}}$	Maximum reactive power at from bus of DC line j (pu)
$q_{j}^{\mathrm{dc,to,max}}$	Maximum reactive power at to bus of DC line j (pu)
$q_{j}^{\mathrm{dc,fr,min}}$	Minimum reactive power at from bus of DC line j (pu)
$q_{j}^{\mathrm{dc,to,min}}$	Minimum reactive power at to bus of DC line j (pu)
$q_{nt}^{\mathrm{qru,min}}$	Exogenous reactive power reserve up requirement for zone n in interval t (pu)
$q_{nt}^{\mathrm{qrd,min}}$	Exogenous reactive power reserve down requirement for zone n in interval t (pu)
r_{j}^{sr}	Series resistance of AC branch j (pu)
$s_{j}^{\max}$	Maximum apparent power flow of branch j (pu)
$s_{j}^{\mathrm{max,ctg}}$	Maximum apparent power flow of branch j in contingencies (pu)
$u_{jyt}^{\mathrm{md,max}}$	Upper bound on mode selection for multi-mode device j mode y in interval t (binary)
$u_{jyy't}^{\mathrm{tr,max}}$	Upper bound on mode transition for multi-mode device j transition (yy') in interval t (binary)
$u_{jyt}^{\mathrm{md,min}}$	Lower bound on mode selection for multi-mode device j mode y in interval t (binary)
$u_{jy}^{\mathrm{md},0}$	Initial mode selection for multi-mode device j mode y (binary)
$u_{jyy't}^{\mathrm{tr,min}}$	Lower bound on mode transition for multi-mode device j transition (yy') in interval t (binary)
$u_{jt}^{\mathrm{on,max}}$	Upper bound on online status indicator for device j in interval t (binary)
$u_{jt}^{\mathrm{on,min}}$	Lower bound on online status indicator for device j in interval t (binary)
$u_{j}^{\mathrm{on},0}$	Initial on-off status of device j (binary)
$u_{w}^{\mathrm{su,max}}$	Maximum startups for multi-interval maximum startups constraint w (integer)

Table 7: Continued

Symbol	Description
$u_j^{\text{sh,max}}$	Maximum number of activated steps of shunt j (integer)
$u_j^{\text{sh,min}}$	Minimum number of activated steps of shunt j (integer)
$v_i^{\max}$	Maximum voltage magnitude at bus i (pu)
$v_i^{\min}$	Minimum voltage magnitude at bus i (pu)
x_j^{sr}	Series reactance of AC branch j (pu)
α_i	Participation factor of bus i in resolving system real power imbalance in contingencies (dimensionless)
β_{jt}	Sensitivity of reactive power with respect to real power of dispatchable device j with equality constraint linking real and reactive power in interval t (pu/pu)
$\beta_{jt}^{\max}$	Sensitivity of upper bound on reactive power with respect to real power of dispatchable device j with inequality constraint modeling upper bounds on reactive power depending on real power in interval t (pu/pu)
$\beta_{jt}^{\min}$	Sensitivity of lower bound on reactive power with respect to real power of dispatchable device j with inequality constraint modeling upper bounds on reactive power depending on real power in interval t (pu/pu)
σ_{nt}^{rgd}	Fraction of total cleared power consumption in zone n forming regulation down requirement for zone n interval t (dimensionless)
σ_{nt}^{rgu}	Fraction of total cleared power consumption in zone n forming regulation up requirement for zone n interval t (dimensionless)
σ_{nt}^{spr}	Fraction of largest cleared power production in zone n forming spinning reserve requirement for zone n interval t (dimensionless)
σ_{nt}^{nsp}	Fraction of largest cleared power production in zone n forming non-spinning reserve requirement for zone n interval t (dimensionless)
$\tau_j^{\max}$	Maximum winding ratio of AC branch j (dimensionless)
$\tau_j^{\min}$	Minimum winding ratio of AC branch j (dimensionless)
$\phi_j^{\max}$	Maximum phase difference of variable phase difference transformer j (rad)
$\phi_j^{\min}$	Minimum phase difference of variable phase difference transformer j (rad)

Table 8: Variables

Symbol	Description
b^{sh}_{jt}	Susceptance of shunt j in interval t. (pu)
g^{sh}_{jt}	Conductance of shunt j in interval t. (pu)
p_{it}	Real power signed mismatch, i.e. load and other real power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t. (pu)
p^{+}_{it}	Real power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t. (pu)
p_{jt}	Real power of non-branch device j in interval t. Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)
p^{δ}_{jtk}	Approximate change from base case value to post-contingency value of the real power flow on branch j in interval t in contingency k, directed from the from bus to the to bus. (pu)
p_{jtm}	Real power of dispatchable device j in interval t in energy cost or benefit block m. (pu)
p_{jytm}	Real power of multi-mode device j in energy cost or benefit block m in mode y in interval t (pu)
p^{fr}_{jt}	Real power at from bus of branch j in interval t. Oriented from the bus into the device. (pu)
p^{rgu}_{jt}	Regulation up provided by device j in interval t
p^{rgd}_{jt}	Regulation down provided by device j in interval t
p^{spr}_{jt}	Spinning reserve provided by device j in interval t
p^{nsp}_{jt}	Non-spinning reserve provided by device j in interval t
$p^{\text{rru,on}}_{jt}$	ramping reserve up provided by device j in interval t when online
$p^{\text{rru,off}}_{jt}$	ramping reserve up provided by device j in interval t when offline
$p^{\text{rrd,on}}_{jt}$	ramping reserve down provided by device j in interval t when online
$p^{\text{rrd,off}}_{jt}$	ramping reserve down provided by device j in interval t when offline
$p^{\text{rgu,req}}_{nt}$	Regulation up requirement for reserve zone n in interval t
$p^{\text{rgd,req}}_{nt}$	Regulation down requirement for reserve zone n in interval t
$p^{\text{spr,req}}_{nt}$	Spinning reserve requirement for reserve zone n in interval t
$p^{\text{nsp,req}}_{nt}$	Non-spinning reserve requirement for reserve zone n in interval t
$p^{\text{rru,req}}_{nt}$	ramping reserve up requirement for reserve zone n in interval t
$p^{\text{rrd,req}}_{nt}$	ramping reserve down requirement for reserve zone n in interval t

Table 8: Continued

Symbol	Description
$p_{nt}^{\text{rgu},+}$	Regulation up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rgd},+}$	Regulation down shortfall for reserve zone n in interval t
$p_{nt}^{\text{spr},+}$	Spinning reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{nsp},+}$	Non-spinning reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{rru},+}$	ramping reserve up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rrd},+}$	ramping reserve down shortfall for reserve zone n in interval t
p_{jt}^{to}	Real power at to bus of branch j in interval t. Oriented from the bus into the device. (pu)
q_{it}	Reactive power signed mismatch, i.e. load and other reactive power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t. (pu)
q_{it}^{+}	Reactive power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t. (pu)
q_{jt}	Reactive power of non-branch device j in interval t. Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)
q_{jt}^{fr}	Reactive power at from bus of branch j in interval t. Oriented from the bus into the device. (pu)
q_{jt}^{to}	Reactive power at to bus of branch j in interval t. Oriented from the bus into the device. (pu)
q_{jt}^{qru}	Reactive power up reserve provided by device j in interval t
q_{jt}^{qrd}	Reactive power down reserve provided by device j in interval t
$q_{nt}^{\text{qru,req}}$	Reactive power up reserve requirement for reserve zone n in interval t
$q_{nt}^{\text{qrd,req}}$	Reactive power down reserve requirement for reserve zone n in interval t
$q_{nt}^{\text{qru},+}$	Reactive power up reserve shortfall for reserve zone n in interval t
$q_{nt}^{\text{qrd},+}$	Reactive power down reserve shortfall for reserve zone n in interval t
s_{jt}^{+}	Branch j apparent power overload in interval t. (pu)
s_{jtk}^{+}	Branch j (approximate) apparent overload in interval t contingency k. (pu)
u_{jyt}^{md}	Mode selection indicator, equal to 1 if multi-mode device j is in mode y in interval t. (binary)
$u_{jyy't}^{\text{tr}}$	Mode transition indicator, equal to 1 if multi-mode device j transitions from mode y to mode y' in interval t. (binary)

Table 8: Continued

Symbol	Description
u_{jt}^{on}	On indicator of device j in interval t. The value 1 indicates the device is online or closed, 0 else. (binary)
u_{jt}^{sd}	Shutdown indicator of device j in interval t. The value 1 indicates the device is shutting down, transitioning from online to offline or from closed to open, 0 else. (binary)
u_{jt}^{sh}	Number of steps activated for shunt j in interval t. (integer)
u_{jt}^{su}	Startup indicator of device j in interval t. The value 1 indicates the device is starting up, transitioning from offline to online or from open to closed, 0 else. (binary)
v_{it}	Voltage magnitude at bus i in interval t. (pu)
$z_{jt}^{\text{md,on}}$	Mode fixed cost incurred by multi-mode device j in interval t (\$)
z^{ms}	Total maximization objective, i.e. base case plus contingencies. Net market surplus, including benefits minus costs and penalties. (\$)
z_{t}^{ms}	Base case market surplus objective in interval t. (\$)
z_{tk}^{ms}	Contingency market surplus objective in interval t contingency k. (\$)
z_{jt}^{en}	Energy cost or benefit of producing or consuming device j in interval t. (\$)
z_{jt}^{on}	Online status cost of device j in interval t. (\$)
z_{it}^{p}	Real power mismatch cost of bus i in interval t. (\$)
z_{it}^{q}	Reactive power mismatch cost of bus i in interval t. (\$)
z_{jt}^{sd}	Shutdown cost of device j in interval t. (\$)
z_{jt}^{su}	Startup cost of device j in interval t. (\$)
z_{jt}^{s}	Branch overload cost of branch j in interval t. (\$)
z_{jtk}^{s}	Branch overload cost of branch j in interval t. (\$)
z_{jt}^{tr}	Mode transition cost incurred by configuration device j in interval t (\$)
z_{jt}^{rgu}	Cost of Regulation up provided by device j in interval t (\$)
z_{jt}^{rgd}	Cost of Regulation down provided by device j in interval t (\$)
z_{jt}^{spr}	Cost of Spinning reserve provided by device j in interval t and not counting as regulation up (\$)
z_{jt}^{nsp}	Cost of Non-spinning reserve provided by device j in interval t (\$)
z_{jt}^{rru}	Cost of ramping reserve up provided by device j in interval t (\$)
z_{jt}^{rrd}	Cost of ramping reserve down provided by device j in interval t (\$)
z_{jt}^{qru}	Cost of Reactive power up reserve provided by device j in interval t (\$)

Table 8: Continued

Symbol	Description
z^{qrd}_{jt}	Cost of Reactive power down reserve provided by device j in interval t (\$)
z^{rgu}_{nt}	Regulation up shortfall penalty for reserve zone n in interval t (\$)
z^{rgd}_{nt}	Regulation down shortfall penalty for reserve zone n in interval t (\$)
z^{spr}_{nt}	Spinning reserve shortfall penalty for reserve zone n in interval t (\$)
z^{nsp}_{nt}	Non-spinning reserve shortfall penalty for reserve zone n in interval t (\$)
z^{rru}_{nt}	ramping reserve up shortfall penalty for reserve zone n in interval t (\$)
z^{rrd}_{nt}	ramping reserve down shortfall penalty for reserve zone n in interval t (\$)
z^{qru}_{nt}	Reactive power up reserve shortfall penalty for reserve zone n in interval t (\$)
z^{qrd}_{nt}	Reactive power down reserve shortfall penalty for reserve zone n in interval t (\$)
θ_{it}	Voltage angle at bus i in interval t. (rad)
θ^{δ}_{itk}	Approximate change from base case value to post-contingency value of the voltage angle at bus i in interval t in contingency k. (rad)
τ_{jt}	Winding ratio of AC branch j in interval t. (dimensionless)
ϕ_{jt}	Phase difference of AC branch j in interval t. (rad)

2 Formulation constraints

2.1 Market surplus objective

Total market surplus objective for maximization. Includes benefits minus costs and penalties, base case and contingencies.

$$z^{\mathrm{ms}} = \sum_{t \in T} \left(z^{\mathrm{ms}}_{t} + \min_{k \in K} z^{\mathrm{ms}}_{tk} + 1/|K| \sum_{k \in K} z^{\mathrm{ms}}_{tk} \right) \tag{1}$$

Market surplus, i.e. in the base case. When a distinction between base case and contingencies is not specified, base case is implied, except if explicitly indicated otherwise.

$$\begin{aligned} z^{\mathrm{ms}}_{t} = & \sum_{j \in J^{\mathrm{cs}}} z^{\mathrm{en}}_{jt} - \sum_{j \in J^{\mathrm{pr}}} z^{\mathrm{en}}_{jt} - \sum_{j \in J} (z^{\mathrm{on}}_{jt} + z^{\mathrm{su}}_{jt} + z^{\mathrm{sd}}_{jt}) - \sum_{j \in J^{\mathrm{md}}} (z^{\mathrm{md}}_{jt} + z^{\mathrm{tr}}_{jt}) - \sum_{j \in J^{\mathrm{br}}} z^{\mathrm{s}}_{jt} \\ & - \sum_{j \in J^{\mathrm{ds}}} (z^{\mathrm{rgu}}_{jt} + z^{\mathrm{rgd}}_{jt} + z^{\mathrm{spr}}_{jt} + z^{\mathrm{nsp}}_{jt} + z^{\mathrm{rru}}_{jt} + z^{\mathrm{rrd}}_{jt} + z^{\mathrm{qru}}_{jt} + z^{\mathrm{qrd}}_{jt}) - \sum_{i \in I} (z^{\mathrm{p}}_{it} + z^{\mathrm{q}}_{it}) \\ & - \sum_{n \in N^{\mathrm{p}}} (z^{\mathrm{rgu}}_{nt} + z^{\mathrm{rgd}}_{nt} + z^{\mathrm{spr}}_{nt} + z^{\mathrm{nsp}}_{nt} + z^{\mathrm{rru}}_{nt} + z^{\mathrm{rrd}}_{nt}) - \sum_{n \in N^{\mathrm{q}}} (z^{\mathrm{qru}}_{nt} z^{\mathrm{qrd}}_{nt}) \ \forall t \in T \end{aligned} \tag{2}$$

Post-contingency market surplus includes branch limit overload costs.

$$z^{\mathrm{ms}}_{tk} = -\sum_{j \in J^{\mathrm{br}}} z^{\mathrm{s}}_{jtk} \ \forall t \in T, k \in K \tag{3}$$

2.2 Bus power mismatch and penalized mismatch definitions

For each bus i and each interval t, the penalized real and reactive power mismatches p^{+}_{it} and q^{+}_{it} are the absolute values of the mismatches p_{it} and q_{it}:

$$p^{+}_{it} \geq p_{it} \ \forall t \in T, i \in I \tag{4}$$
$$p^{+}_{it} \geq -p_{it} \ \forall t \in T, i \in I \tag{5}$$
$$q^{+}_{it} \geq q_{it} \ \forall t \in T, i \in I \tag{6}$$
$$q^{+}_{it} \geq -q_{it} \ \forall t \in T, i \in I \tag{7}$$

2.3 Bus power mismatch penalty

For each bus i and interval t the bus real and reactive power mismatch penalties z^{p}_{it} and z^{q}_{it} are given by linear penalty functions of the penalized mismatches:

$$z^{\mathrm{p}}_{it} = d_t c^{\mathrm{p}} p^{+}_{it} \ \forall t \in T, i \in I \tag{8}$$
$$z^{\mathrm{q}}_{it} = d_t c^{\mathrm{q}} q^{+}_{it} \ \forall t \in T, i \in I \tag{9}$$

2.4 Real and reactive power balance

For each bus i and each interval t, real and reactive power balance are enforced by:

$$\sum_{j \in J^{\mathrm{cs}}_i} p_{jt} + \sum_{j \in J^{\mathrm{sh}}_i} p_{jt} + \sum_{j \in J^{\mathrm{fr}}_i} p^{\mathrm{fr}}_{jt} + \sum_{j \in J^{\mathrm{to}}_i} p^{\mathrm{to}}_{jt} = \sum_{j \in J^{\mathrm{pr}}_i} p_{jt} + p_{it} \ \forall t \in T, i \in I \tag{10}$$
$$\sum_{j \in J^{\mathrm{cs}}_i} q_{jt} + \sum_{j \in J^{\mathrm{sh}}_i} q_{jt} + \sum_{j \in J^{\mathrm{fr}}_i} q^{\mathrm{fr}}_{jt} + \sum_{j \in J^{\mathrm{to}}_i} q^{\mathrm{to}}_{jt} = \sum_{j \in J^{\mathrm{pr}}_i} q_{jt} + q_{it} \ \forall t \in T, i \in I \tag{11}$$

In Eqs. (10) and (11) power withdrawals at bus i, including consuming devices, shunts, and branches, appear on the left, while injections, including producing devices and power mismatch, appear on the right.

2.5 Reserve shortfall domains

For each reserve product, in each reserve zone and time interval, a nonnegative variable represents the shortfall of procured reserve relative to the reserve requirement:

$$p^{\text{rgu},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{12}$$

$$p^{\text{rgd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{13}$$

$$p^{\text{spr},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{14}$$

$$p^{\text{nsp},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{15}$$

$$p^{\text{rru},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{16}$$

$$p^{\text{rrd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{17}$$

$$p^{\text{qru},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{q}} \tag{18}$$

$$p^{\text{qrd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{q}} \tag{19}$$

2.6 Reserve shortfall penalties

Reserve shortfall penalties are given by a linear penalty function of the reserve shortfall:

$$z^{\text{rgu}}_{nt} = d_t c^{\text{rgu}}_n p^{\text{rgu},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{20}$$

$$z^{\text{rgd}}_{nt} = d_t c^{\text{rgd}}_n p^{\text{rgd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{21}$$

$$z^{\text{spr}}_{nt} = d_t c^{\text{spr}}_n p^{\text{spr},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{22}$$

$$z^{\text{nsp}}_{nt} = d_t c^{\text{nsp}}_n p^{\text{nsp},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{23}$$

$$z^{\text{rru}}_{nt} = d_t c^{\text{rru}}_n p^{\text{rru},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{24}$$

$$z^{\text{rrd}}_{nt} = d_t c^{\text{rrd}}_n p^{\text{rrd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{p}} \tag{25}$$

$$z^{\text{qru}}_{nt} = d_t c^{\text{qru}}_n p^{\text{qru},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{q}} \tag{26}$$

$$z^{\text{qrd}}_{nt} = d_t c^{\text{qrd}}_n p^{\text{qrd},+}_{nt} \geq 0 \ \forall t \in T, n \in N^{\text{q}} \tag{27}$$

2.7 Reserve requirements

For each reserve product, in each zone and each interval, the reserve requirement is given by a variable that is constrained from below by either an exogenous bound given by data or an endogenous bound formed in a prescribed fashion from the power dispatch:

$$p^{\text{rgu,req}}_{nt} \geq \sigma^{\text{rgu}}_{nt} \sum_{j \in J^{\text{cs}}_n} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{28}$$

$$p^{\text{rgd,req}}_{nt} \geq \sigma^{\text{rgd}}_{nt} \sum_{j \in J^{\text{cs}}_n} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{29}$$

$$p^{\text{spr,req}}_{nt} \geq \sigma^{\text{spr}}_{nt} p_{jt} \ \forall t \in T, n \in N^{\text{p}}, j \in J^{\text{pr}}_n \tag{30}$$

$$p^{\text{nsp,req}}_{nt} \geq \sigma^{\text{nsp}}_{nt} p_{jt} \ \forall t \in T, n \in N^{\text{p}}, j \in J^{\text{pr}}_n \tag{31}$$

$$p^{\text{rru,req}}_{nt} \geq p^{\text{rru,min}}_{nt} \ \forall t \in T, n \in N^{\text{p}} \tag{32}$$

$$p_{nt}^{\text{rrd,req}} \geq p_{nt}^{\text{rrd,min}} \ \forall t \in T, n \in N^{\text{p}} \tag{33}$$

$$q_{nt}^{\text{qru,req}} \geq q_{nt}^{\text{qru,min}} \ \forall t \in T, n \in N^{\text{q}} \tag{34}$$

$$q_{nt}^{\text{qrd,req}} \geq q_{nt}^{\text{qrd,min}} \ \forall t \in T, n \in N^{\text{q}} \tag{35}$$

The requirements for ramping reserve and reactive power reserve up and down are modeled exogenously in Eqs. (32) to (35). The requirements for regulation up and down are modeled endogenously in Eqs. (28) and (29) as prescribed factors of the total real power over all consuming devices. The requirements for spinning and non-spinning reserves are modeled endogenously in Eqs. (30) and (31) as a prescribed factors of the largest real power over all producing devices.

2.8 Reserve balance

For each reserve product

$$\sum_{j \in J_n^{\text{ds}}} p_{jt}^{\text{rgu}} + p_{nt}^{\text{rgu},+} \geq p_{nt}^{\text{rgu,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{36}$$

$$\sum_{j \in J_n^{\text{ds}}} p_{jt}^{\text{rgd}} + p_{nt}^{\text{rgd},+} \geq p_{nt}^{\text{rgd,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{37}$$

$$\sum_{j \in J_n^{\text{ds}}} (p_{jt}^{\text{rgu}} + p_{jt}^{\text{spr}}) + p_{nt}^{\text{spr},+} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{spr,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{38}$$

$$\sum_{j \in J_n^{\text{ds}}} (p_{jt}^{\text{rgu}} + p_{jt}^{\text{spr}} + p_{jt}^{\text{nsp}}) + p_{nt}^{\text{nsp},+} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{spr,req}} + p_{nt}^{\text{nsp,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{39}$$

$$\sum_{j \in J_n^{\text{ds}}} (p_{jt}^{\text{rru,on}} + p_{jt}^{\text{rru,off}}) + p_{nt}^{\text{rru},+} \geq p_{nt}^{\text{rru,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{40}$$

$$\sum_{j \in J_n^{\text{ds}}} (p_{jt}^{\text{rrd,on}} + p_{jt}^{\text{rrd,off}}) + p_{nt}^{\text{rrd},+} \geq p_{nt}^{\text{rrd,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{41}$$

$$\sum_{j \in J_n^{\text{ds}}} q_{jt}^{\text{qru}} + p_{nt}^{\text{qru},+} \geq q_{nt}^{\text{qru,req}} \ \forall t \in T, n \in N^{\text{q}} \tag{42}$$

$$\sum_{j \in J_n^{\text{ds}}} q_{jt}^{\text{qrd}} + p_{nt}^{\text{qrd},+} \geq q_{nt}^{\text{qrd,req}} \ \forall t \in T, n \in N^{\text{q}} \tag{43}$$

2.9 Voltage limits

Voltage magnitude limits at each bus in each interval are modeled by hard constraints:

$$v_i^{\text{min}} \leq v_{it} \leq v_i^{\text{max}} \ \forall t \in T, in \in I \tag{44}$$

2.10 Shunts

Shunt real and reactive power are:

$$p_{jt} = g^{\text{sh}}_{jt} v^2_{it} \; \forall t \in T, j \in J^{\text{sh}}, i = i_j \tag{45}$$

$$q_{jt} = -b^{\text{sh}}_{jt} v^2_{it} \; \forall t \in T, j \in J^{\text{sh}}, i = i_j \tag{46}$$

Shunt conductance and susceptance are:

$$g^{\text{sh}}_{jt} = g^{\text{sh}}_{j} u^{\text{sh}}_{jt} \; \forall t \in T, j \in J^{\text{sh}} \tag{47}$$

$$b^{\text{sh}}_{jt} = b^{\text{sh}}_{j} u^{\text{sh}}_{jt} \; \forall t \in T, j \in J^{\text{sh}} \tag{48}$$

Shunt steps integral

$$u^{\text{sh}}_{jt} \in \{\dots, -1, 0, 1, \dots\} \; \forall t \in T, j \in J^{\text{sh}} \tag{49}$$

Shunt steps bound

$$u^{\text{sh,min}}_{j} u^{\text{on}}_{jt} \le u^{\text{sh}}_{jt} \le u^{\text{sh,max}}_{j} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{sh}} \tag{50}$$

2.11 AC branch controls

Bounds on the AC branch controls of phase difference and winding ratio are:

$$\phi^{\text{min}}_{j} \le \phi_{jt} \le \phi^{\text{max}}_{j} \; \forall t \in T, j \in J^{\text{ac}} \tag{51}$$

$$\tau^{\text{min}}_{j} \le \tau_{jt} \le \tau^{\text{max}}_{j} \; \forall t \in T, j \in J^{\text{ac}} \tag{52}$$

2.12 DC lines

Bounds on DC line real and reactive power absorption at from and to buses are:

$$-p^{\text{dc,max}}_{j} u^{\text{on}}_{jt} \le p^{\text{fr}}_{jt} \le p^{\text{dc,max}}_{j} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{dc}} \tag{53}$$

$$-p^{\text{dc,max}}_{j} u^{\text{on}}_{jt} \le p^{\text{to}}_{jt} \le p^{\text{dc,max}}_{j} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{dc}} \tag{54}$$

$$q^{\text{dc,min,fr}}_{j} u^{\text{on}}_{jt} \le q^{\text{fr}}_{jt} \le q^{\text{dc,max,fr}}_{j} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{dc}} \tag{55}$$

$$q^{\text{dc,min,to}}_{j} u^{\text{on}}_{jt} \le q^{\text{to}}_{jt} \le q^{\text{dc,max,to}}_{j} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{dc}} \tag{56}$$

DC line real power losses are equal to 0:

$$p^{\text{fr}}_{jt} + p^{\text{to}}_{jt} = 0 \; \forall t \in T, j \in J^{\text{dc}} \tag{57}$$

2.13 AC branch flows

AC branch real and reactive power flows at the from and to buses, directed from the bus into the device, are:

$$\begin{aligned}
&p^{\mathrm{fr}}_{jt} = u^{\mathrm{on}}_{jt}((g^{\mathrm{sr}}_j + g^{\mathrm{fr}}_j)v^2_{it}/\tau^2_{jt} + (-g^{\mathrm{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad - b^{\mathrm{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \; \forall t \in T, j \in J^{\mathrm{ac}}, i = i^{\mathrm{fr}}_j, i' = i^{\mathrm{to}}_j \qquad (58)
\end{aligned}$$

$$\begin{aligned}
&q^{\mathrm{fr}}_{jt} = u^{\mathrm{on}}_{jt}((-b^{\mathrm{sr}}_j - b^{\mathrm{fr}}_j - b^{\mathrm{ch}}_j/2)v^2_{it}/\tau^2_{jt} + (b^{\mathrm{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad - g^{\mathrm{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \; \forall t \in T, j \in J^{\mathrm{ac}}, i = i^{\mathrm{fr}}_j, i' = i^{\mathrm{to}}_j \qquad (59)
\end{aligned}$$

$$\begin{aligned}
&p^{\mathrm{to}}_{jt} = u^{\mathrm{on}}_{jt}((g^{\mathrm{sr}}_j + g^{\mathrm{to}}_j)v^2_{i't} + (-g^{\mathrm{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad + b^{\mathrm{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \; \forall t \in T, j \in J^{\mathrm{ac}}, i = i^{\mathrm{fr}}_j, i' = i^{\mathrm{to}}_j \qquad (60)
\end{aligned}$$

$$\begin{aligned}
&q^{\mathrm{to}}_{jt} = u^{\mathrm{on}}_{jt}((-b^{\mathrm{sr}}_j - b^{\mathrm{to}}_j - b^{\mathrm{ch}}_j/2)v^2_{i't} + (b^{\mathrm{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad + g^{\mathrm{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \; \forall t \in T, j \in J^{\mathrm{ac}}, i = i^{\mathrm{fr}}_j, i' = i^{\mathrm{to}}_j \qquad (61)
\end{aligned}$$

2.14 Branch overload penalties

Branch overload is modeled by a nonnegative variable s^+_{jt}, which results in a penalty z^{s}_{jt} appearing in the objective:

$$0 \le s^+_{jt} \; \forall t \in T, j \in J^{\mathrm{br}} \qquad (62)$$

$$z^{\mathrm{s}}_{jt} = d_t c^{\mathrm{s}} s^+_{jt} \; \forall t \in T, j \in J^{\mathrm{br}} \qquad (63)$$

2.15 Branch flow limits

Branch flow limits at from and to buses are enforced by soft constraints:

$$((p^{\mathrm{fr}}_{jt})^2 + (q^{\mathrm{fr}}_{jt})^2)^{1/2} \le s^{\max}_j + s^+_{jt} \; \forall t \in T, j \in J^{\mathrm{br}} \qquad (64)$$

$$((p^{\mathrm{to}}_{jt})^2 + (q^{\mathrm{to}}_{jt})^2)^{1/2} \le s^{\max}_j + s^+_{jt} \; \forall t \in T, j \in J^{\mathrm{br}} \qquad (65)$$

2.16 Network connectivity requirements

In each time interval, in the base case and in each contingency, the graph consisting of all buses and in service online branches must be connected:

$$\text{The bus-branch graph on } (I, \{j \in J^{\mathrm{br}}_t : u^{\mathrm{on}}_{jt} = 1\}) \text{ is connected } \forall t \in T \qquad (66)$$

$$\text{The bus-branch graph on } (I, \{j \in J^{\mathrm{br}}_{tk} : u^{\mathrm{on}}_{jt} = 1\}) \text{ is connected } \forall t \in T, k \in K \qquad (67)$$

2.17 Post-contingency power flow, balance, limits

Penalty on post-contingency branch overload Post contingency branch overload is modeled by a nonnegative variable s^+_{jtk}, which results in a penalty z^{s}_{jtk} appearing in the objective:

$$s^+_{jtk} \ge 0 \; \forall t \in T, k \in K, j \in J^{\mathrm{br}}_{tk} \qquad (68)$$

$$z^{\mathrm{s}}_{jtk} = d_t c^{\mathrm{s}} s^+_{jtk} \; \forall t \in T, k \in K, j \in J^{\mathrm{br}}_{tk} \qquad (69)$$

Post-contingency power flow limits. We model the real power delta from the base case solution and neglect the reactive power delta. Post-contingency branch apparent power flow limits are formulated as soft second order cone constraints:

$$((p^{\text{fr}}_{jt} + p^{\delta}_{jtk})^2 + (q^{\text{fr}}_{jt})^2)^{1/2} \leq s^{\text{max,ctg}}_{j} + s^{+}_{jtk} \; \forall t \in T, k \in K, j \in J^{\text{br}}_{tk} \tag{70}$$

$$((p^{\text{to}}_{jt} - p^{\delta}_{jtk})^2 + (q^{\text{to}}_{jt})^2)^{1/2} \leq s^{\text{max,ctg}}_{j} + s^{+}_{jtk} \; \forall t \in T, k \in K, j \in J^{\text{br}}_{tk} \tag{71}$$

Post-contingency real power flow delta DC lines remaining in service in a contingency are assumed to have no change in power flow relative to the base case:

$$p^{\delta}_{jtk} = 0 \; \forall t \in T, k \in K, j \in J^{\text{dc}}_{tk} \tag{72}$$

For AC branches, the post-contingency real power flow delta from the base case is related to the bus angle delta by a DC power flow model:

$$p^{\delta}_{jtk} = -u^{\text{on}}_{jt} b^{\text{sr}}_{j} (\theta^{\delta}_{itk} - \theta^{\delta}_{i'tk}) \; \forall t \in T, k \in K, j \in J^{\text{ac}}_{tk}, i = i^{\text{fr}}_{j}, i' = i^{\text{to}}_{j} \tag{73}$$

Balance of post-contingency real power delta Real power delta on AC branches remaining in service in a contingency redistributes lost base case power flow on branches going out of service, with a distributed slack accounting for pre-contingency losses on outaged branches:

$$\begin{aligned} &\sum_{j \in J^{\text{out}}_{tk} \cap J^{\text{fr}}_{i}} p^{\text{fr}}_{jt} + \sum_{j \in J^{\text{out}}_{tk} \cap J^{\text{to}}_{i}} p^{\text{to}}_{jt} - \alpha_i \sum_{j \in J^{\text{out}}_{tk}} (p^{\text{fr}}_{jt} + p^{\text{to}}_{jt}) \\ &\quad = \sum_{j \in J^{\text{ac}}_{tk} \cap J^{\text{fr}}_{i}} p^{\delta}_{jtk} - \sum_{j \in J^{\text{ac}}_{tk} \cap J^{\text{to}}_{i}} p^{\delta}_{jtk} \; \forall t \in T, k \in K, i \in I \end{aligned} \tag{74}$$

2.18 On-off status and transitions

On indicator binary

$$u^{\text{on}}_{jt} \in \{0, 1\} \; \forall t \in T, j \in J \tag{75}$$

Startup binary

$$u^{\text{su}}_{jt} \in \{0, 1\} \; \forall t \in T, j \in J \tag{76}$$

Shutdown binary

$$u^{\text{sd}}_{jt} \in \{0, 1\} \; \forall t \in T, j \in J \tag{77}$$

Devices are offline when outaged in a given time interval:

$$u^{\text{on}}_{jt} = 0 \; \forall t \in T, j \in J^{\text{out}}_{t} \tag{78}$$

Bounds on on-off status (must run, no run, maintenance, etc)

$$u^{\text{on,min}}_{jt} \leq u^{\text{on}}_{jt} \leq u^{\text{on,max}}_{jt} \; \forall t \in T, j \in J \tag{79}$$

On/off evolution, first time interval.

$$u^{\text{on}}_{jt} - u^{\text{on},0}_{j} = u^{\text{su}}_{jt} - u^{\text{sd}}_{jt} \; \forall t = t^{\min}, j \in J \tag{80}$$

On/off evolution, after first time interval.

$$u^{\text{on}}_{jt} - u^{\text{on}}_{j,t-1} = u^{\text{su}}_{jt} - u^{\text{sd}}_{jt} \; \forall t > t^{\min}, j \in J \tag{81}$$

2.19 On-off status and transition costs

On-off status, startup, and shutdown indicators result in costs that appear in the objective:

$$z^{\text{on}}_{jt} = d_t c^{\text{on}}_{jt} u^{\text{on}}_{jt} \; \forall t \in T, j \in J \tag{82}$$
$$z^{\text{su}}_{jt} = c^{\text{su}}_{jt} u^{\text{su}}_{jt} \; \forall t \in T, j \in J \tag{83}$$
$$z^{\text{sd}}_{jt} = c^{\text{sd}}_{jt} u^{\text{sd}}_{jt} \; \forall t \in T, j \in J \tag{84}$$

2.20 Energy costs and benefits of dispatchable devices

Dispatchable device real power p_{jt} results in an objective term z^{en}_{jt} that is either a cost (for producing devices) or a benefit (for consuming devices). The value of this energy cost (or benefit) term is determined by a piecewise linear convex (or concave) cost (or benefit) function, evaluated at $p_{jt} - p_{jt}$. To model this relationship, we split $p_{jt} - p_{jt}$ into offer (or bid) blocks and apply an objective coefficient to each block:

$$0 \le p_{jtm} \le p^{\text{en,max}}_{jtm} \; \forall t \in T, j \in J^{\text{ds}} \setminus J^{\text{md}}, m \in M_{jt} \tag{85}$$
$$p_{jt} = p^{\min}_{jt} u^{\text{on}}_{jt} + \sum_{m \in M_{jt}} p_{jtm} \; \forall t \in T, j \in J^{\text{ds}} \setminus J^{\text{md}} \tag{86}$$
$$z^{\text{en}}_{jt} = d_t \sum_{m \in M_{jt}} c^{\text{en}}_{jtm} p_{jtm} \; \forall t \in T, j \in J^{\text{ds}} \setminus J^{\text{md}} \tag{87}$$

Note that Eqs. (85) to (87) apply only to devices that are not multi-mode.

2.21 Producing and consuming device power bounds

Dispatchable device real and reactive power maximum and minimum values depend on on-off status:

$$p^{\min}_{jt} u^{\text{on}}_{jt} \le p_{jt} \le p^{\max}_{jt} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{ds}} \tag{88}$$
$$q^{\min}_{jt} u^{\text{on}}_{jt} \le q_{jt} \le q^{\max}_{jt} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{ds}} \tag{89}$$

2.22 linear constraints linking device real and reactive power

Some dispatchable devices have linear constraints linking real and reactive power, enforcing either a constant marginal power factor (in Eq. (90)) or a trapezoidal approximation of a reactive power capability curve (in Eqs. (91) and (92)):

$$q_{jt} = q^{\text{p0}}_{jt} + \beta_{jt} p_{jt} \ \forall t \in T, j \in J^{\text{pqe}} \tag{90}$$

$$q_{jt} \le q^{\text{max,p0}}_{jt} + \beta^{\text{max}}_{jt} p_{jt} \ \forall t \in T, j \in J^{\text{pqmax}} \tag{91}$$

$$q_{jt} \ge q^{\text{min,p0}}_{jt} + \beta^{\text{min}}_{jt} p_{jt} \ \forall t \in T, j \in J^{\text{pqmin}} \tag{92}$$

2.23 Ramping limits

Real power ramping limits depend on on-off status and startup:

$$p_{jt} - p^0_j \le d_t (p^{\text{ru,max}}_{jt} (u^{\text{on}}_{jt} - u^{\text{su}}_{jt}) + p^{\text{ru,max,su}}_{jt} u^{\text{su}}_{jt}) \ \forall t = t^{\text{min}}, j \in J^{\text{ds}} \tag{93}$$

$$p_{jt} - p_{j,t-1} \le d_t (p^{\text{ru,max}}_{jt} (u^{\text{on}}_{jt} - u^{\text{su}}_{jt}) + p^{\text{ru,max,su}}_{jt} u^{\text{su}}_{jt}) \ \forall t > t^{\text{min}}, j \in J^{\text{ds}} \tag{94}$$

$$p_{jt} - p^0_j \ge -d_t (p^{\text{rd,max}}_{jt} (1 - u^{\text{sd}}_{jt}) + p^{\text{rd,max,sd}}_{jt} u^{\text{sd}}_{jt}) \ \forall t = t^{\text{min}}, j \in J^{\text{ds}} \tag{95}$$

$$p_{jt} - p_{j,t-1} \ge -d_t (p^{\text{rd,max}}_{jt} (1 - u^{\text{sd}}_{jt}) + p^{\text{rd,max,sd}}_{jt} u^{\text{sd}}_{jt}) \ \forall t > t^{\text{min}}, j \in J^{\text{ds}} \tag{96}$$

2.24 Limits on up/down time

Minimum downtime. If startup then no recent shutdown.

$$u^{\text{su}}_{jt} \le 1 - \sum_{t^{\text{min,sd,su}}_{jt} \le t' < t} u^{\text{sd}}_{jt'} \ \forall t \in T, t \ge t^{\text{min,dn}}_j, j \in J \tag{97}$$

Minimum uptime. If shutdown then no recent startup.

$$u^{\text{sd}}_{jt} \le 1 - \sum_{t^{\text{min,su,sd}}_{jt} \le t' < t} u^{\text{su}}_{jt'} \ \forall t \in T, t \ge t^{\text{min,up}}_j, j \in J \tag{98}$$

2.25 Maximum starts over multiple intervals

$$\sum_{t^{\text{min}}_{jw} \le t \le t^{\text{max}}_{jw}} u^{\text{su}}_{jt} \le u^{\text{su,max}}_{jw} \ \forall j \in J, w \in W^{\text{su,max}}_j \tag{99}$$

2.26 Maximum/minimum energy over multiple intervals

$$\sum_{t^{\text{min}}_{jw} \le t \le t^{\text{max}}_{jw}} d_t p_{jt} \le e^{\text{max}}_{jw} \ \forall j \in J, w \in W^{\text{en,max}}_j \tag{100}$$

$$\sum_{t^{\text{min}}_{jw} \le t \le t^{\text{max}}_{jw}} d_t p_{jt} \ge e^{\text{min}}_{jw} \ \forall j \in J, w \in W^{\text{en,min}}_j \tag{101}$$

2.27 Mode selection and transition

Device mode selection and transition are indicated by binary variables u^{md}_{jyt} and $u^{\text{tr}}_{jyy't}$:

$$u^{\text{md}}_{jyt} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{md}}, y \in Y_j \tag{102}$$

$$u^{\text{tr}}_{jyy't} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{md}}, y \in Y^{\text{tr}}_j \tag{103}$$

2.28 Mode selection uniqueness

Exactly one mode is selected for each device and each time interval:

$$\sum_{y \in Y_j} u^{\mathrm{md}}_{jyt} = 1 \ \forall t \in T, j \in J^{\mathrm{md}} \tag{104}$$

2.29 Mode transition network flow model

Consistency between mode and transitions and selections in successive time intervals is ensured by network flow constraints:

$$u^{\mathrm{md}}_{jyt} = \sum_{(y'y) \in Y^{\mathrm{tr}}_j} u^{\mathrm{tr}}_{jy'yt} \ \forall t \in T, j \in J^{\mathrm{md}}, y \in Y_j \tag{105}$$

$$u^{\mathrm{md},0}_{jy} = \sum_{(yy') \in Y^{\mathrm{tr}}_j} u^{\mathrm{tr}}_{jyy't} \ \forall t = t^{\mathrm{min}}, j \in J^{\mathrm{md}}, y \in Y_j \tag{106}$$

$$u^{\mathrm{md}}_{j,y,t-1} = \sum_{(yy') \in Y^{\mathrm{tr}}_j} u^{\mathrm{tr}}_{jyy't} \ \forall t > t^{\mathrm{min}}, j \in J^{\mathrm{md}}, y \in Y_j \tag{107}$$

Eq. (105) ensures that each mode is entered by a transition from some previous mode, while Eqs. (106) and (107) ensure that each mode is exited by a transition to some next mode.

2.30 Mode dwell time limits

Maximum dwell time If a unit is in a given mode, then it must have not been in that mode in at least one interval in a prescribed prior duration.

$$u^{\mathrm{md}}_{jyt} \le \sum_{t^{\mathrm{min,md,off}}_{jyt} \le t' < t} (1 - u^{\mathrm{md}}_{jyt'}) \ \forall j \in J^{\mathrm{md}}, y \in Y_j, t \ge t^{\mathrm{min,md,off}}_{jy} \tag{108}$$

Minimum dwell time If a unit is in a transition from one mode to a different mode then it must have been in the former mode in all intervals in a prescribed prior duration.

$$(t - t^{\mathrm{min,md,on}}_{jyt}) \sum_{y' \ne y : (yy') \in Y^{\mathrm{tr}}_j} u^{\mathrm{tr}}_{jyy't} \le \sum_{t^{\mathrm{min,md,on}}_{jyt} \le t' < t} u^{\mathrm{md}}_{jyt'}$$
$$\forall j \in J^{\mathrm{md}}, y \in Y_j, t \ge t^{\mathrm{min,md,on}}_{jy} \tag{109}$$

2.31 Mode sub-device on-off status requirements

Any sub-device of a multi-mode main device is online if and only if the main device is in one of the modes in which the sub-device is specified to be on:

$$u^{\mathrm{on}}_{j't} = \sum_{y \in Y_j : j' \in J^{\mathrm{sub,on}}_{jy}} u^{\mathrm{md}}_{jyt} \ \forall t \in T, j \in J^{\mathrm{md}}, j' \in J^{\mathrm{sub}}_j \tag{110}$$

2.32 Mode/transition discrete costs

Mode selection and transition indicators carry costs that appear in the objective:

$$z_{jt}^{\mathrm{md}} = d_t \sum_{y \in Y_j} c_{jyt}^{\mathrm{md}} u_{jyt}^{\mathrm{md}} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{111}$$

$$z_{jt}^{\mathrm{tr}} = \sum_{(yy') \in Y_j^{\mathrm{tr}}} c_{jyy't}^{\mathrm{tr}} u_{jyy't}^{\mathrm{tr}} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{112}$$

2.33 Multi-mode device real and reactive power limits

Multi-mode device real and reactive power upper and lower bounds depend on the selected mode:

$$\sum_{y \in Y_j} p_{jyt}^{\min} u_{jyt}^{\mathrm{md}} \le p_{jt} \le \sum_{y \in Y_j} p_{jyt}^{\max} u_{jyt}^{\mathrm{md}} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{113}$$

$$\sum_{y \in Y_j} q_{jyt}^{\min} u_{jyt}^{\mathrm{md}} \le q_{jt} \le \sum_{y \in Y_j} q_{jyt}^{\max} u_{jyt}^{\mathrm{md}} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{114}$$

2.34 Multi-mode device real power ramping limits

Multi-mode device real power ramping limits depend on the selected mode:

$$p_{jt} - p_j^0 \le d_t \sum_{y \in Y_j} p_{jyt}^{\mathrm{ru,max}} u_{jyt}^{\mathrm{md}} \ \forall t = t^{\min}, j \in J^{\mathrm{md}} \tag{115}$$

$$p_{jt} - p_{j,t-1} \le d_t \sum_{y \in Y_j} p_{jyt}^{\mathrm{ru,max}} u_{jyt}^{\mathrm{md}} \ \forall t > t^{\min}, j \in J^{\mathrm{md}} \tag{116}$$

$$p_{jt} - p_j^0 \ge -d_t \sum_{y \in Y_j} p_{jyt}^{\mathrm{rd,max}} u_{jyt}^{\mathrm{md}} \ \forall t = t^{\min}, j \in J^{\mathrm{md}} \tag{117}$$

$$p_{jt} - p_{j,t-1} \ge -d_t \sum_{y \in Y_j} p_{jyt}^{\mathrm{rd,max}} u_{jyt}^{\mathrm{md}} \ \forall t > t^{\min}, j \in J^{\mathrm{md}} \tag{118}$$

2.35 Multi-mode device energy cost/benefit

Multi-mode device energy cost and benefit values are modeled in a similar fashion to non-multi-mode dispatchable devices, but with a separate cost or benefit function for each mode:

$$0 \le p_{jytm} \le p_{jytm}^{\mathrm{en,max}} u_{jyt}^{\mathrm{md}} \ \forall t \in T, j \in J^{\mathrm{md}}, y \in Y_j, m \in M_{jyt}^{\mathrm{en}} \tag{119}$$

$$p_{jt} = \sum_{y \in Y_j} p_{jyt}^{\min} u_{jyt}^{\mathrm{md}} + \sum_{y \in Y_j, m \in M_{jyt}^{\mathrm{en}}} p_{jytm} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{120}$$

$$z_{jt}^{\mathrm{en}} = d_t \sum_{y \in Y_j, m \in M_{jyt}^{\mathrm{en}}} c_{jytm}^{\mathrm{en}} p_{jytm} \ \forall t \in T, j \in J^{\mathrm{md}} \tag{121}$$

2.36 Absolute reserve limits

Based on ramp rate, duration, and on-off status. At each time scale, all the reserve products consuming ramping capability up to that time are included.

$$p^{\text{rgu}}_{jt} \le p^{\text{rgu,max}}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{122}$$

$$p^{\text{rgd}}_{jt} \le p^{\text{rgd,max}}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{123}$$

$$p^{\text{rgu}}_{jt} + p^{\text{spr}}_{jt} \le p^{\text{spr,max}}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{124}$$

$$p^{\text{nsp}}_{jt} \le p^{\text{nsp,max}}_{jt} (1 - u^{\text{on}}_{jt}) \ \forall t \in T, j \in J^{\text{ds}} \tag{125}$$

$$p^{\text{rgu}}_{jt} + p^{\text{spr}}_{jt} + p^{\text{rru,on}}_{jt} \le p^{\text{rru,on,max}}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{126}$$

$$p^{\text{nsp}}_{jt} + p^{\text{rru,off}}_{jt} \le p^{\text{rru,off,max}}_{jt} (1 - u^{\text{on}}_{jt}) \ \forall t \in T, j \in J^{\text{ds}} \tag{127}$$

$$p^{\text{rgd}}_{jt} + p^{\text{rrd,on}}_{jt} \le p^{\text{rrd,on,max}}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{128}$$

$$p^{\text{rrd,off}}_{jt} \le p^{\text{rrd,off,max}}_{jt} (1 - u^{\text{on}}_{jt}) \ \forall t \in T, j \in J^{\text{ds}} \tag{129}$$

The absolute reserve limits might typically be determined as an applicable ramp rate times the ramping duration for the lowest quality product considered. In general these limits are supplied as part of the reserve offer of a device and might differ from this simple formula for various reasons. Therefore we model these as parameters provided by the data.

2.37 Relative reserve limits, producing devices

Based on p/q max/min, dispatch, and on-off status, for producing devices.

$$p_{jt} + p^{\text{rgu}}_{jt} + p^{\text{spr}}_{jt} + p^{\text{rru,on}}_{jt} \le p^{\max}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{pr}} \tag{130}$$

$$p_{jt} - p^{\text{rgd}}_{jt} - p^{\text{rrd,on}}_{jt} \ge p^{\min}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{pr}} \tag{131}$$

$$q_{jt} + q^{\text{qru}}_{jt} \le q^{\max}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{pr}} \tag{132}$$

$$q_{jt} - q^{\text{qrd}}_{jt} \ge q^{\min}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{pr}} \tag{133}$$

$$p^{\text{nsp}}_{jt} + p^{\text{rru,off}}_{jt} \le p^{\max}_{jt} (1 - u^{\text{on}}_{jt}) \ \forall t \in T, j \in J^{\text{pr}} \tag{134}$$

$$p^{\text{rrd,off}}_{jt} = 0 \ \forall t \in T, j \in J^{\text{pr}} \tag{135}$$

2.38 Relative reserve limits, consuming devices

Based on p/q max/min, dispatch, and on-off status, for consuming devices.

$$p_{jt} + p^{\text{rgd}}_{jt} + p^{\text{rrd,on}}_{jt} \le p^{\max}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{cs}} \tag{136}$$

$$p_{jt} - p^{\text{rgu}}_{jt} - p^{\text{spr}}_{jt} - p^{\text{rru,on}}_{jt} \ge p^{\min}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{cs}} \tag{137}$$

$$q_{jt} + q^{\text{qrd}}_{jt} \le q^{\max}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{cs}} \tag{138}$$

$$q_{jt} - q^{\text{qru}}_{jt} \ge q^{\min}_{jt} u^{\text{on}}_{jt} \ \forall t \in T, j \in J^{\text{cs}} \tag{139}$$

$$p^{\text{rrd,off}}_{jt} \le p^{\max}_{jt} (1 - u^{\text{on}}_{jt}) \ \forall t \in T, j \in J^{\text{cs}} \tag{140}$$

$$p^{\text{nsp}}_{jt} + p^{\text{rru,off}}_{jt} = 0 \ \forall t \in T, j \in J^{\text{cs}} \tag{141}$$

2.39 Device reserve variable domains

Device reserve variables are nonnegative

$$p^{\text{rgu}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{142}$$
$$p^{\text{rgd}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{143}$$
$$p^{\text{spr}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{144}$$
$$p^{\text{nsp}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{145}$$
$$p^{\text{rru,on}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{146}$$
$$p^{\text{rru,off}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{147}$$
$$p^{\text{rrd,on}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{148}$$
$$p^{\text{rrd,off}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{149}$$
$$q^{\text{qru}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{150}$$
$$q^{\text{qrd}}_{jt} \geq 0 \ \forall t \in T, j \in J^{\text{ds}} \tag{151}$$

2.40 Device reserve costs

Devices may include a cost in their reserve offers. This is modeled as a cost coefficient on each device reserve provision variable.

$$z^{\text{rgu}}_{jt} = d_t c^{\text{rgu}}_{jt} p^{\text{rgu}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{152}$$
$$z^{\text{rgd}}_{jt} = d_t c^{\text{rgd}}_{jt} p^{\text{rgd}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{153}$$
$$z^{\text{spr}}_{jt} = d_t c^{\text{spr}}_{jt} p^{\text{spr}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{154}$$
$$z^{\text{nsp}}_{jt} = d_t c^{\text{nsp}}_{jt} p^{\text{nsp}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{155}$$
$$z^{\text{rru}}_{jt} = d_t (c^{\text{rru,on}}_{jt} p^{\text{rru,on}}_{jt} + c^{\text{rru,off}}_{jt} p^{\text{rru,off}}_{jt}) \ \forall t \in T, j \in J^{\text{ds}} \tag{156}$$
$$z^{\text{rrd}}_{jt} = d_t (c^{\text{rrd,on}}_{jt} p^{\text{rrd,on}}_{jt} + c^{\text{rrd,off}}_{jt} p^{\text{rrd,off}}_{jt}) \ \forall t \in T, j \in J^{\text{ds}} \tag{157}$$
$$z^{\text{qru}}_{jt} = d_t c^{\text{qru}}_{jt} p^{\text{rgu}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{158}$$
$$z^{\text{qrd}}_{jt} = d_t c^{\text{qrd}}_{jt} p^{\text{rgu}}_{jt} \ \forall t \in T, j \in J^{\text{ds}} \tag{159}$$

2.41 Absolute reserve limits, multi-mode devices

Reserve provision by online multi-mode devices is subject to an absolute upper bound that depends on the selected mode. Offline reserve provision does not need further bounds.

$$p^{\text{rgu}}_{jt} \leq \sum_{y \in Y_j} p^{\text{rgu,max}}_{jyt} u^{\text{md}}_{jyt} \ \forall t \in T, j \in J^{\text{md}} \tag{160}$$
$$p^{\text{rgd}}_{jt} \leq \sum_{y \in Y_j} p^{\text{rgd,max}}_{jyt} u^{\text{md}}_{jyt} \ \forall t \in T, j \in J^{\text{md}} \tag{161}$$
$$p^{\text{rgu}}_{jt} + p^{\text{spr}}_{jt} \leq \sum_{y \in Y_j} p^{\text{spr,max}}_{jyt} u^{\text{md}}_{jyt} \ \forall t \in T, j \in J^{\text{md}} \tag{162}$$

$$p_{jt}^{\text{rgu}} + p_{jt}^{\text{spr}} + p_{jt}^{\text{rru,on}} \leq \sum_{y \in Y_j} p_{jyt}^{\text{rru,on,max}} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{md}} \tag{163}$$

$$p_{jt}^{\text{rgd}} + p_{jt}^{\text{rrd,on}} \leq \sum_{y \in Y_j} p_{jyt}^{\text{rrd,on,max}} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{md}} \tag{164}$$

2.42 Relative reserve limits, multi-mode producing devices

Bounds on real and reactive power output imposing relative reserve limits on online producing devices depend on the selected mode for multi-mode devices. No further bounds are needed for offline reserves.

$$p_{jt} + p_{jt}^{\text{rgu}} + p_{jt}^{\text{spr}} + p_{jt}^{\text{rru,on}} \leq \sum_{y \in Y_j} p_{jyt}^{\max} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{md}} \tag{165}$$

$$p_{jt} - p_{jt}^{\text{rgd}} - p_{jt}^{\text{rrd,on}} \geq \sum_{y \in Y_j} p_{jyt}^{\min} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{md}} \tag{166}$$

$$q_{jt} + q_{jt}^{\text{qru}} \leq \sum_{y \in Y_j} q_{jyt}^{\max} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{md}} \tag{167}$$

$$q_{jt} - q_{jt}^{\text{qrd}} \geq \sum_{y \in Y_j} q_{jyt}^{\min} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{md}} \tag{168}$$

2.43 Relative reserve limits, multi-mode consuming devices

Bounds on real and reactive power output imposing relative reserve limits on online consuming devices depend on the selected mode for multi-mode devices. No further bounds are needed for offline reserves.

$$p_{jt} + p_{jt}^{\text{rgd}} + p_{jt}^{\text{rrd,on}} \leq \sum_{y \in Y_j} p_{jyt}^{\max} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{md}} \tag{169}$$

$$p_{jt} - p_{jt}^{\text{rgu}} - p_{jt}^{\text{spr}} - p_{jt}^{\text{rru,on}} \geq \sum_{y \in Y_j} p_{jyt}^{\min} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{md}} \tag{170}$$

$$q_{jt} + q_{jt}^{\text{qrd}} \leq \sum_{y \in Y_j} q_{jyt}^{\max} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{md}} \tag{171}$$

$$q_{jt} - q_{jt}^{\text{qru}} \geq \sum_{y \in Y_j} q_{jyt}^{\min} u_{jyt}^{\text{md}} \; \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{md}} \tag{172}$$